UNIT 4 TEST REVIEW

Key

Functions & Limits Unit

Set 1: Give the domain, range, x and y intercepts of the following functions.

1. $f(x)=\dfrac{5x^2-30x}{10x}$

$y=\dfrac{\cancel{5x}(x-6)}{\cancel{5x}(2)}$ Hole: (0,-3)

No VA, No HA

D: $(-\infty,0)\cup(0,\infty)$
R: $(-\infty,-3)\cup(-3,\infty)$
x-int: (6,0) y-int: none

2. $g(x)=\dfrac{x-3}{(\sqrt{x}-1)}$

$x\neq 1$

VA: x = 1
HA: y = 0

x-int: (3,0)
y-int: none
D: $(1,\infty)$
R: $(-\infty,\infty)$

* Domain CANNOT be $[1,\infty)$ because x=1 gives ÷ by 0

Set 2: Determine the type of discontinuities in the functions and state them. Then list any horizontal asymptotes.

3. $f(x)=\dfrac{x^2-16}{x^3-64}$

Bottom degree > Top degree → y = 0 HA

$\dfrac{\cancel{(x-4)}(x+4)}{\cancel{(x-4)}(x^2+4x+16)}$ $y=\dfrac{4+4}{48}$

Hole: (4, 1/6)
(Removable Disc.)

HA: y=0 (NonRemovable, Infinite Disc.)

4. $g(x)=\dfrac{x-3}{2x^2+x-21}=\dfrac{\cancel{x-3}}{(2x+7)\cancel{(x-3)}}$

Hole: (3, 1/13)
(Removable Disc.)

$y=\dfrac{1}{2\cdot3+7}$

VA: x = -7/2 or x = -3.5
(NonRemovable, Infinite Disc.)

HA: y = 0
Bottom degree > Top degree

Set 3:

5. Using the following function, $f(x)=\dfrac{x^2-3x-18}{x^2+x-42}$, $\dfrac{(x+3)\cancel{(x-6)}}{(x+7)\cancel{(x-6)}}$
list the domain, range, all discontinuities, and x and y intercepts.

VA: x = -7
Hole: (6, 9/13)

$y=\dfrac{6+3}{6+7}$

D: $(-\infty,-7)\cup(-7,6)\cup(6,\infty)$
R: $(-\infty,9/13)\cup(9/13,1)\cup(1,\infty)$

HA: y = 1
x-int: (-3,0)
y-int: (0, 3/7)

6. For the above function, find the limits:

$\lim_{x\to\infty}=1$ (look at HA)

$\lim_{x\to-\infty}=1$ (look at HA)

$\lim_{x\to6}=9/13$

x-value of hole ↑ y-value of hole ↑

$\lim_{x\to-7}=$ DNE

Set 4:

7. Find the x and y intercepts of the function: $f(x)=\dfrac{3x-5}{2x+7}$

$0=\dfrac{3x-5}{2x+7}$ $0=3x-5$ x-int: (5/3, 0)

$y=\dfrac{3\cdot0-5}{2\cdot0+7}$ y-int: (0, -5/7)

8. For the following function f(x)=x^3+2x^2-7x+3: Determine all local maximums and minimums. Determine the increasing and decreasing intervals.

Local max: 17.519 at x = $-2.\overline{3}$

Local min: -1 at x = 1

Incr: $(-\infty,-2.\overline{3}]\cup[1,\infty)$
Decr: $[-2.\overline{3},1]$

9. Write a function that has a horizontal asymptote at y=⅔, an infinite discontinuity at 4 and a removable point of discontinuity at 7.

Example: $y=\dfrac{2x(x-7)}{3(x-7)(x-4)}$

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Set 5: Given $f(x) = 4x^2 - x + 3$ and $g(x) = \sqrt{x+1}$;

10. Find. $(g \circ f)(x)$ and state its domain in interval notation.

$g(4x^2-x+3) = \sqrt{(4x^2-x+3)+1} = \boxed{\sqrt{4x^2-x+4} = g(f(x))}$

$\boxed{D: (-\infty, \infty)}$

11. Find $(f(g(5))$.

$f(\sqrt{5+1}) = f(\sqrt{6}) = 4(\sqrt{6})^2 - \sqrt{6} + 3 = 24 + 3 - \sqrt{6} = \boxed{27 - \sqrt{6}}$

12. Find $g(x+1) - f(4)$.

$\sqrt{(x+1)+1} - (4(4)^2 - 4 + 3)$

$\sqrt{x+2} - (64-1)$ $\boxed{\sqrt{x+2} - 63}$

Set 6: State whether the function is odd, even, or neither. Support graphically and confirm algebraically.

13. $f(x) = \sqrt{x^3 + x - 3}$

$f(-x) = \sqrt{(-x)^3 + (-x) - 3}$

$f(-x) = \sqrt{-x^3 - x - 3}$

Not Even because $f(-x) \neq f(x)$

$-f(x) = -\sqrt{x^3 + x - 3}$ $\boxed{\text{Neither}}$

Not Odd because $f(-x) \neq -f(x)$

14. f(x) = $\dfrac{x^2 + x^3}{x^3}$

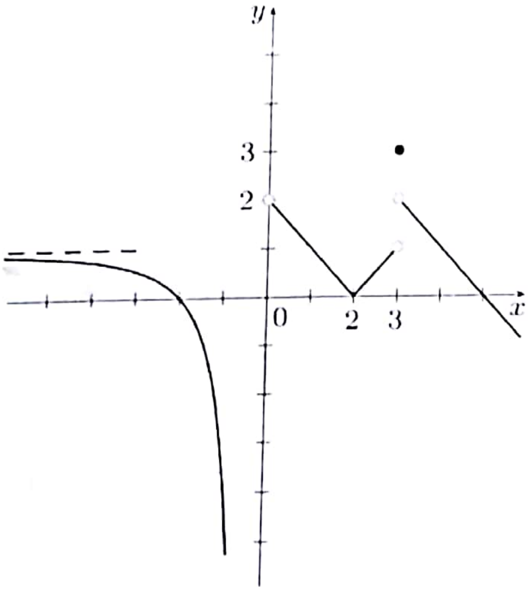
$f(-x) = \dfrac{(-x)^2 + (-x)^3}{(-x)^3}$ $-f(x) = \dfrac{-(x^2 + x^3)}{x^3}$

$f(-x) = \dfrac{x^2 - x^3}{-x^3} = \dfrac{-x^2 + x^3}{x^3}$ $\boxed{\text{Neither}}$ $-f(x) = \dfrac{-x^2 - x^3}{x^3}$

$f(-x) \neq f(x)$ so Not Even & $f(-x) \neq -f(x)$ So not odd

Set 7: Find the following limits based on the function below.

15.



(a) $f(0) =$ DNE

(b) $f(2) =$ 0 } Find y-value when x = 2

(c) $f(3) =$ 3 } Find y-value when x = 3

(d) $\lim_{x \to 0^-} f(x) = -\infty$

(e) $\lim_{x \to 0} f(x) =$ DNE

(f) $\lim_{x \to 3^+} f(x) =$ 2

(g) $\lim_{x \to 3} f(x) =$ DNE

(h) $\lim_{x \to -\infty} f(x) =$ 1

*Remember, for a limit you are finding the y-value that the function approaches as the x's change as indicated