Unit 6 Day 8:
2.3 Methods of Apportionment

Warm-Up Apportionment = Half Sheet Handout

Art, Bethany, Carl, and Denise are heirs to an estate consisting of four cars (a ’57 Chevy Impala, a ’54 Ford pickup, a ’62 Mustang convertible, and a ’60 VW bug) and $7500 in cash. Without collusion, each person submits sealed bids on the cars. For each heir, state his or hers fair share, the car or cars received, and the amount of cash paid or received.

	Art	Bethany	Carl	Denise
’57 Chevy Impala	3500	7000	6000	3000
’54 Ford pickup	2000	5500	4500	2500
’62 Mustang Convertible	5000	4000	4200	4500
’60 VW Bug	4000	5000	4800	5500

HW Questions?

Notes 2.3 Methods of Apportionment

Use Packet p. 10-13 for help completing your notes today ☺

Fair Division:

- When a fair division problem is strictly discrete, the problem can be impossible to solve in a way that treats all parties fairly.
  (An estate involves discrete objects, but it also involves cash.)

Apportionment:

- One of the most politically charged fair distribution problems in the U.S. involves the apportionment of seats in the U.S. House of Representatives.
  (The House is reapportioned every 10 years after a new census is taken.)
- Unlike estate division, the value of a seat in the House is not subjective. All seats must be distributed among the states according to population.
- The method by which these seats are apportioned can be controversial.

Apportionment:

- The first presidential veto occurred in 1792 by George Washington. After much debate, Congress approved a bill for a 120 member House and Hamilton's method to apportion the seats among the states. Hamilton's method won over Jefferson's method.
- The House, unable to override the veto, passed a bill for a new 105 member house and Jefferson's method to apportion the seats among the states (this method was used until 1840.)
- **Current US House of Reps: 435 seats, 50 states**

Example:

- A country has 6 states with populations:
  27,774; 25,178; 19,947; 14,614; 9,225; and 3,292.
- Its House of Representatives has 36 seats.
- Find the apportionment using the methods of Hamilton, Jefferson, Webster, and Hill.

- All methods obtain a standard divisor/ **ideal ratio**

$$s = \frac{\text{total population}}{\text{number of seats}}$$

\# of seats = __36__; total population = __100,030__; s = __2778.6__

Hint: Store into calc.

Ideal Ratio / Quotas:

- To obtain **quotas**, divide the population of each state by the ideal ratio (s).

$$\text{quota} = \frac{\text{population}}{\text{ideal ratio}}$$

State	Population	Quota
A	27,774	9.99564
B	25,178	9.06136
C	19,947	7.17876
D	14,614	5.25946
E	9,225	3.32000
F	3,292	1.18476

The Hamilton Method:

- Round each quota down to get a tentative apportionment. (i.e. truncate the decimal)
- Since the resulting house size is too small (by 2 seats) consider the 2 quotas with the largest decimal values. Increase their apportionments by 1.

total:36 seats

State	Quota	Tentative Apportionment	Final Apportionment
A	9.99564*	9	10
B	9.06136	9	9
C	7.17877	7	7
D	5.25946	5	5
E	3.32000*	3	4
F	1.18476	1	1

34 seats

Quota Condition:

- The Hamilton method always satisfies the quota condition. (i.e., each states apportionment is equal to either its lower quota or its upper quota.)

The Jefferson Method:

- The tentative apportionment is the same as the Hamilton method (found by dividing each states population by s and rounding down). Since the resulting house size is again too small, calculate the adjusted ratio for each state.

$$\text{Jefferson Adjusted Ratio} = \frac{\text{state size}}{\text{tentative apportionment} + 1}$$

- Give the state with the adjusted ratio closest to s (that is the state with the largest adjusted ratio) an additional seat.
- Recompute the state's adjusted ratio based on its new tentative apportionment. If more seats are to be given out, give the state with the largest adjusted ratio another seat. Continue in this manner until all seats are allocated

The Jefferson Method:

(tends to favor large states)

Remember s = 2778.611

State	Tentative Apportionment	Jefferson Adjusted Ratio	Next Tentative Apportionment	Final Apportionment
A	9	2777.4 *	10	11
B	9	2517.8		9
C	7	2493.38		7
D	5	2435.67		5
E	3	2306.25		3
F	1	1646.0		1
	34 seats		35 seats	36 seats

The Webster Method:

- To obtain the tentative apportionment, round each quota. (round up if the decimal is .5 or higher and round down if it is smaller than .5)
- When too few seats are given (as the case here), compute the adjusted ratio as follows:

$$\text{Webster Adjusted Ratio (for too few seats)} = \frac{\text{state size}}{\text{tentative apportionment} + 0.5}$$

- Choose the state with the largest adjusted ratio (the adjusted ratio closest to "s"). Increase that state's apportionment by 1.
- Recompute the state's adjusted ratio based on its new tentative apportionment. If more seats are to be added, compare the adjusted ratio to s as before. Continue until all seats have been allocated.

The Webster Method:

- When too many seats are given, compute the adjusted ratio as follows:

$$\text{Webster Adjusted Ratio (for too many seats)} = \frac{\text{state size}}{\text{tentative apportionment} - 0.5}$$

- Choose the state with the smallest adjusted ratio (the adjusted ratio closest to "s"). Decrease that state's apportionment by 1.
- Recompute the state's adjusted ratio based on its new tentative apportionment. If more seats are to be taken away, compare the adjusted ratio to s as before. Continue until the house size is reached.

Webster Method:

(favors neither large or small states)

State	Quota	Tentative Apportionment	Webster Adjusted Ratio	Final Apportionment
A	9.996	10	2645.1429	**10**
B	9.061	9	2650.3158	**9**
C	7.179	7	2659.60 *	**8**
D	5.259	5	2657.0909	**5**
E	3.320	3	2635.7143	**3**
F	1.185	1	2194.6667	**1**
		35 seats		36 seats

- The Webster method does not satisfy the Quota Condition

Hill-Huntington Method:

- For each quota, compute the geometric mean as follows:

  Geometric Mean = $\sqrt{lower\ \ quota \cdot upper\ \ quota}$

- To get the tentative apportionment, compare the quota to the geometric mean

  Round the geometric mean:

  UP if the quota is bigger

  DOWN if the quota is smaller

Hill-Huntington Method:

- When too few seats are given (as is the case here), compute the adjusted ratio as follows:

  H–H Adjusted Ratio = $\dfrac{\textit{state size}}{\sqrt{\textit{tentative apportionment}\left(\textit{tentative appportionment}+1\right)}}$

- Choose the state with the largest adjusted ratio and increase that states apportionment by 1 seat. Recompute the adjusted ratio and continue until the desired size is reached.

- When too many seats are given, compute the adjusted ratio as follows:

  H–H Adjusted Ratio = $\dfrac{\textit{state size}}{\sqrt{\textit{tentative apportionment}\left(\textit{tentative appportionment}-1\right)}}$

- Choose the state with the smallest adjusted ratio (closest to s) and decrease that states apportionment by 1 seat. Recompute the adjusted ratio and continue until the desired size is reached.

Hill-Huntington Method:

Remember: Round the geometric mean

UP if the quota is bigger

DOWN if the quota is smaller

State	Quota	Geometric Mean	Tentative Apportionment	Adjusted Ratio	Final Apportionment
A	9.9956	9.487	⇧ 10	2648.15	10
B	9.061	9.487	⇩ 9	2653.99	9
C	7.179	7.48	⇩ 7	2665.53	7
D	5.259	5.477	⇩ 5	2668.14*	6
E	3.320	3.464	⇩ 3	2663.03	3
F	1.185	1.414	⇩ 1	2327.78	1
			35 seats		36 seats

Practice in Packet
p. 14-15

Apportionment Paradoxes

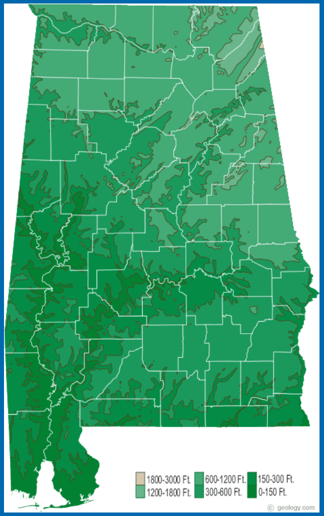
- Any method of apportionment will sometimes produce one of the following undesirable results:

- Violation of Quota i.e. some “state” is given a number of seats that does not equal either the integer part—aka truncated quota-or one more than that

- Alabama Paradox: the loss of a seat by a state when the size of the legislative body increases even if populations do not change.

- *The Alabama Paradox first surfaced after the 1870 census. With 270 members in the House of Representatives, Rhode Island got 2 representatives but when the House size was increased to 280, Rhode Island lost a seat.*

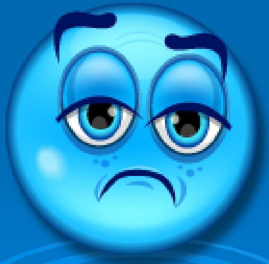
- *After the 1880 census,C. W. Seaton (chief clerk of U. S. Census Office) computed apportionments for all House sizes between 275 and 350 members. He then wrote a letter to Congress pointing out that if the House of Representatives had 299 seats, Alabama would get 8 seats but if the House of Representatives had 300 seats, Alabama would only get 7 seats.*



- Total Number of seats

299 8

300 7



- Population Paradox: the loss of a seat by one state whose population has increased to another whose population has decreased.

- *The Population Paradox was discovered around 1900, when it was shown that a state could lose seats in the House of Representatives as a result of an increase in its population. (Virginia was growing much faster than Maine--about 60% faster--but Virginia lost a seat in the House while Maine gained a seat.)*

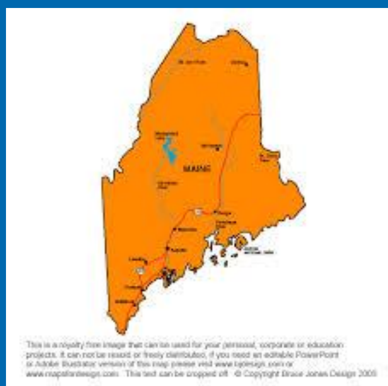
- **The New States Paradox:**
  Adding a new state with its fair share of seats can affect the number of seats due other states.

- *The New States Paradox was discovered in 1907 when Oklahoma became a state. Before Oklahoma became a state, the House of Representatives had 386 seats. Comparing Oklahoma's population to other states, it was clear that Oklahoma should have 5 seats so the House size was increased by five to 391 seats.*

- *The intent was to leave the number of seats unchanged for the other states. However, when the apportionment was mathematically recalculated, Maine gained a seat (4 instead of 3) and New York lost a seat (from 38 to 37).*

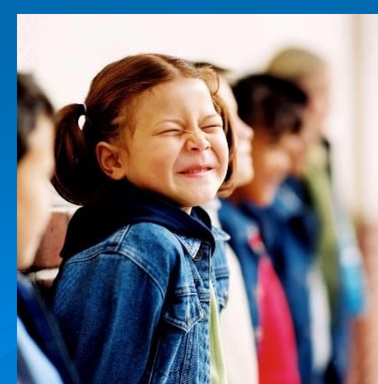

Kaw Lake
Arkansas River
Enid
35
Canadian River
Tulsa
44
Muskogee
40
Oklahoma City
40
44
Canadian River
Eufaula Lake
Lawton
OKLAHOMA

+5



+1

-1



HW = Finish Practice in Packet p. 14-15