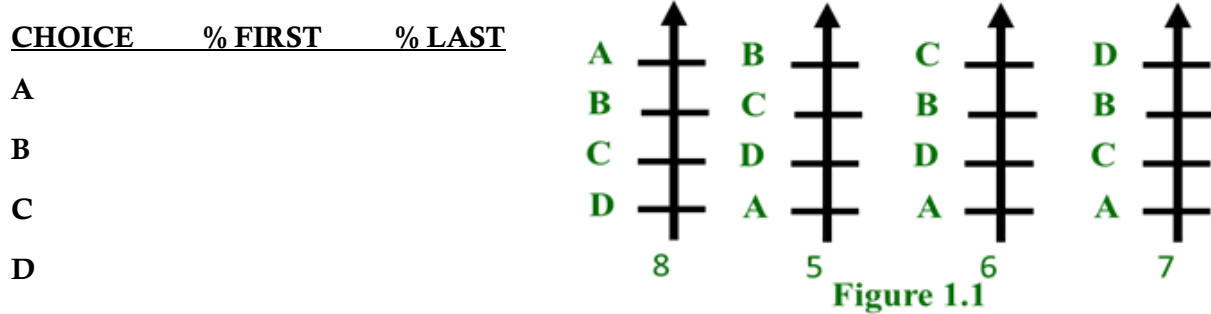| **Day 1: p. 12–14 #5-10, 12 SECTION 1.2** |
|---|

5. For this example determine the percentage of voters that ranked each choice first and last.

   a. Enter the results in a table like the following: Round to the nearest hundredth.

CHOICE	% FIRST	% LAST
A		
B		
C		
D		

Figure 1.1

   b. On the basis of these percentages only, which choice do you think would be the most fair to voters?
      The least fair? Explain your answers.

   c. Which choice do you think most deserves to be ranked first for the group? Explain your reasoning.

   d. Give at least one argument against your choice.

6. The 1998 race for governor of Minnesota has three strong candidates. The following are unofficial results from
   the general election.

Jesse Ventura	768,356
Norm Coleman	713,410
Hubert Humphrey III	581,497
Others	12,017

   a. What percentage of the vote did the winner receive? Is the winner a majority winner?

   b. What is the smallest percentage the plurality winner can receive in a race with exactly three candidates?
       Explain.

7. The Borda method determines a complete group ranking, but the other methods examined in this lesson produce
   only a first. Each of these methods may be extended, however, to produce a complete group ranking.

   a. Describe how the plurality method could be extended to determine a second, third, and so forth. Apply this
       to the example in Figure 1.1 and list the second, third, and fourth that your extension produces.

8. Determine the plurality, Borda, runoff, and sequential runoff winners for the following set of preferences.

	16	20	12	7
	A	B	C	C
	D	D	B	D
	C	A	D	A
	B	C	A	B
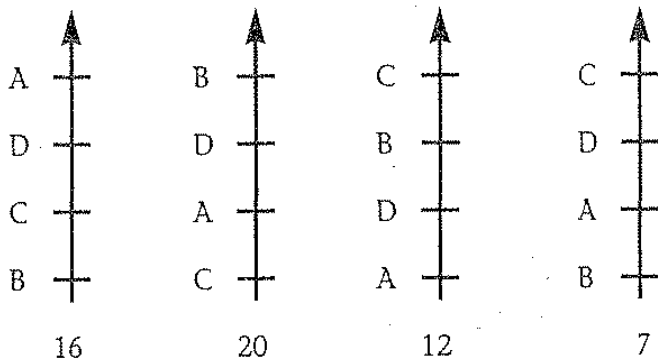
Plurality Winner: _____   Runoff Winner:_____

Borda Winner:_____   Sequential Runoff Winner:_____

9. Each year the Heisman Trophy recognizes one of the country's outstanding college football players. The year 1997 marked the first time a defensive player received the award. The results of the voting follow. Each voter selects a player to rank first, another to rank second, and another to rank third.

	1st	2nd	3rd	Points
Charles Woodson, Michigan	433	209	98	1,815
Peyton Manning, Tennessee	281	263	174	1,543
Ryan Leaf, Washington State	70	205	174	861
Randy Moss, Marshall	17	56	90	253
Ricky Williams, Texas	4	31	61	135
Curtis Enis, Penn State	3	18	20	65
Tim Dwight, Iowa	5	3	11	32
Cade McNown, UCLA	0	7	12	26
Tim Couch, Kentucky	0	5	12	22
Amos Zerouoe, West Virginia	3	1	10	21

a. How many points are awarded for a first-place vote? For a second place? For a third place vote?

b. Would the ranking produced by this system have differed if the plurality method had been used? Explain.

10. When runoff elections are used in the U.S., voters do not rank the candidates and therefore must return to the polls to vote in the runoff. In some countries, such as Ireland, a method commonly called "instant runoff" is used. In an instant runoff, the voters rank the candidates and do not return to the polls. **Examine** the vote totals in the two runoffs below. **Do the totals tell you anything about the merits of the instant runoff? Explain.**

President of Ireland: 1997 Results

	General Election	Runoff
Mary Banotti	372,002	497,516
Mary McAleese	574,424	706,259
Derek Nally	59,529	
Adi Roche	88,423	
Dana Scallon	175,458	

U.S. House Texas District 9: 1996 Results

|	General Election	Runoff
Nick Lampson	83,781	59,217
Steve Stockman	88,171	52,853
Geraldine Sam	17,886	

12. A procedure for solving a problem is called an algorithm. This section has presented various algorithms for determining a group ranking from individual preferences. Algorithms are often written in numbered steps in order to make them easy to apply. The following is an EXAMPLE algorithmic of the runoff method.

1. For each choice, determine the number of preference schedules on which the choice was ranked.
2. Eliminate all choices except the two that were ranked first most often.
3. For each preference schedule, transfer the vote total to the remaining choice that ranks highest on that schedule.
4. Determine the vote total for the preference schedules on which each of the remaining choices is ranked first.
5. The winner is the choice ranked first most.

a. Write a description of the sequential runoff method.

b. Write a description of the Borda method.

9. What are the advantages and disadvantages of approval voting?

10. What is the effect on a group ranking if casting approval votes for all choices? Or casting approval votes for none of the choices?

11. The voters whose preferences are represented below all feel strongly about their first choices bur are not sure about their second and third choices. They all dislike their fourth and fifth choices. Use the Condorcet Method to determine the winner. Show all work.
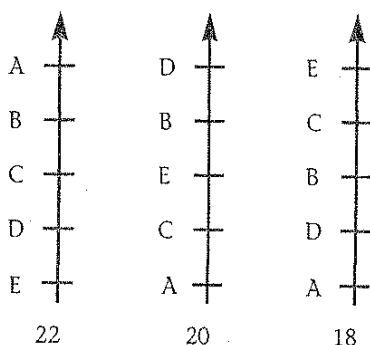
12. Consider the preference list shown with lines added to indicate an "approval line".
    Voters will cast approval votes for candidates above the approval line and will not cast approval votes for candidates below that line. What candidate would win by the approval method? Explain.

	Number of Voters (170 total)				
	33	33	34	36	34
1st	A	B	**E**	D	B
2nd	**D**	E	C	A	C
3rd	C	**C**	D	**B**	A
4th	B	A	A	E	**D**
5th	E	D	B	C	E

13. <u>Kenneth Arrow</u> is an American mathematician and gained worldwide recognition for his mathematical applications to election theory. The many paradoxes in election methods led Mr. Arrow to formulate a list of conditions he thought were necessary for a group ranking to be fair: **1-Non Dictatorship, 2-Individual Sovereignty, 3-Unanamity, 4-Freedom from Irrelevant Choices, 5-Unquieness of Group Ranking.** Use Arrow's Five Conditions for fair ranking methods to answer the following questions.

    a. Suppose that there are only two choices in a list of preferences and that the plurality method is used to decide the group ranking. Which of Arrow's conditions could be violated?

    b. There often are situations in which insincere voting results. Do any of Arrow's conditions state that insincere voting should not be part of a fair group-ranking procedure?

1. Consider a situation in which A, B, and C have 3, 2, and 1 votes, respectively, and in which 4 votes are required to pass an issue.

     a. List all possible coalitions and all winning coalitions

     b. Determine the power index for each voter.

     c. If the number of votes required to pass an issue is increased from 4 to 5, determine the power index of each voter.

2. In a situation with three voters, 51 votes are required to win. A has 49 votes, B has 48, and C has 3.

     a. Determine the power index of each voter

     b. A dictator is a member of a voting body who has all the power. A dummy is a member who has no power. Are there any dictators or dummies in this situation?

3. Four partners decide to start a business. P_1 buys 8 shares, P_2 buys 7 shares, P_3 buys 3 shares and P_4 buys 2 shares. One share = one vote. The quota is set at two-thirds of the total number of votes.

  a. Describe as a weighted voting system.

  b. The partnership above decides the quota is too high and changes the quota to 10 votes. Describe the winning coalitions.

  c. The partnership above decides to make the quota equal 21 votes. Describe the winning coalitions.

4. Weighted voting is commonly used to decide issues at meetings of corporate stockholders. Each member is given one vote for each share of stock held.

  a. A company has four stockholders: A, B, C, and D. They own 26%, 25%, 25%, and 24% of the stock, respectively, and more that 50% of the vote is needed to pass an issue. Determine the power index of each stockholder. Use your results from Exercise 4 as an aid.

  b. Another company has four stockholders. They own 47%, 41%, 7%, and 5% of the stock. Find the power index of each stockholder.

c. Compare the percentage of stock owned by the smallest stockholder in parts a and b. Do the same for the power index of the smallest stock holder in each case.

1. Ten committee members vote by approval voting on four candidates for a new chairperson of the committee. The following table indicates the results; an X denotes approval of the candidate.

Candidate	#1	#2	#3	#4	#5	#6	#7	#8	#9	#10
A	X	X	X	X		X	X			
B	X	X	X	X		X			X	X
C	X	X			X		X	X	X	
D						X	X			

a. Which candidate wins and which finishes last?

b. If committee members #5 and #8 are adamantly opposed to candidates B and D and they have prior knowledge of the others' votes, how might they have voted differently when using the approval voting method in part (a)?

2. Determine whether any voter is a dictator and whether any is a dummy. Explain your choice.

   20 votes are needed to pass an issue

   A: 10 votes   B: 8 votes   C: 6 votes   D: 3 votes

3. Consider the weighted voting situation in which voters A, B, C, and D have 15, 12, 6, and 3 votes respectively, and 24 votes are needed to pass an issue.

   a. Is the coalition formed by B, C, and D a winning coalition?

   b. Which players are essential in the coalition {A, B, C; 33}?

   c. Which players are essential in the coalition {A, B, C, D; 36}?

   d. List all the winning coalitions.

   e. Find the power index for each voter.

   f. Will each voter think that he or she has a fair share of the power? If not, who received more and who received less? Explain.

1. A basketball ranking poll is trying to rank the top teams in the nation. The leading contenders are: Villanova (V), Illinois (I), UConn (C), or Duke (D). The preference ballots are organized in the following preference schedule.

NUMBER OF VOTERS	13	11	4	11	7
1ST CHOICE	V	I	D	C	C
2ND CHOICE	I	V	I	I	D
3RD CHOICE	D	D	V	V	I
4TH CHOICE	C	C	C	D	V

a). How many votes were cast?_____

b). How many votes would be needed for there to be a majority winner? _____

    Is there a majority winner? _____   If so, who is it?_____

c). Find the winner by the plurality method. _____

d). Find the winner by the 4-3-2-1 Borda Count Method. _____

       POINT TOTAL FOR V: _____

       POINT TOTAL FOR I: _____

       POINT TOTAL FOR C: _____

       POINT TOTAL FOR D: _____

e). Find the winner by the runoff method. _____

f). Find the winner by the sequential runoff method. _____

g). Find the winner by the Condorcet method. _____

2. Suppose that this election is conducted by the approval method instead and that each voter approves of the choices marked on the chart below.

VOTERS	A	B	C	D	E	G	H	I	J	K
Villanova	X		X			X		X		
Illinois	X	X		X		X	X			X
U Conn	X		X		X			X		
Duke	X		X	X	X		X	X	X	X

a) Determine the approval winner. _____

b) If any team with 50% or more of the votes can be selected and they will have a run off, which teams would be in this run-off?

3. How many possible preference schedules are there if there are 7 choices to choose from in an election? _____

4. Name and describe each of Arrow's Conditions.

5. Consider the weighted voting situation in which voters A, B, C, & D have 11, 8, 6, & 2 votes, respectively, and 14 votes are needed to pass an issue.

   a) Is the coalition formed by A and C a winning coalition? _____

   b) Which players are essential in the coalition {A, B, C, D; 27}? _____ Explain your reasoning.

   c) List all the winning coalitions (USE CORRECT NOTATION – see question above).

   d) Find the Power index for each voter.
                              A:____ B:____ C:____ D:____

   e) Are there any voters who are dummies or dictators? Explain.

   f) Do the power indices reflect the distribution of the votes? Explain.

1. a. The application of any fair division algorithm requires certain assumptions, or axioms. For example, the success of the estate division algorithm requires that each heir be capable of placing a value on each object in the estate. If any heir considers an object priceless or is otherwise capable of placing a dollar value on an object, the algorithm fails. Give at least one other axiom that you think is necessary for the success of the algorithm.

3. Amy, Bart, and Carol are heirs to an estate that consists of a valuable painting, a motorcycle, a World Series ticket, and $5,000 in cash. They submit the bids shown:

	Painting	Motorcycle	Ticket	Fair Share Calculations
AMY	$2, 000	$4,000	$500	
BART	$5,000	$2,000	$100	
CAROL	$3,000	$3,000	$300	

a. Use the estate division algorithm to divide the estate among the heirs. For each heir, state the fair share, the items received, the amount of cash, and the final settlement. Show all of your work and summarize your results in a table.

You may wish to use a similar table:

Cash in the estate	$5,000
Received from Amy	
Received from Bart	
Paid to Carol	
Cash remaining	

b. It is common for one or more heirs to pay into an estate. This lesson's algorithm fails if an heir who must pay into the estate cannot do so. Suggest a way the algorithm could be modified to account for situations in which one or more heirs cannot raise the cash necessary to complete the division.

5. If two heirs submit an identical highest bid for an item, how would you resolve the tie?

7. Could the estate division algorithm of this lesson encourage insincerity by any of the heirs? Explain.

9. Two friends have decided to share an apartment in order to obtain a nicer apartment than either could afford individually. They choose a two-bedroom that rents for $900 monthly, including utilities. One bedroom is larger than the other. Propose a procedure for deciding which of the friends gets the nicer bedroom.

Section 2.3-2.4: Methods of Apportionment

Example: A country has 6 states with **populations 27774, 25178, 19947, 14614, 9225, and 3292.**
Its House of Representatives has 36 seats. Find the apportionment using the methods
of Hamilton, Jefferson, Webster, and Hill.

** All methods obtain a standard divisor / ideal ratio

$$s = \frac{total\ population}{number\ of\ seats}$$

# of seats = _____ ;   total population = _____;   s = _____

(store in calc)

♦ To obtain quotas, divide the population of each state by the idea ratio (s).

State	Population	Quota
A		
B		
C		
D		
E		
F		

♦ **THE HAMILTON METHOD** – each state receives either its lower quota or its upper quota.

State	Quota	Tentative Apportionment	Final Apportionment
A			
B			
C			
D			
E			
F			

♦ Round each quota down to get a tentative apportionment. Since the resulting house size is too small (by 2), consider the two quotas with the largest decimal values. Increase their apportionments by 1.
♦ The Hamilton method always satisfies the <u>quota condition</u> (each states apportionment is equal to either its lower quota or its upper quota).

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♦ **THE JEFFERSON METHOD** (tends to favor large states)

Remember s = 2778.611

State	Tentative Apportionment	Jefferson Adjusted Ratio	Next Tentative Apportionment	Final Apportionment
A				
B				
C				
D				
E				
F				

♦ The tentative apportionment is the same as the Hamilton method (found by dividing each states population by s and rounding down). Since the resulting house size is too small, calculate the adjusted ratio for each state.

$$\text{Jefferson Adjusted Ratio} = \frac{state\ size}{tentative\ apportionment + 1}$$

♦ Give the state with the adjusted ratio closest to s (that is the state with the largest adjusted ratio) an additional seat.

♦ Recompute the state's adjusted ratio based on its new tentative apportionment. If more seats are to be given out, give the state with the largest adjusted ratio another seat. Continue in this manner until all seats are allocated

♦ The Jefferson method may not satisfy the Quota Condition

♦ **THE WEBSTER METHOD** (favors neither large nor small states)

State	Quota	Tentative Apportionment	Webster Adjusted Ratio	Final Apportionment
A	9.996			
B	9.061			
C	7.179			
D	5.259			
E	3.320			
F	1.185			

♦ To obtain the tentative apportionment, round each quota (round up if the decimal is .5 or higher and round down if it is smaller than .5)

♦ When too few seats are given (as is the case here), compute the adjusted ratio as follows:

$$\text{Webster Adjusted Ratio} = \frac{state\ size}{tentative\ apportionment + 0.5}$$

(for too few seats)

♦ Choose the state with the largest adjusted ratio (that is, the adjusted ratio that is closest to "s"). Increase that state's apportionment by 1.

♦ Recompute the state's adjusted ratio based on its new tentative apportionment. If more seats are to be added, compare the adjusted ratio to s as before. Continue in this manner until all seats have been allocated.

♦ When too many seats are given, compute the adjusted ratio as follows:

$$\text{Webster Adjusted Ratio} = \frac{state\ size}{tentative\ apportionment - 0.5}$$

(for too many seats)

♦ Choose the state with the smallest adjusted ratio (that is, the adjusted ratio that is closest to "s"). Decrease that state's apportionment by 1.

♦ Recompute the state's adjusted ratio based on its new tentative apportionment. If more seats are to be taken away, compare the adjusted ratio to s as before. Continue in this manner until the house size is reached.

♦ The Webster method does not satisfy the quota condition.

Hill-Huntington Method (tends to favor small states)

s = 2778.6111

State	Quota	Geometric Mean	Tentative Apportionment	Adjusted Ratio	Final Apportionment
A	9.9956				
B	9.061				
C	7.179				
D	5.259				
E	3.320				
F	1.185				

♦ For each quota, compute the geometric mean as follows:

$$\text{Geometric Mean} = \sqrt{\text{lower quota} \cdot \text{upper quota}}$$

♦ To get the tentative apportionment, compare the quota to the geometric mean

Round the geometric mean:

UP if the quota is bigger

DOWN if the quota is smaller

♦ When too few seats are given (as is the case here), compute the adjusted ratio as follows:

$$\text{Hill} - \text{Huntington Adjusted Ratio} = \frac{\text{state size}}{\sqrt{\text{tentative apportionment}\,(\text{tentative apportionment} + 1)}}$$

♦ Choose the state with the largest adjusted ratio and increase that states apportionment by 1 seat. Recompute the adjusted ratio and continue until the desired size is reached.

♦ When too many seats are given, compute the adjusted ratio as follows:

$$\text{Hill} - \text{Huntington Adjusted Ratio} = \frac{\text{state size}}{\sqrt{\text{tentative apportionment}\,(\text{tentative apportionment} - 1)}}$$

♦ Choose the state with the smallest adjusted ratio (closest to s) and decrease that states apportionment by 1 seat. Recompute the adjusted ratio and continue until the desired size is reached.

Hamilton, Jefferson, Webster, and Hill

Ex. 1) City College is made up of 5 different departments: communications, accounting, marketing, psychology, and technology. A total of 110 teaching positions are to be apportioned based on the school's enrollment as shown below.

Department	Communications	Accounting	Marketing	Psychology	Technology
Enrollment	2425	745	497	230	1053

a. Find the total enrollment.

b. Find the ideal ratio, s. What does the ideal ratio represent in this problem?

c. Find the number of faculty members apportioned to each department using the methods of Hamilton, Jefferson, Webster, and Hill-Huntington and record in the table below. **The table columns are split in two to represent initial apportionment and final apportionment!**

Initial & Final Apportionments

Department	Initial Quota	Hamilton		Jefferson		Webster		Hill-Huntington	
Communications									
Accounting									
Marketing									
Psychology									
Technology									

For each method that requires an adjusted ratio, state the adjusted ratio that you use.

Jefferson's Adjusted Ratios: C:_____   A:_____   M:_____   P:_____   T:_____

Webster's Adjusted Rations: C:_____   A:_____   M:_____   P:_____   T:_____

Hill-Huntington Geometric Means: C:_____   A:_____   M:_____   P:_____   T:_____

Hill-Huntington Adjusted Ratios: C:_____   A:_____   M:_____   P:_____   T:_____

Which department prefers each apportionment method and why?

Do any methods violate the quota condition?

Ex. 2) A country has six states with populations 27,770; 25,193; 19,418; 14,612; 9,217; 3,790. It's House of Representatives has 40 seats.

    a. Find the apportionment using the methods of Hamilton, Jefferson, Hill-Huntington, and Webster and record in the table below. For Jefferson, Hill-Huntington, and Webster, state the adjusted ratio that you use.

Initial & Final Apportionments

Population	Initial Quota	Hamilton	Jefferson	Webster	Hill-Huntington
A: 27,770					
B: 25,193					
C: 19,418					
D: 14,612					
E: 9,217					
F: 3,790					

    b.   Jeff. Adjusted Ratios: A:_____B:_____ C:_____   D:_____   E:_____ F:_____
    c.   Webs. Adjusted Ratios: A:_____B:_____ C:_____   D:_____   E:_____ F:_____
    d.   H-H Geometric Means: A:_____B:_____ C:_____   D:_____   E:_____ F:_____
    e.   H-H Adjusted Ratios: A:_____ B:_____ C:_____   D:_____   E:_____ F:_____

Ex. 3) Central High School has sophomore, junior, and senior classes of 464, 240, and 196 students respectively. The 20 seats on the school's student council are divided among the classes according to population.

    a. Find the apportionment using the methods of Hamilton, Jefferson, Hill-Huntington, and Webster and record in the table below.

Class Size	Initial Quota	Hamilton	Jefferson	Webster	Hill-Huntington
Sophomores 464					
Juniors 240					
Seniors 196					

    b. Jeff. Adjusted Ratios: Sophomores:_____   Juniors:_____   Seniors:_____
    c. Webs. Adjusted Ratios: Sophomores:_____   Juniors:_____   Seniors:_____
    d. H-H Geometric Means: Sophomores:_____   Juniors:_____   Seniors:_____
    e. H-H Adjusted Ratios: Sophomores:_____   Juniors:_____   Seniors:_____
    f. Did any methods violate quota condition? If so, which ones?