Unit 6 Day 3

Section 1.5
Weighted Voting & Voting Power

Warm-up Day 3

1) Given these preference schedules, identify the Condorcet, Runoff, and Sequential Runoff winners.

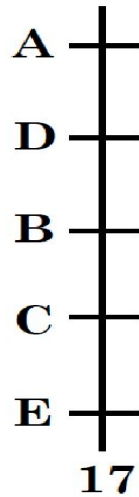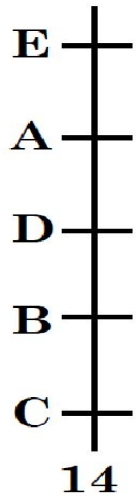| E	A	C	C	D	B
A	D	B	D	E	A
D	B	A	B	A	C
B	C	D	E	C	E
C	E	E	A	B	D
14	17	19	12	16	22
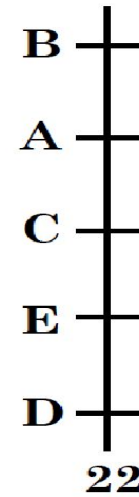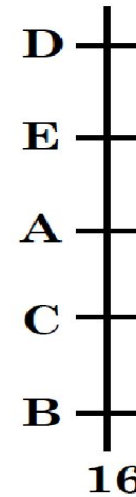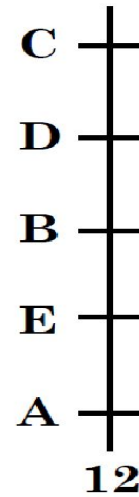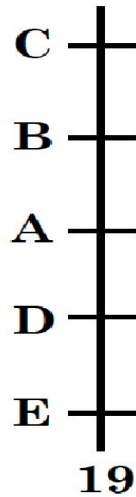
Condorcet:
Runoff:
Seq.Runoff:

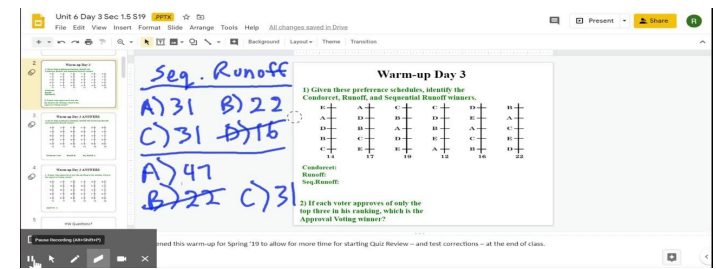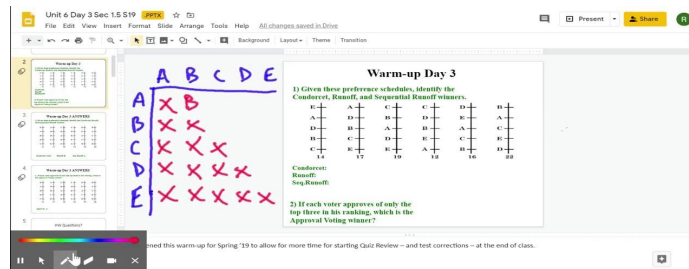**2) If each voter approves of only the top three in his ranking, which is the Approval Voting winner?**

Warm-up Day 3 ANSWERS

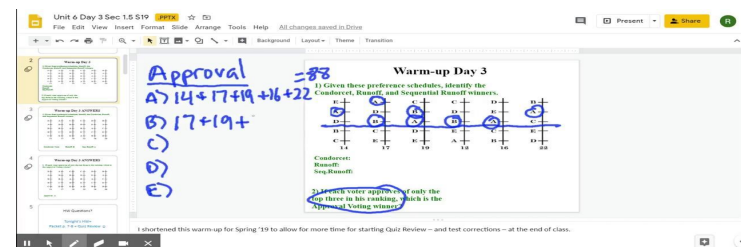**1) Given these preference schedules, identify the Condorcet, Runoff, and Sequential Runoff winners.**

E	A	C	C	D	B
A	D	B	D	E	A
D	B	A	B	A	C
B	C	D	E	C	E
C	E	E	A	B	D
14	17	19	12	16	22
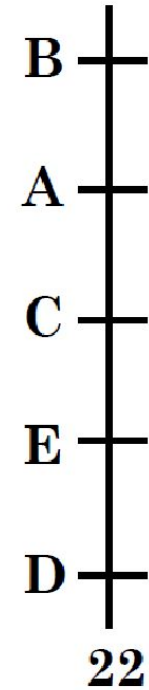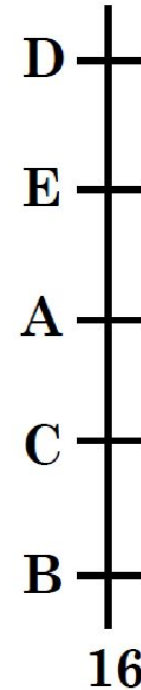
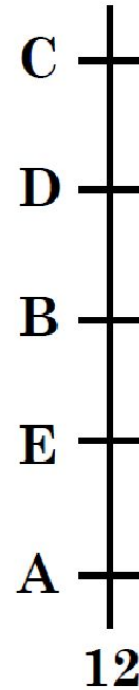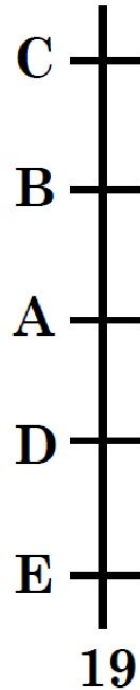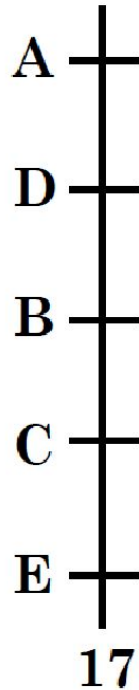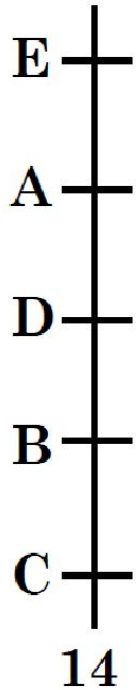**Condorcet: None   Runoff: B   Seq. Runoff: A**

Warm-up Day 3 ANSWERS

2. If each voter approves of only the top three in his ranking, which is the Approval Voting winner?

E	A	C	C	D	B
A	D	B	D	E	A
D	B	A	B	A	C
B	C	D	B	C	E
C	E	E	A	B	D
14	17	19	12	16	22
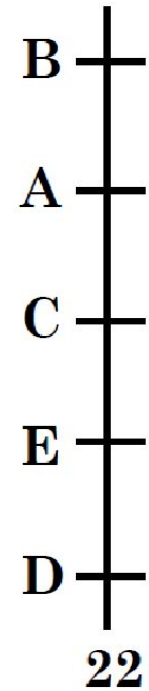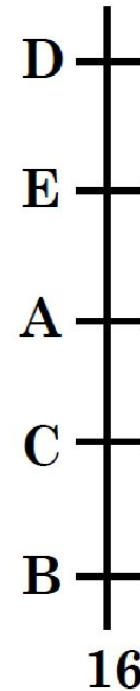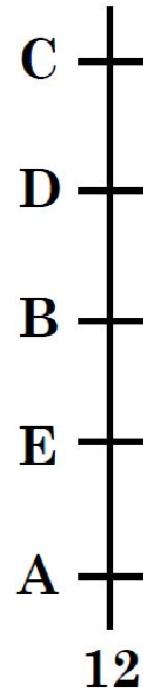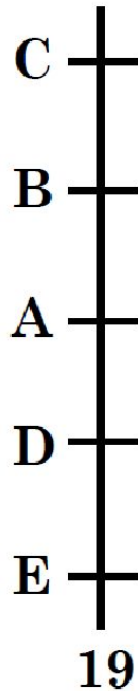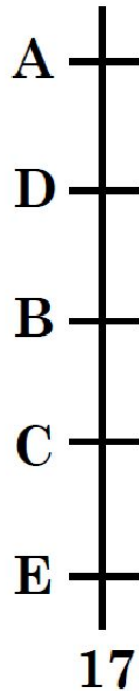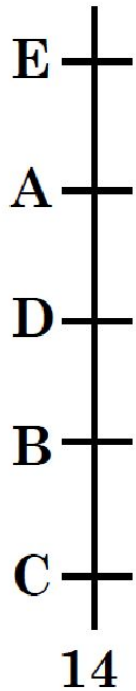
Approval: A

HW Questions?

Tonight's HW=

Packet p. 5-6 😊

Practice: Determine the winner by the Condorcet Method

	Number of votes received			
Place	390	360	300	450
1st	Shawn	Gail	Gail	Ricco
2nd	Twanda	Twanda	Twanda	Twanda
3rd	Ricco	Ricco	Shawn	Shawn
4th	Gail	Shawn	Ricco	Gail

Unit 6   Day 3 Notes
Weighted Voting and Voting Power

Section 1.5

WEIGHTED VOTING

- Definition: Method of voting when some members of a voting body have more votes than others.

- Why might it be fair for some members or groups to have more weight than others?

- Examples: Electoral College, Corporate stockholder meetings, Mom, etc.
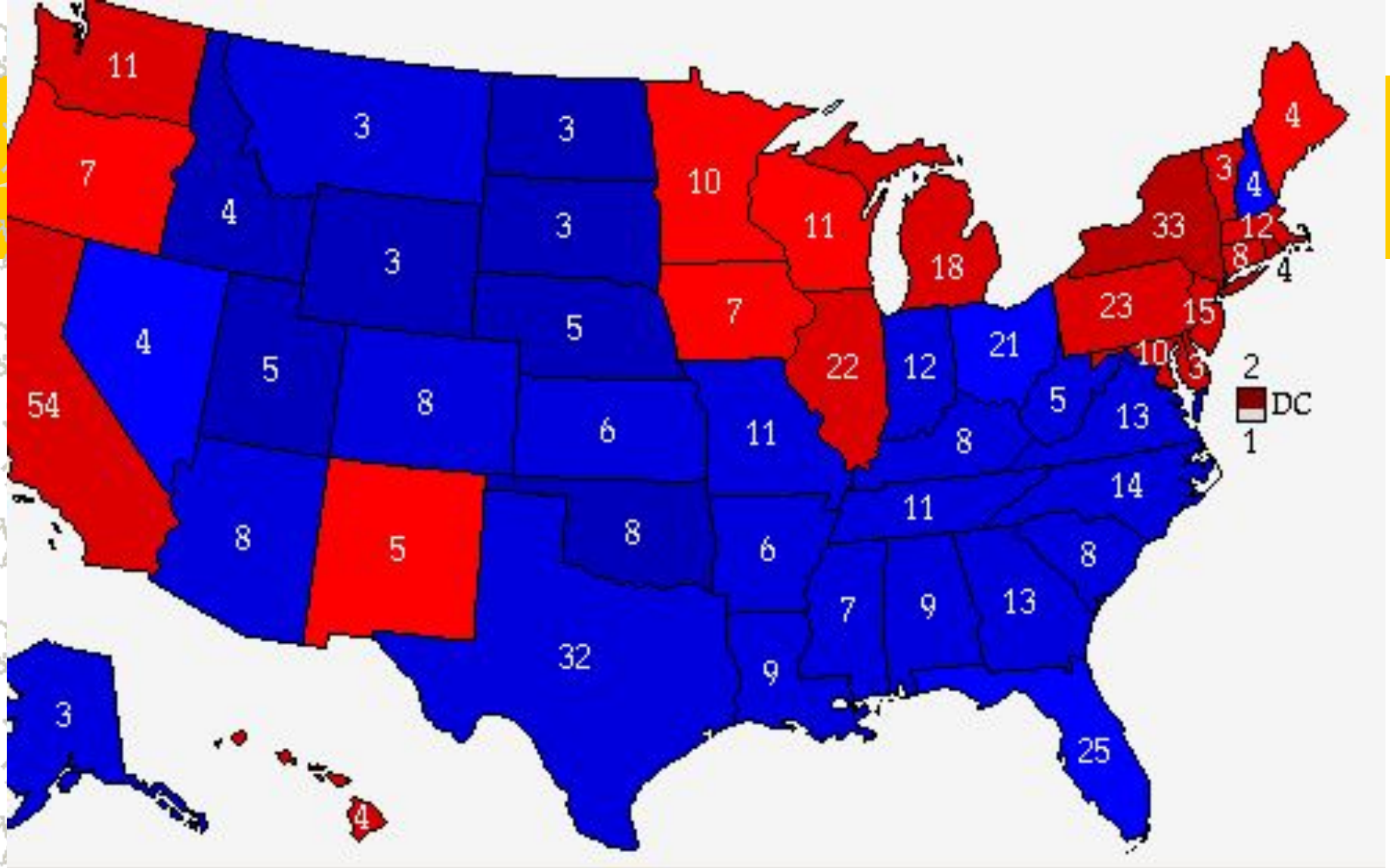
the # of votes is based on population in each state

Example:

A student council has 1 representative per class, but there are 500 sophomores, 300 juniors, and 300 seniors. How could votes be weighted to represent everyone fairly?

Every 100 students = 1 vote

How many votes would be needed to pass an issue? If not given, go for majority.

WINNING COALITIONS:

- Each collection is known as a **<u>Coalition</u>**.

  {So;5}, {Jr;3}, {So,Jr;8}, {Jr,Sr;6} …

- Those with enough votes to pass an issue are known as **<u>Winning Coalitions</u>**. (A winning coalition is like an alliance in "Survivor.")

- A voter is **<u>essential</u>** when their vote is NECESSARY to win. (i.e. if you remove it, the winning coalition becomes a losing coalition.)

COALITIONS:

- Sophomores get 5 votes
- Juniors get 3 votes
- Seniors get 3 votes

Notation: {So;5}, {Jr;3}, {Sr;3}

There are a total of 11 votes and they need 6 votes to win.

Could any group win by themselves?

NO!!

So, join forces!

WINNING COALITIONS:
(like an alliance on <u>Survivor</u>)

- Sophomores get 5 votes.
- Juniors get 3 votes.
- Seniors get 3 votes.
- You need 6 votes to WIN so a winning coalition must have 6 votes.

What are all the possible coalitions?

$\{$ none; $0\}$ $\{$ So; $5\}$ $\{$ Jr; $3\}$ $\{$ Sr; $3\}$

$\{$ So,Jr; $8\}$ $\{$ So,Sr; $8\}$ $\{$ Jr,Sr; $6\}$

$\{$ So,Jr,Sr; $11\}$

Of these coalitions which ones are winners?

$\{$ So,Jr; $8\}$ $\{$ So,Sr; $8\}$ $\{$ Jr,Sr; $6\}$

$\{$ So,Jr,Sr; $11\}$

$$\{So, Jr; 8\} \quad \{So, Sr; 8\} \quad \{Jr, Sr; 6\} \quad \{So, Jr, Sr; 11\}$$

Notice that the sophomore, junior, and senior representatives are each essential to 2 of the coalitions.

This is a **PARADOX**: Although the votes have been distributed to give more power to the sophomores, the outcome is that all members have the same amount of power.

Banzhaf Power Index:
determines the power of a member of a voting body

- **Step 1.** Make a list of all possible coalitions
- **Step 2.** Determine which of them are winning coalitions
- **Step 3.** In each winning coalition, determine which of the players are *essential* players
- **Step 4.** Count the total number of times each player is essential
- You just found the Banzhaf Power Index of each player!

- Consider the weighted voting situation:

Voter A – 15 votes;   Voter B – 12 votes

Voter C – 6 votes;   Voter D – 3 votes

20 *votes are needed to pass…*

List all of the possible coalitions:

$\{$ none; $0\}$ $\{$A;$15\}$ $\{$B;$12\}$ $\{$C;$6\}$ $\{$D;$3\}$

$\{$A,B;$27\}$ $\{$A,C;$21\}$ $\{$A,D;$18\}$

$\{$B,C;$18\}$ $\{$B,D;$15\}$ $\{$C,D;$9\}$

$\{$A,B,C;$33\}$ $\{$A,B,D;$30\}$

$\{$A,C,D;$24\}$ $\{$B,C,D;$21\}$

$\{$A,B,C,D;$36\}$

List all of the winning coalitions:

$\{$A,B;$27\}$ $\{$A,C;$21\}$ $\{$A,B,C;$33\}$ $\{$A,B,D;$30\}$

$\{$A,C,D;$24\}$ $\{$B,C,D;$21\}$ $\{$A,B,C,D;$36\}$

SO….

A – 15 votes; B – 12 votes; C – 6 votes; D – 3 votes

***20** votes are needed to pass*

Winning coalitions:

$$\{A,B;27\}\ \{A,C;21\}\ \{A,B,C;33\}\ \{A,B,D;30\}$$

$$\{A,C,D;24\}\ \{B,C,D;21\}\ \{A,B,C,D;36\}$$

Determine the POWER INDEX for each voter:

A is NEEDED for __**5**__ coalitions

B is NEEDED for __**3**__ coalitions

C is NEEDED for __**3**__ coalitions

D is NEEDED for __**1**__ coalitions

**Power Index:
Number of winning coalitions to which the voter is essential.**

One More Example

- Consider the weighted voting situation:

Voter A – 7 votes;   Voter B – 3 votes

Voter C – 3 votes;

7 *votes are needed to pass…*

List all of the possible coalitions:

$\{\text{none};0\}\ \{A;7\}\ \{B;3\}\ \{C;3\}$

$\{A,B;10\}\ \{A,C;10\}\ \{B,C;6\}$

$\{A,B,C;13\}$

List all of the winning coalitions:

$\{A;7\}\ \{A,B;10\}\ \{A,C;10\}\ \{A,B,C;13\}$

Power Indices: A _____ **4**

B _____ **0**

C _____ **0**

Dummies and Dictators…

- **A <u>Dictator</u>** has all the "power" in a voting body. He is essential to EVERY winning coalition.

- **A <u>Dummy</u>** has no power in a voting body. He is not essential to ANY winning coalitions.

You Try

Consider a situation in which voters A, B, C, and D have 4, 3, 3, and 2 votes, respectively, and 7 votes are needed to pass an issue.

a. Identify all winning coalitions and their vote totals.

b. Find the power index for each voter.

c. Do the power indices reflect the distribution of votes?

d. Suppose the number of votes necessary to pass an issue is increased from 7 to 8. How does this change the power indices of the voters?

Consider a situation in which voters A, B, C, and D have 4, 3, 3, and 2 votes, respectively, and 7 votes are needed to pass an issue.

a. Identify all winning coalitions and their vote totals.

$\{A,B;7\}$ $\{A,C;7\}$ $\{A,B,C;10\}$ $\{A,B,D;9\}$ $\{A,C,D;9\}$ $\{B,C,D;8\}$ $\{A,B,C,D;12\}$

b. Find the power index for each voter.

A:5, B:3, C:3, D:1

c. Do the power indices reflect the distribution of votes?

No, A's power is disproportionately high, while D's is low.

d. Suppose the number of votes necessary to pass an issue is increased from 7 to 8 . How does this change the power indices of the voters?

All voters now have equal power.

Classwork

Packet p. 5 #1 and #2
Packet p. 6

HW Questions?

Tonight's HW=

Packet p. 5-6 😊