

Unit 5 Day 2

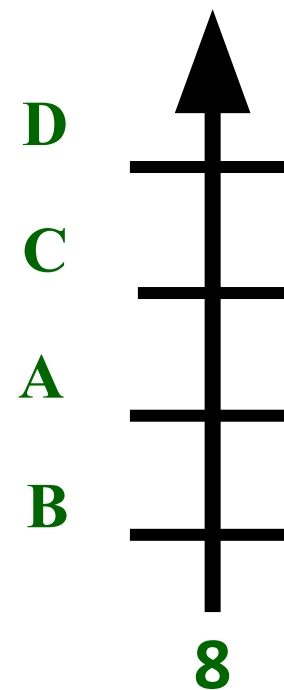
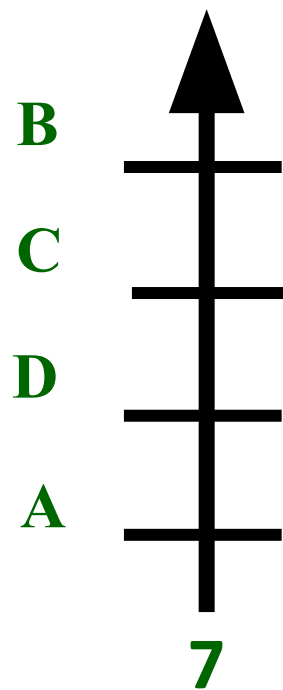
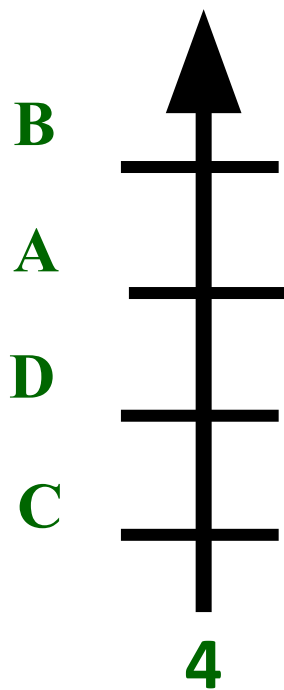
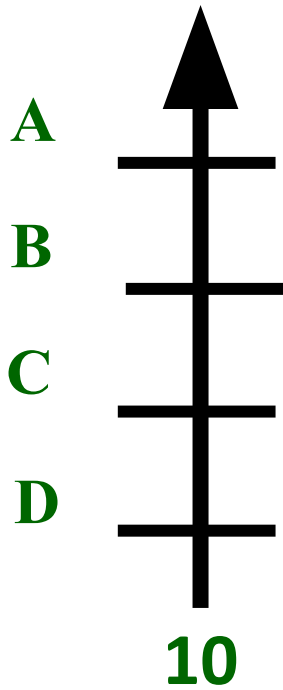
1.3 and 1.4

More Group Ranking Methods And Approval Voting

Warm Up Day 2

Determine the winners by each method

1. Plurality
2. Majority
3. Borda
4. Runoff
5. Sequential Runoff



Warm Up Day 2

Determine the winners by each method

1. Plurality
2. Majority
3. Borda
4. Runoff
5. Sequential Runoff

① B with 11 first place

Warm Up Day 2

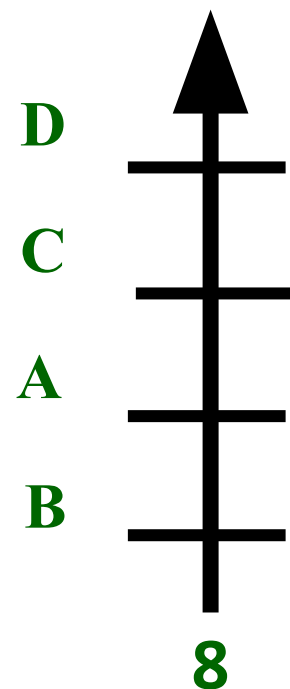
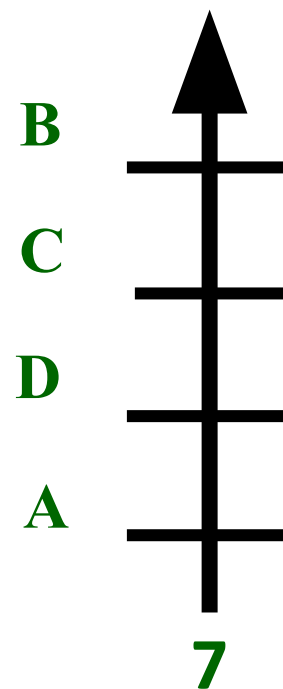
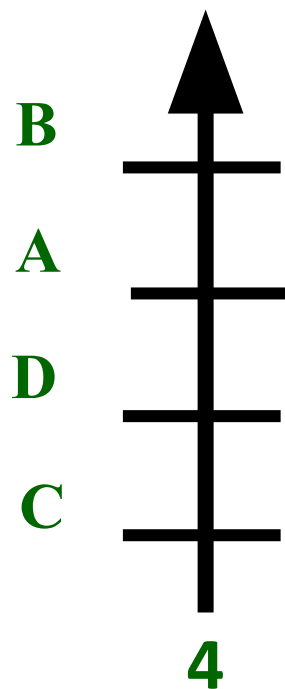
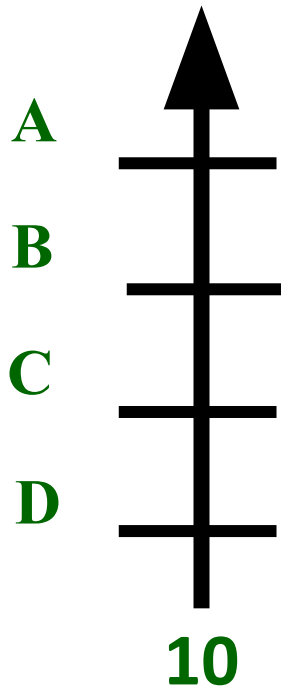
Determine the winners by each method

3. Borda

$A: 10(4) + 4(3) + 7(1) + 8(2) = 77$
 $B: 10(3) + 4(4) + 7(4) + 8(1) = 82$
 $C: 10(2) + 4(1) + 7(3) + 8(3) = 69$

Warm Up Day 2 ANSWERS

Determine the Plurality, Majority, Borda, Runoff, and Sequential Runoff winners.



Plurality: B
Runoff: A

Majority: None
Seq. Runoff: A

Borda: B

HW Questions?

Homework Day 2

- Packet p. 4

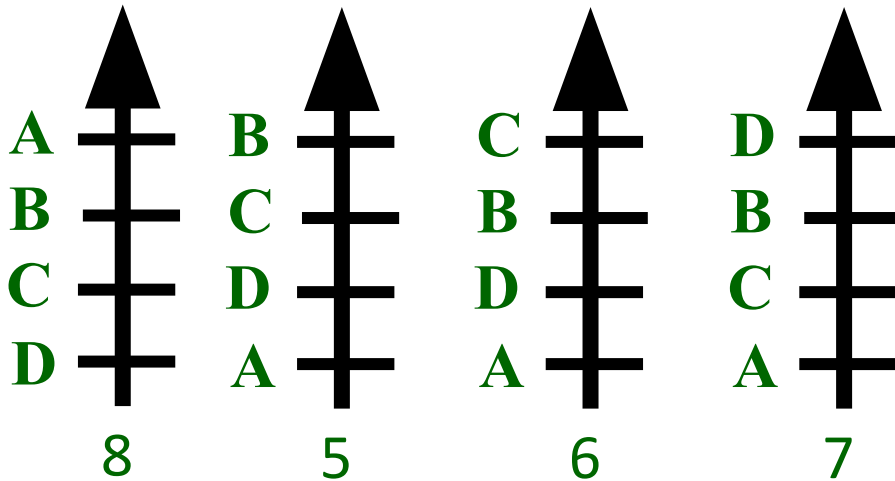
Notes 1.3:
More Group Ranking
Methods and Paradoxes

Pairwise Voting

- Once all of the ballots are submitted, we consider all of the different pairings of two candidates against one another
- If there are three candidates, there are three pairings: A vs. B, A vs. C, and B vs. C
- If there are four candidates, there are six pairings: A&B, A&C, A&D, B&C, B&D, C&D

Condorcet

The Marquis de Condorcet was a friend of Jean-Charles de Borda. He believed that a choice that could obtain a *head-to-head* majority over every other choice should win (using pairwise voting)



*Make an educated guess for the winner and compare with other candidates.

A vs B (B wins)

B vs C (B wins)

B vs D (B wins)

Compare each choice with every other choice. Record the wins and losses in a table.

Since **B** wins *head-to-head* over every other choice, it is the Condorcet winner.

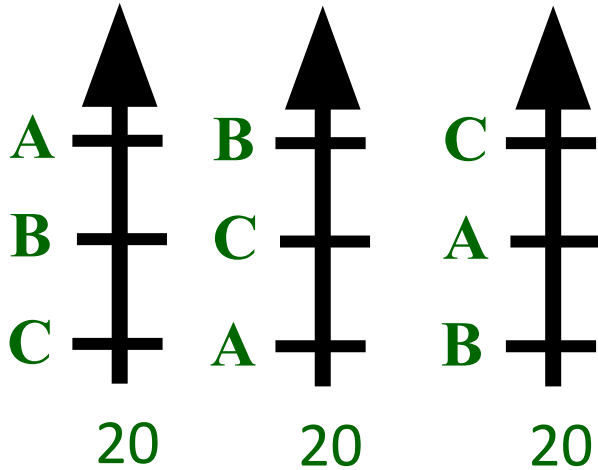
The screenshot shows a presentation slide titled "Condorcet". The text on the slide reads: "The Marquis de Condorcet was a friend of Jean-Charles de Borda. He believed that a choice that could obtain a *head-to-head* majority over every other choice should win (using pairwise voting)". Below the text are the same four pairwise voting tallies shown in the main image. To the right of the tallies is a handwritten table with columns A, B, C, D and rows A, B, C, D. The table contains 'X' marks indicating wins and 'O' marks indicating losses. The table is as follows:

| | A | B | C | D |
|---|---|---|---|---|
| A | | | | |
| B | X | | | |
| C | | X | | |
| D | | X | X | |

Below the table, the slide text says: "Compare each choice with every other choice. Record the wins and losses in a table." and "Since wins *head-to-head* over every other choice, it is the Condorcet winner."

The Condorcet method has a flaw.

Consider this set of preference schedules.



Example Head-to-head:

A vs B (A wins)

B vs C (B wins)

A vs C (C wins)

Another Example head-to-head:

A vs B (tie)

B vs C (tie)

C vs D (tie)

Condorcet sometimes fails to produce a winner.

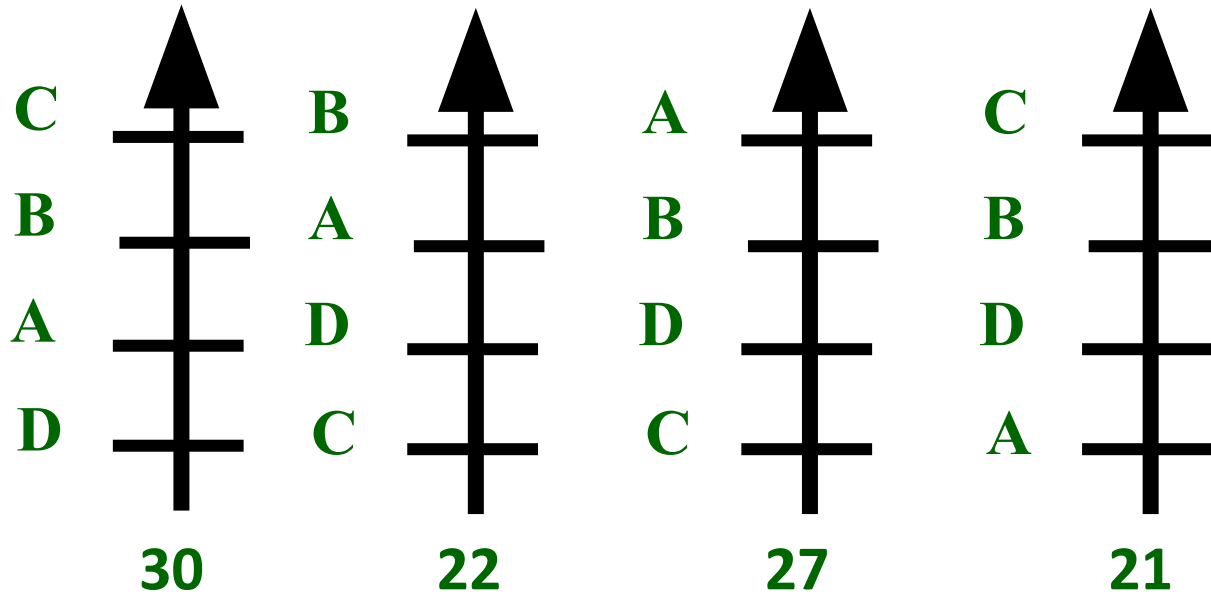
This is known as a Paradox.

The screenshot shows the same content as the previous slides, but with handwritten annotations in blue and red. The Condorcet paradox is circled in blue. The head-to-head results are also annotated: 'A vs B (A wins)' is circled in blue, 'B vs C (B wins)' is underlined in blue, and 'A vs C (C wins)' is circled in blue. The second set of head-to-head results (A vs B tie, B vs C tie, C vs D tie) is circled in red. A handwritten table on the right side of the slide shows the pairwise comparisons:

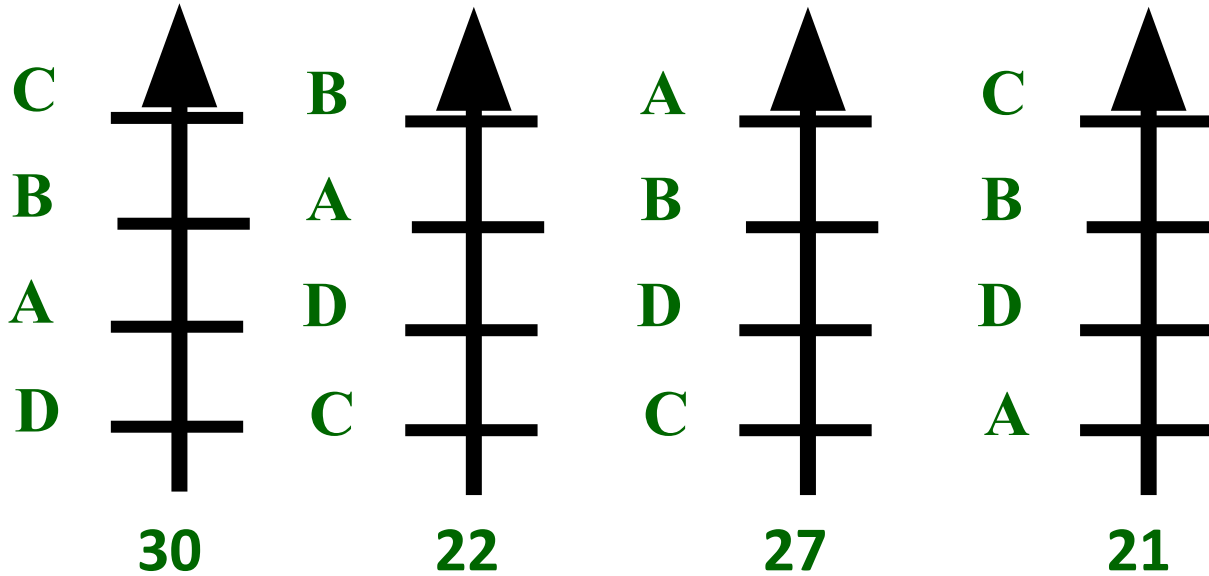
| | | | |
|---|----|----|----|
| | A | B | C |
| A | | 20 | 20 |
| B | 20 | | 20 |
| C | 20 | 20 | |

Group ranking methods may violate the Transitive Property.

You Try! Find the winner using **Condorcet**, **majority**, **plurality**, **runoff**, **sequential runoff** and **Borda**:



You Try! Find the winner using Condorcet, majority, plurality, runoff, sequential runoff and Borda:



You Try! Find the winner using Condorcet, majority, plurality, runoff, sequential runoff and Borda:

$A: 27$
 $B: 30+22=52$
 $A: 27+27=54$
 $C: 30+21=51$
 $A: 30+22+27=79$
 $D: 21$

| | | | |
|---|----|----|----|
| C | B | A | C |
| B | A | B | B |
| A | D | D | D |
| D | C | C | A |
| | 30 | 22 | 27 |

ABCD
 XBCA
 XX
 XXXX
 XXXX

You Try! Find the winner using Condorcet, majority, plurality, runoff, sequential runoff and Borda:

$C: 51$
 $B: 52$
 $A: 27$

| | | | |
|---|----|----|----|
| C | B | A | C |
| B | A | B | B |
| A | D | D | D |
| D | C | C | A |
| | 30 | 22 | 27 |

$B: 22+27=49$
 $C: 30+21=51$

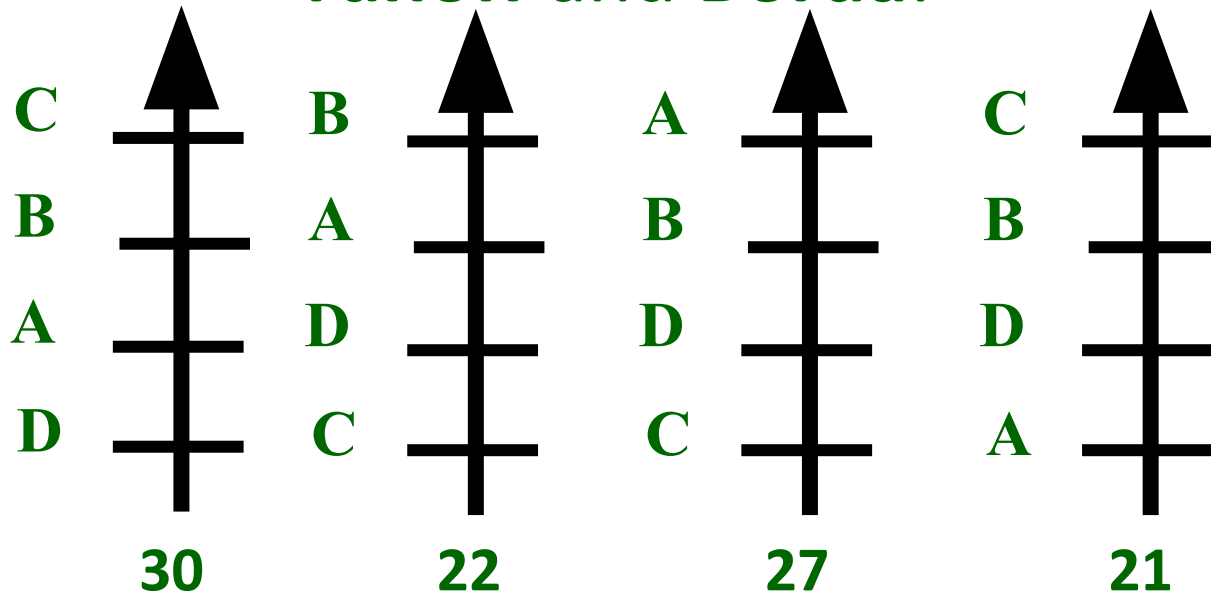
total
 $30+22+27+21=100$
 Majority: C

You Try! Find the winner using Condorcet, majority, plurality, runoff, sequential runoff and Borda:

| | | | |
|-----|-----|-----|-----|
| 4 C | 3 B | 2 A | 1 C |
| 3 B | 2 A | 2 B | 2 B |
| 2 A | 2 D | 2 D | 2 D |
| 1 D | 1 C | 1 C | 1 A |
| | 30 | 22 | 27 |

$A: 30(2) + 22(3) + 27(4) + 21(1) = 255$
 $B: 30(3) + 22(4) + 27(3) + 21(2) = 322$
 $C:$
 $D:$

You Try ANSWERS Find the winner using
**Condorcet, majority, plurality, runoff, sequential
runoff and Borda:**



Condorcet: **C**

Majority: **C** Plurality: **C**

Runoff: **C**

Sequential runoff: **C**

Borda: **B**

Day 2

**Notes 1.4 continued:
Arrow's Conditions &
Approval Voting**

Arrow's 5 Conditions Necessary for a Fair Group Ranking Method

Kenneth Arrow is an American economist and mathematician. He gained worldwide recognition for his mathematical applications to election theory.

The many paradoxes in election methods led Mr. Arrow to formulate a list of conditions he thought were necessary for a group ranking to be fair.



ONLINE EDITION

Take a few minutes to read this article.

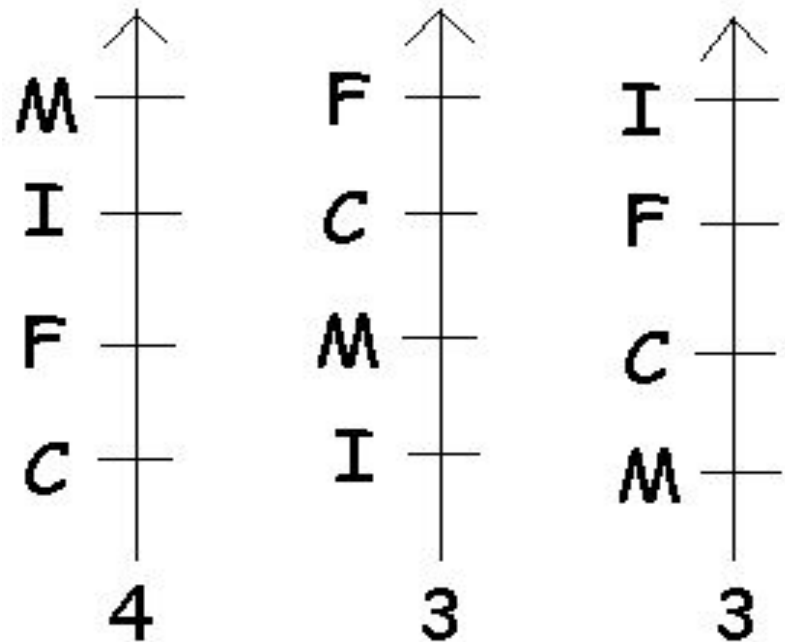
<https://tinyurl.com/hex8ven>



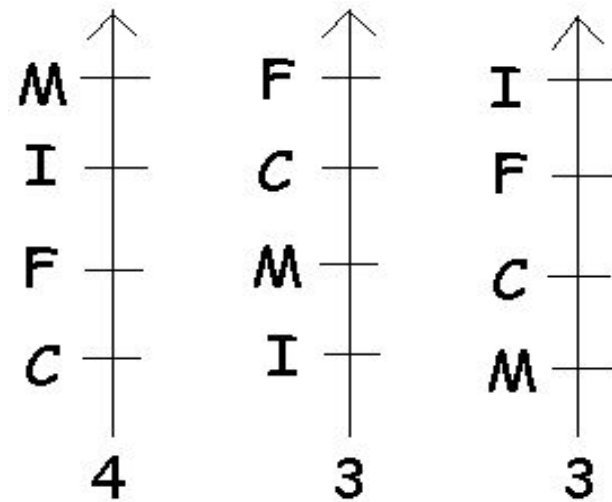
Ten representatives of the language clubs at Central High School are meeting to select a location for the clubs annual joint dinner. They must choose between a Chinese, French, Italian, or Mexican restaurant.

You Try this example.
Using pairwise voting,
try to find a winner.

After trying this example,
you should have found
that this scenario is
problematic. More on
the next slide...



- Racquel suggests that because the last 2 dinners have been held at Mexican and Chinese restaurants, this year's dinner should be at either an Italian or French restaurant. They vote 7 to 3 in favor of the Italian restaurant.



- Martin doesn't like Italian food and says that the new Mexican restaurant is really good. He proposes that the group choose between Italian and Mexican. They voted 7 to 3 to hold the dinner at the Mexican restaurant.

This is an example of Pairwise Voting and Mr. Arrow considers this group ranking method to be flawed.

- Sarah's parents own a Chinese restaurant and say that she can get a group discount. The group votes between the Mexican and Chinese restaurant and selects the Chinese restaurant by a 6 to 4 margin.

* If we look back at their original preferences, we see that French food was preferred to Chinese food in every case, yet they voted for Chinese food.

Example: Determine the winner by the Condorcet Method

| | Number of votes received | | | |
|-----------------|--------------------------|--------|--------|--------|
| Place | 390 | 360 | 300 | 450 |
| 1 st | Shawn | Gail | Gail | Ricco |
| 2 nd | Twanda | Twanda | Twanda | Twanda |
| 3 rd | Ricco | Ricco | Shawn | Shawn |
| 4 th | Gail | Shawn | Ricco | Gail |

Example ANSWER: Determine the winner by the Condorcet Method

| | Number of votes received | | | |
|-----------------|--------------------------|--------|--------|--------|
| Place | 390 | 360 | 300 | 450 |
| 1 st | Shawn | Gail | Gail | Ricco |
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Twanda



Example: Determine the winner by the Condorcet Method

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Arrow's 5 Conditions Necessary for a Fair Group Ranking Method



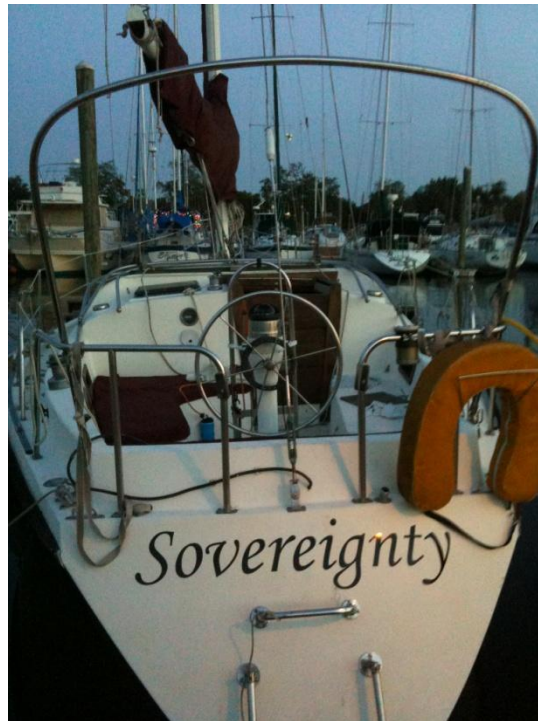
1. Non-Dictatorship

- The preference of a single individual should not become the group ranking without considering the preferences of others.



2. Individual Sovereignty

- Each individual should be allowed to order the choices in any way and to indicate ties.



3. Unanimity

- If everyone prefers one choice over another, then the group ranking should do the same.
- Example:
 - If every voter ranks candidate A higher than candidate B, then the final ranking should place candidate A higher than candidate B.



"Then we are agreed nine to one that we will say our previous vote was unanimous!"

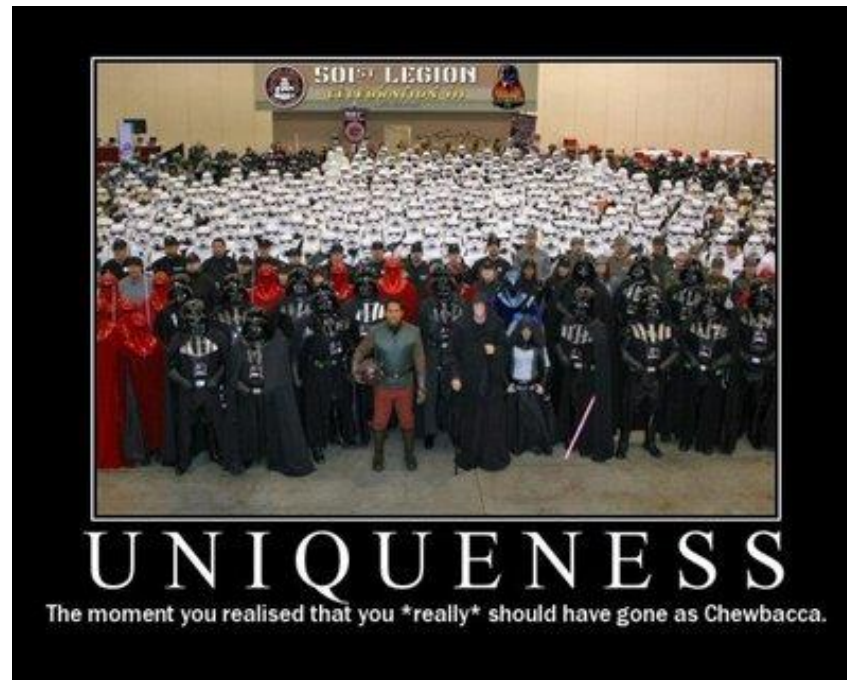
4. Freedom from Irrelevant Alternatives

- The winning choice should still win if one of the other choices is removed.
- The choice that is removed is known as an irrelevant alternative.



5. Uniqueness of the Group Ranking

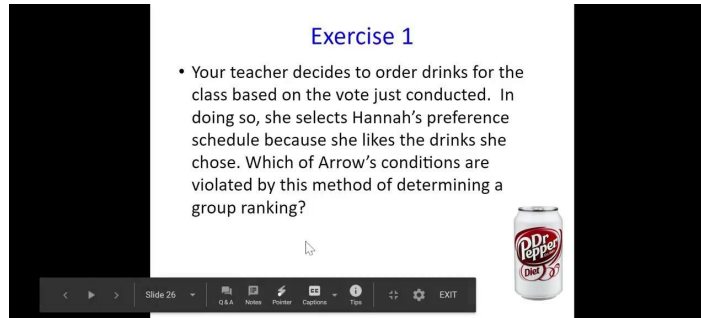
- The method of producing the group ranking should give the same result whenever it is applied to a given set of preferences.



Exercise 1

- Your teacher decides to order drinks for the class based on the vote just conducted. In doing so, she selects Hannah's preference schedule because she likes the drinks she chose. Which of Arrow's conditions are violated by this method of determining a group ranking?

Non-Dictatorship



Exercise 1

- Your teacher decides to order drinks for the class based on the vote just conducted. In doing so, she selects Hannah's preference schedule because she likes the drinks she chose. Which of Arrow's conditions are violated by this method of determining a group ranking?

Slide 26 | G.S.A. | Notes | Printer | Captions | Text | EXIT



Exercise 2

- Instead of selecting the preference schedule of a single student, your teacher places all of the individual preferences in a hat and draws one. If this method were repeated, would the same group ranking result? Which of Arrow's conditions does this violate?

Uniqueness of the Group Ranking



Exercise 2

- Instead of selecting the preference schedule of a single student, your teacher places all of the individual preferences in a hat and draws one. If this method were repeated, would the same group ranking result? Which of Arrow's conditions does this violate?



Exercise 3

- Do any of Arrow's conditions require that the voting mechanism include a secret ballot? Is a secret ballot desirable in all group ranking situations? Explain why or why not.



Exercise 3

- Do any of Arrow's condition's require that the voting mechanism include a secret ballot? Is a secret ballot desirable in all group ranking situations? Explain why or why not.

Approval Voting:

- Kenneth Arrow proved that **no method**, known or unknown, could always obey **all 5 conditions**.
(Any group-ranking method will violate at least one of Arrow's conditions in certain situations)
- Although a perfect group ranking will never be found, current methods can still be improved.
- A new system is called Approval Voting:

Approval Voting

**In Approval Voting,
you may vote for as many choices as you like, but
you do not rank them.**

You mark all those of which you approve.

**For example, if there are five choices, you may vote for
as few as none or as many as five.**

Advantages of Approval Voting?

- It gives voters more flexible options
- It reduces negative campaigning
- It increases voter turnout
- It give minority candidates their proper due

What are some disadvantages?

- Approval voting forces voters to cast equally weighted votes for candidates they approve of.
- Voting for your second choice candidate can in some cases lead to the defeat of your favorite candidate.

Approval Voting Practice

The participants in a summer school recreation program decided to vote on which activity they preferred, Running Track, Softball, Badminton, or Swimming. The winning activity was determined by Approval Voting.

The following summarizes the responses of the participants:

12 participants voted for Swimming and Badminton.

5 participants voted for Badminton, Running Track, and Softball.

10 participants voted for Running Track and Badminton.

13 participants voted for Softball and Badminton.

1. How many total votes did **Swimming** receive?
2. How many total votes did **Badminton** receive?
3. How many total votes did **Running Track** receive?
4. How many total votes did **Softball** receive?
5. Which activity is selected by the summer school participants using Approval Voting?

The screenshot shows a presentation slide with the following text and handwritten notes:

Approval Voting Practice
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10 participants voted for Running Track and Badminton.
13 participants voted for Softball and Badminton.

Handwritten notes on the slide include:
1. How many total votes did Swimming receive? 12
2. How many total votes did Badminton receive? 12 + 5 + 10 + 13 = 40
3. How many total votes did Running Track receive? 5 + 10 = 15
4. How many total votes did Softball receive? 5 + 13 = 18
5. Which activity is selected by the summer school participants using Approval Voting? Badminton: 40 votes

Approval Voting Practice **ANSWERS**

The participants in a summer school recreation program decided to vote on which activity they preferred, Running Track, Softball, Badminton, or Swimming. The winning activity was determined by Approval Voting.

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1. How many total votes did **Swimming** receive? **12**
2. How many total votes did **Badminton** receive? **40**
3. How many total votes did **Running Track** receive? **15**
4. How many total votes did **Softball** receive? **18**
5. Which activity is selected by the summer school participants using Approval Voting? **Badminton**

You Try! Frisbee Club members decided to let the participants vote on the color of the T-shirt, using Approval Voting. The possible colors are Steel Gray, Robin's Egg Blue, Eggshell, Candy Apple Red, and Sunflower Yellow.

Approval Voting Practice

Here is a summary of the results:

12 participants voted for Steel Gray.

7 participants voted for Steel Gray and Sunflower Yellow.

20 participants voted for Eggshell and Candy Apple Red.

18 participants voted for Robin's Egg Blue, Eggshell, and Candy Apple Red

23 participants voted for Sunflower Yellow and Robin's Egg Blue.

25 participants voted for Candy Apple Red.

Use Approval Voting to determine the color of the t-shirt.

You Try! ANSWERS Frisbee Club members decided to let the participants vote on the color of the T-shirt, using Approval Voting. The possible colors are Steel Gray, Robin's Egg Blue, Eggshell, Candy Apple Red, and Sunflower Yellow.

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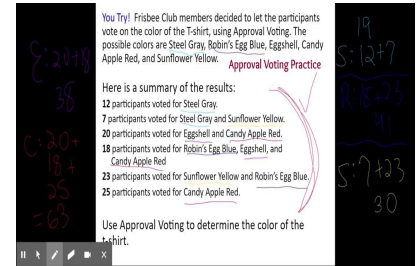
18 participants voted for Robin's Egg Blue, Eggshell, and Candy Apple Red

23 participants voted for Sunflower Yellow and Robin's Egg Blue.

25 participants voted for Candy Apple Red.

Use Approval Voting to determine the color of the t-shirt.

Candy Apple Red wins with 63 votes



Homework Day 2

- Packet p. 4