Unit 7 Day 6

Section 5.2
Traveling Salesman Problem
& Section 5.3
Finding the Shortest Route

Warm Up Day 7:

1. Draw a planar graph with 6 vertices.

2. Is a $K_{9,3}$ graph planar? Explain your reasoning.

3. Does a $K_{9,3}$ graph have an Euler circuit? Explain your reasoning.

4. What is the chromatic number of a K_{12} graph ?

5. Is a K_9 graph planar? Explain your reasoning.

6. Does a K_{13} graph have an Euler circuit? Explain your reasoning.

7. How many edges are in a $K_{6,4}$ graph ?

8. What is the chromatic number of a $K_{6,7}$ graph ?

Warm Up Day 7 ANSWERS:

1. Draw a planar graph with 6 vertices.

2. Is a $K_{9,3}$ graph planar? Explain your reasoning.

No. This graph will contain a $K_{3,3}$ as a subgraph. So, it is not planar.

3. Does a $K_{9,3}$ graph have an Euler circuit? Explain your reasoning.

No. The degrees are odd.

4. What is the chromatic number of a K_{12} graph ? 12

5. Is a K_9 graph planar? Explain your reasoning.

No. Any complete graph with more than 4 vertices will not be planar.

6. Does a K_{13} graph have an Euler circuit? Explain your reasoning.

Yes, it would have all even degrees.

7. How many edges are in a $K_{6,4}$ graph ? $6 \times 4 = 24$ edges

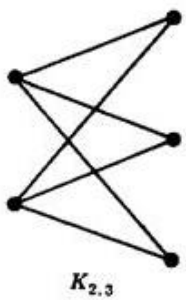
8. What is the chromatic number of a $K_{6,7}$ graph ? 2

Homework Answers

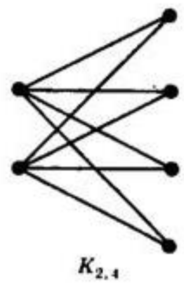
p. 11-12

1. Planar 2. Nonplanar 3. Planar

10. a.



b.



11. a. {A, B,C, D, E, F} {G} b. {M, S, Q} {N, R, P}
    c. {T, W, Y, Z} {U, V, X}

12. a. Yes, bipartite b. Not bipartite c. Yes, bipartite

14. $K_{2,3}$ = 6 edges, $K_{4,3}$ = 12 edges, $K_{m,n}$ = m x n edges

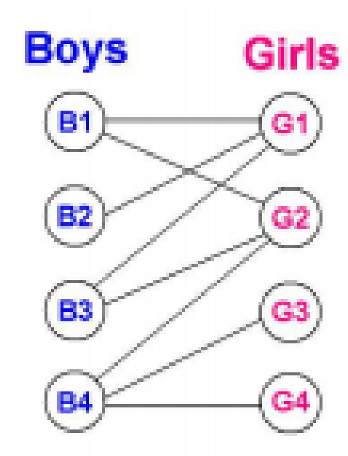
15. A bipartite graph $K_{m,n}$ have an Euler circuit when all of the vertices have even degrees.

Homework Answers

16. Chromatic number of a $K_{m,n}$ graph is 2 because it is a bipartite graph.

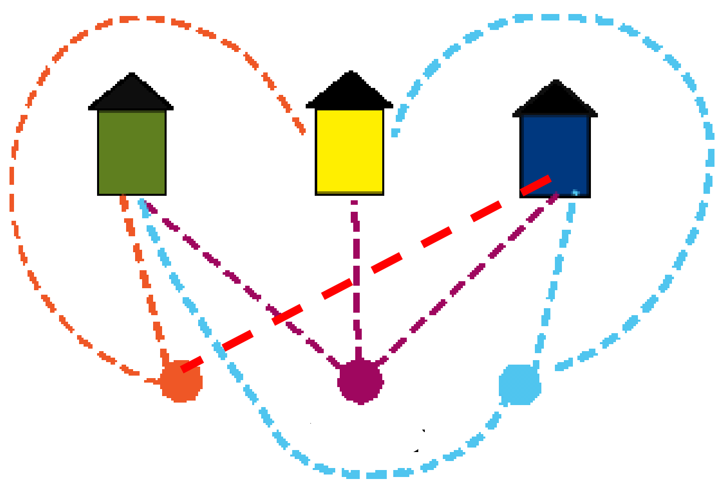
17. 30 handshakes. It is a bipartite $K_{5,6}$ graph.

18. Answers will vary. Example →



*4 girls have the choice of inviting 4 boys to a school dance. The boys (who are all gentlemen) accept all invitations.

19. Not possible.



For Tonight's HW

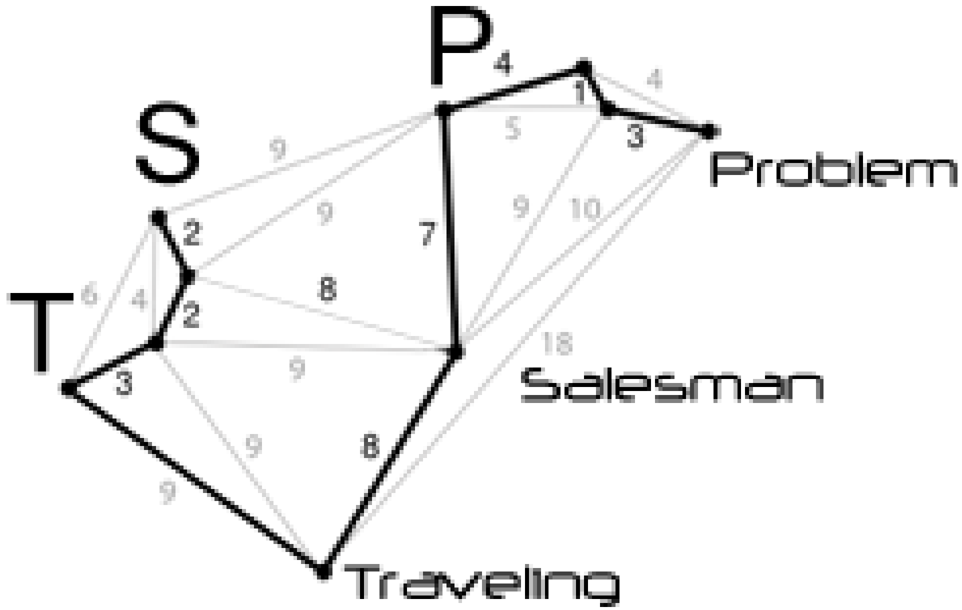
Packet p. 13-14

Some heads-up….

- #5 -> all you need to do is divide 9! by 1 million and do the same for each scenario

- #6 refers to the algorithm we discussed in class. Just describe your method of finding the shortest path by explaining each step.

Notes Section 5.2:

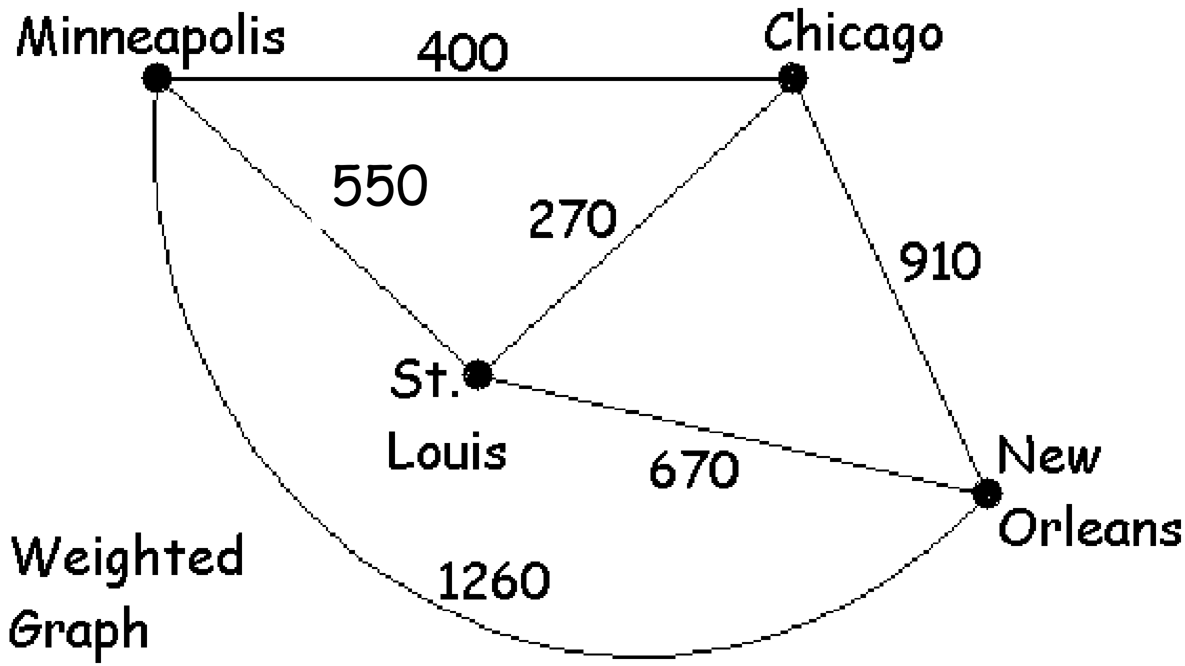


The Traveling Salesperson Problem (TSP) is a common scenario in the worlds of math and business.

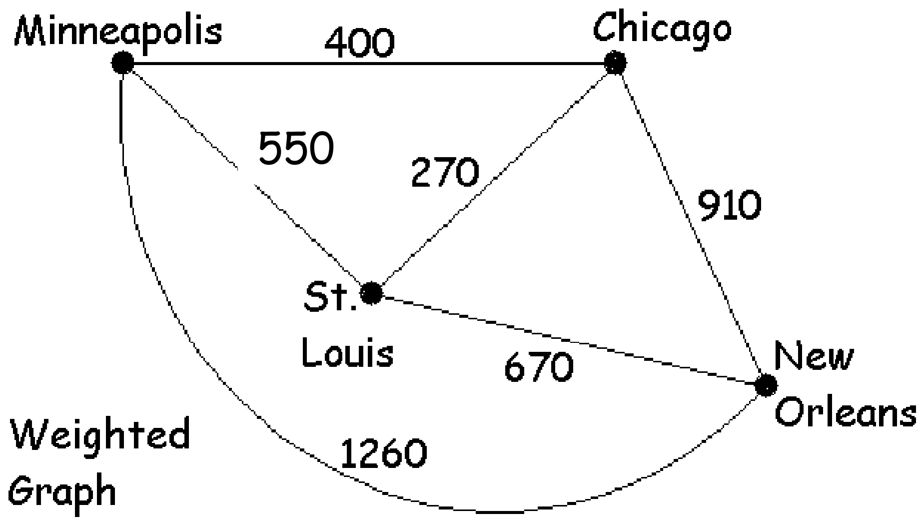
In business, it’s desirable to minimize things like costs, time, and distance.

TSP – Traveling Salesperson Problem

Below is a weighted graph (edges are assigned a value). Find the cheapest route from St. Louis to each of the other cities and back to St. Louis if the numbers (weights) on the edges of the graph represent the cost of making the trip between two cities.



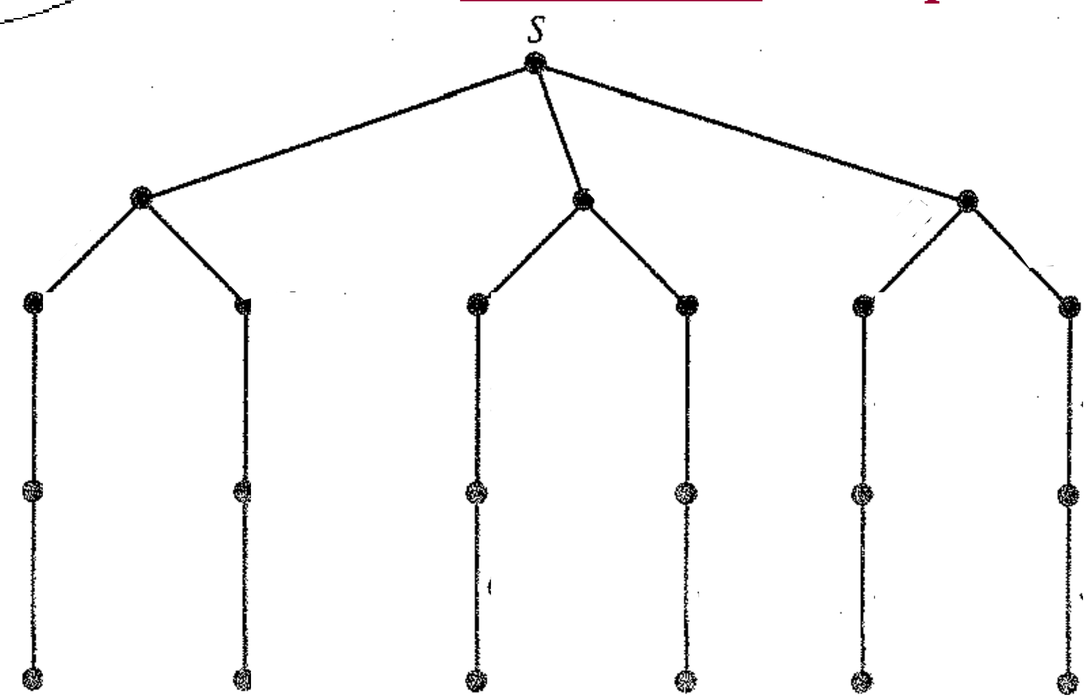
One way to find the minimum-cost route is to list every possible circuit along with its cost.



The Manual Tree Technique.

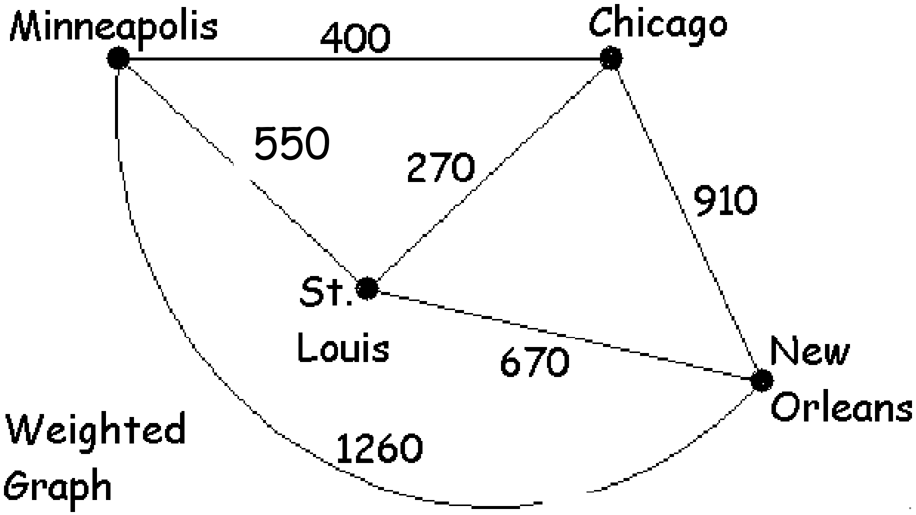
This is the Tree Technique. It shows ALL the possible paths, so you can "see" the shortest one.

Notice, this is a HAMILTONIAN CIRCUIT concept.



The minimum-cost route is ________ or its reverse order ________ .

This problem might get a little tough when more vertices exist.



The Manual Tree Technique.

This is the <u>Tree Technique</u>. It shows ALL the possible paths, so you can "see" the shortest one.

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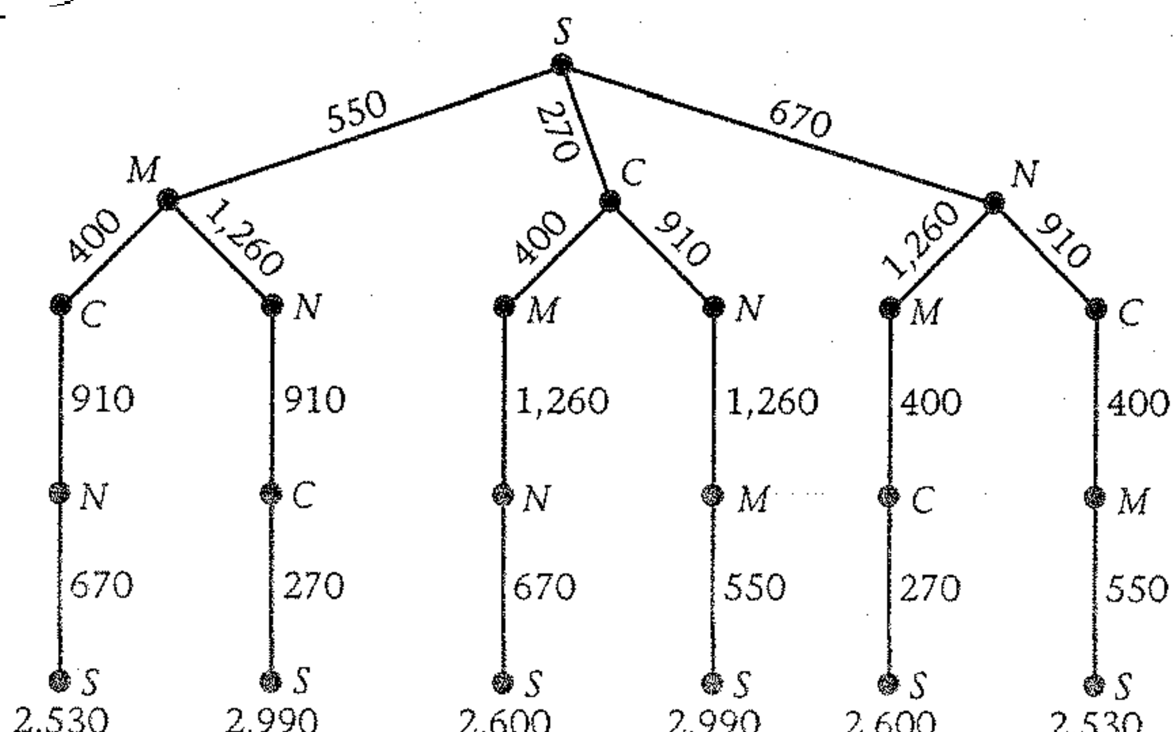
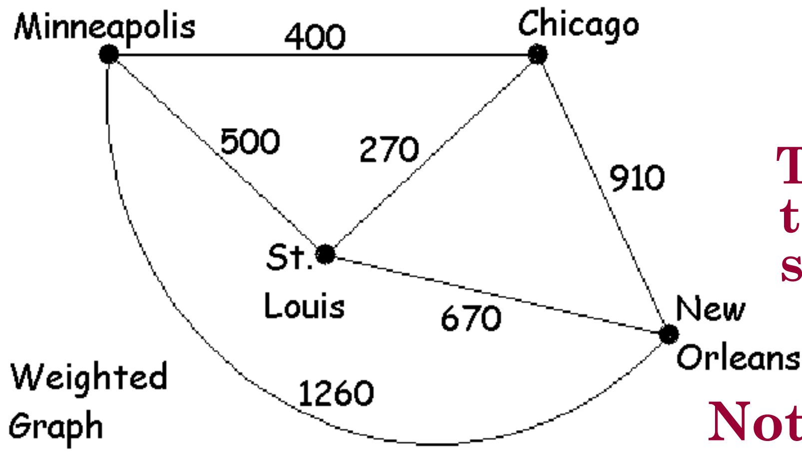


The minimum-cost route is SMCNS or its reverse order SNCMS .

This problem might get a little tough when more vertices exist.



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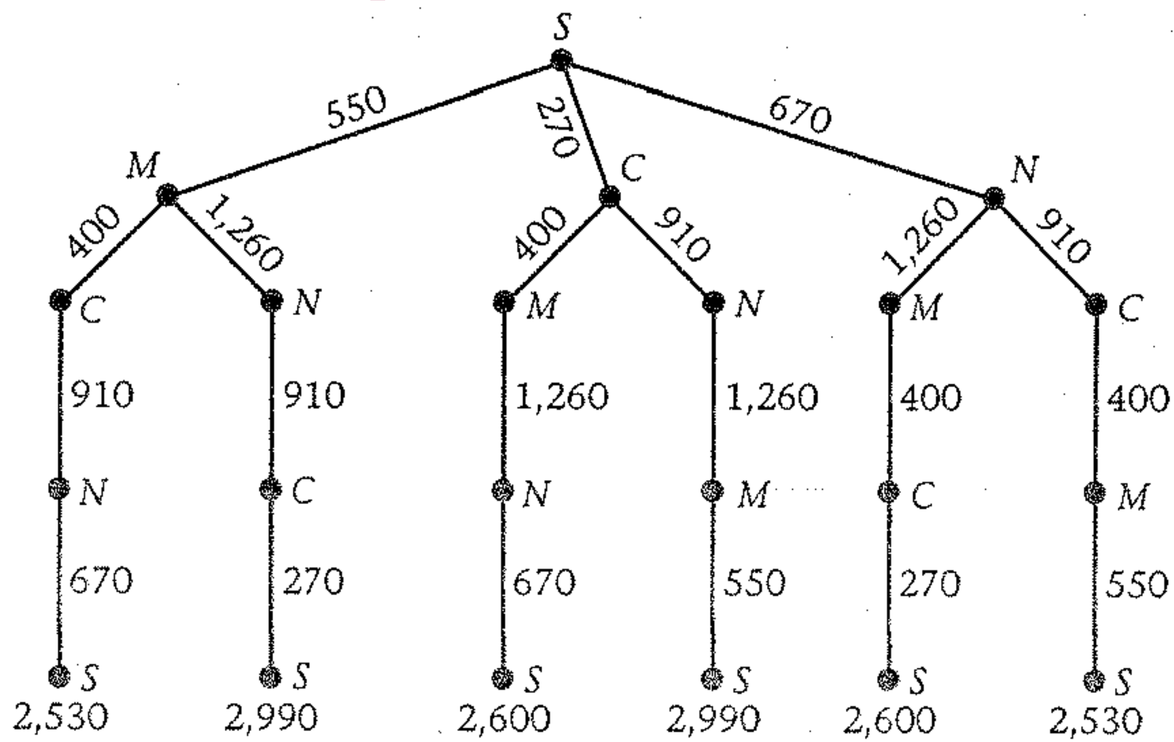
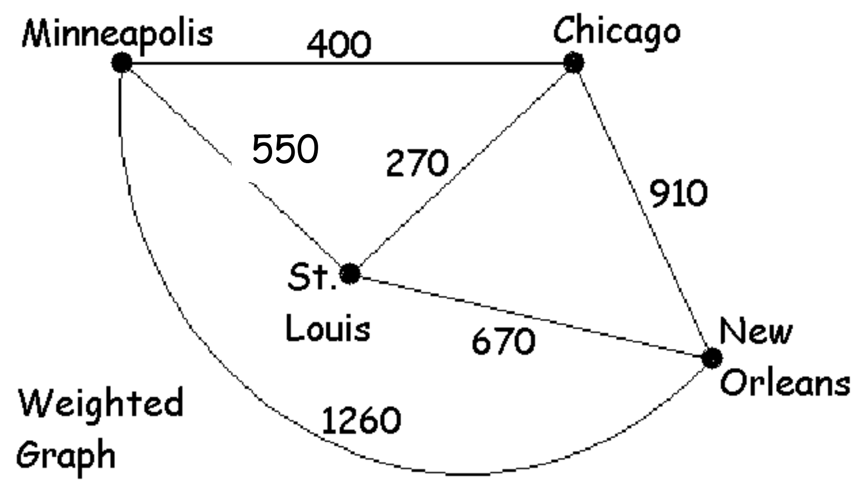


The minimum-cost route is SMCNS or its reverse order SNCMS .

This problem might get a little tough when more vertices exist.

More Efficient Algorithms

- Nearest-Neighbor Algorithm
  Beginning at St. Louis, move to the nearest neighboring vertex (Chicago,) then to the nearest vertex not yet visited, and return to St. Louis when all other cities have been visited.



- 270 + 400 + 1260 + 670 = 2600

This technique is faster but does not come up with the optimal answer. It's only a "close-to-optimal" answer.

A method such as this, which produces a quick and close-to-optimal solution, is known as a <u>HEURISTIC METHOD</u>.

TSP – Traveling Salesperson Problem

There is no best method for solving the TSP.

The choice of method becomes a trade-off.

Inspecting every possible path guarantees the optimal route, but becomes terribly difficult as the number of vertices increases.

The Nearest-Neighbor method is quick but does not always produce a good solution.

Mathematicians and scientists continue to discover more efficient algorithms.

Solutions to the TSP are of great interest to businesses because they translate into savings of millions of dollars for certain areas of the economy.

TSP – Traveling Salesperson Problem

Practical situations that could involve the TSP.

- A mailman's delivery route
- Automated mail delivery in an office
- Bank courier
- Robot moving in a warehouse
- Robot drilling holes in a circuit board
- and, of course, a salesman traveling to his clients

TSP Traveling Salesperson Problem
Practice with both TSP Methods

In Exercises 1 through 4,

a. Construct a tree diagram showing all possible circuits that begin at *S*, visit each vertex of the graph exactly once, and end at *S*.

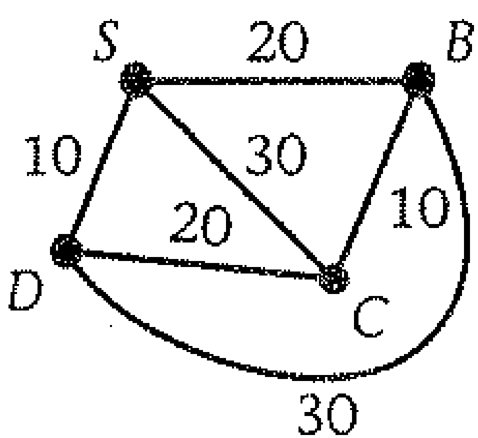
b. Find the total weight of each route.

c. Identify the shortest circuit.

d. Use the nearest-neighbor algorithm to find the shortest circuit.

e. Does the nearest-neighbor algorithm produce the optimal solution?

1.



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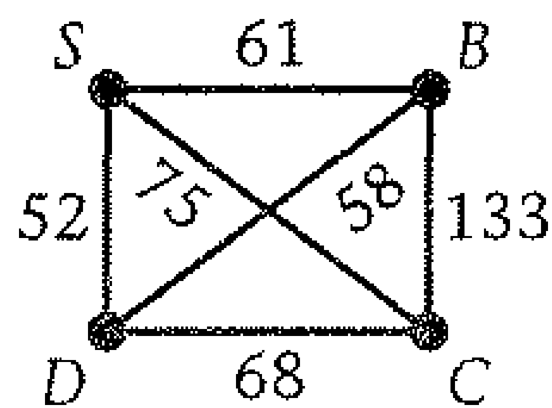
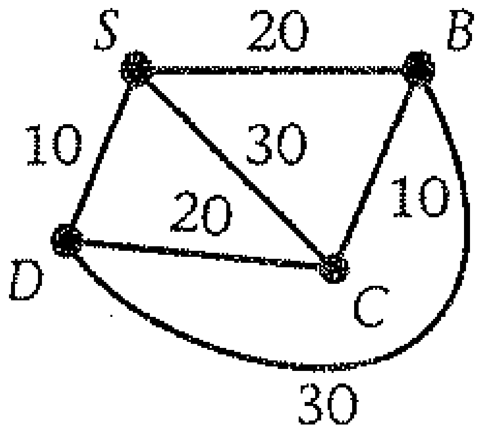
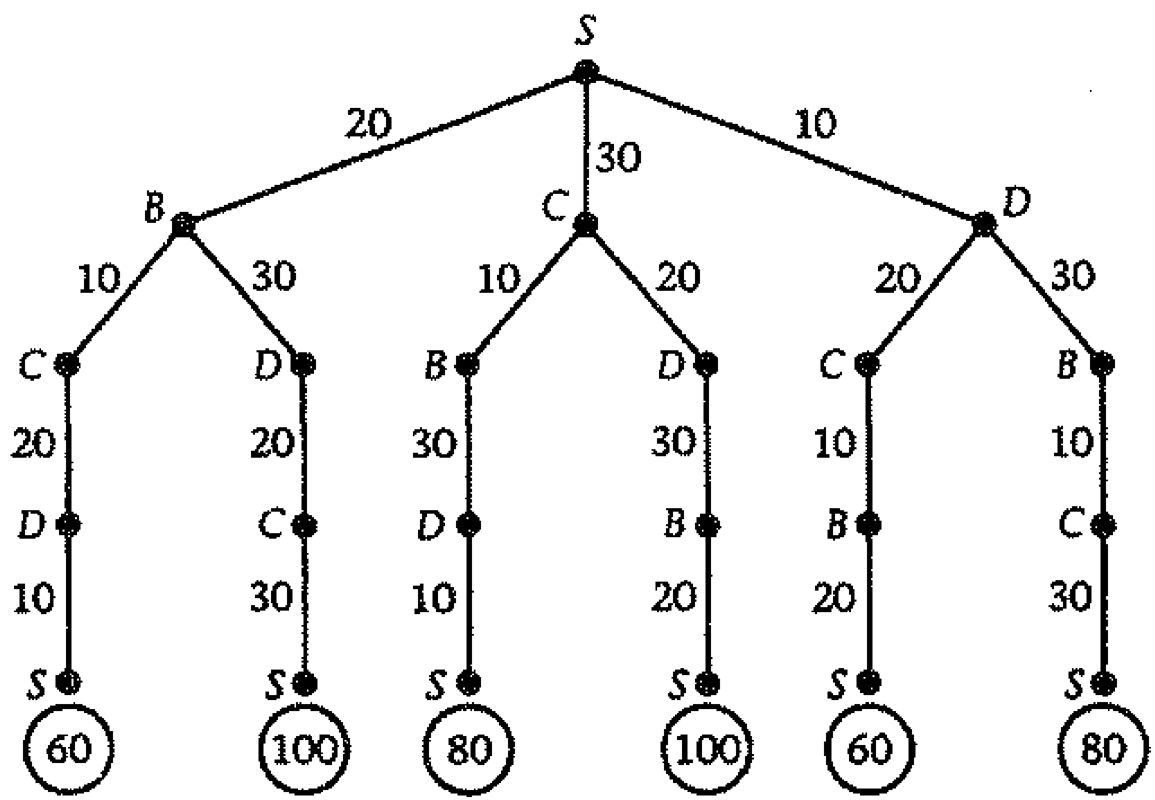


1.





c. SDCBS or SBCDS with a weight of 60

d. SDCBS - weight is 60.

e. Yes

In Exercises 1 through 4,

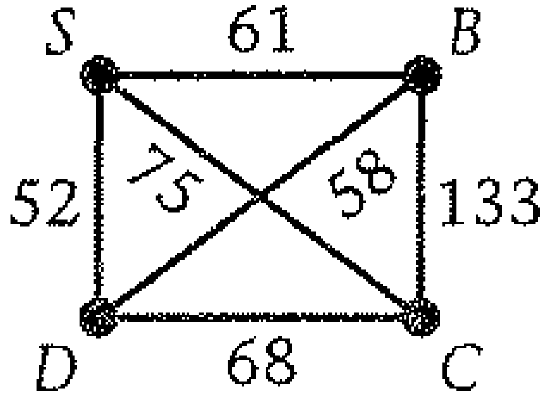
a. Construct a tree diagram showing all possible circuits that begin at *S*, visit each vertex of the graph exactly once, and end at *S*.

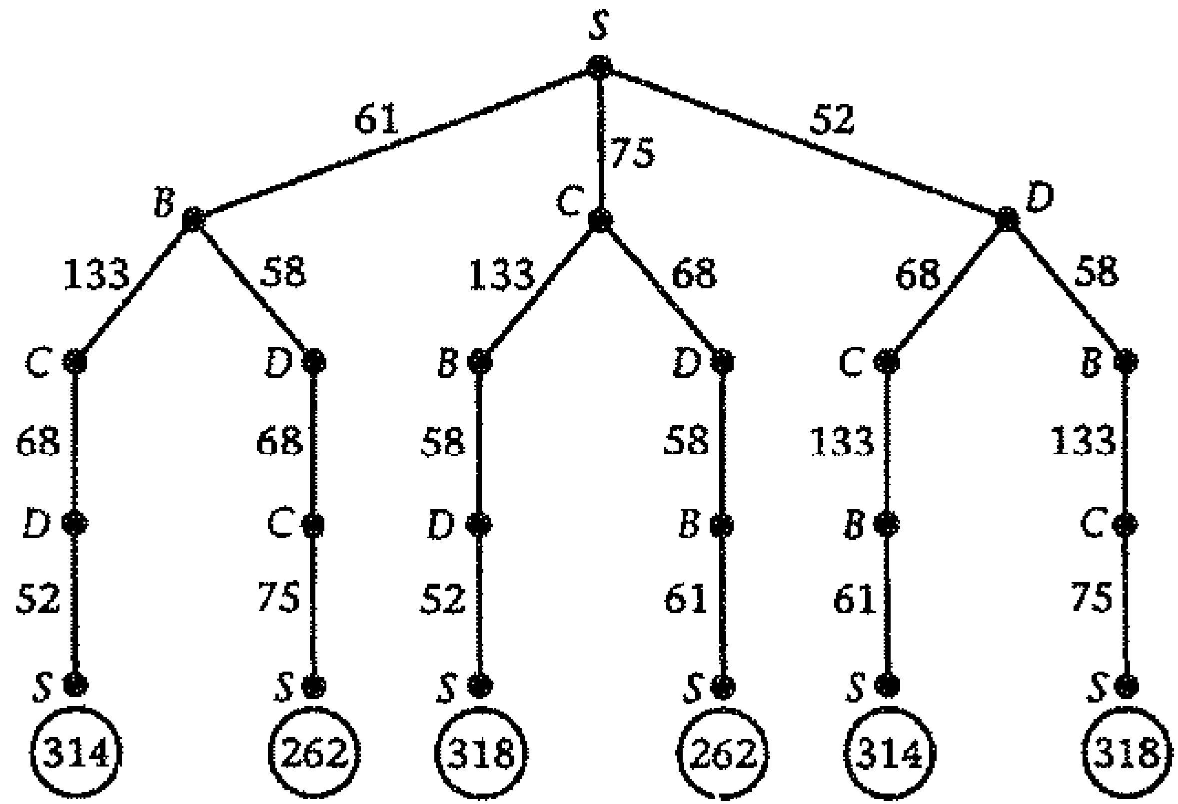
b. Find the total weight of each route.

c. Identify the shortest circuit.

d. Use the nearest-neighbor algorithm to find the shortest circuit.

e. Does the nearest-neighbor algorithm produce the optimal solution?

2. 



c. SBDCS. Or SCDBS with a weight of 262

d. SDBCS. Weight is 318.

e. No.

TSP Traveling Salesperson Problem

MORE Practice with both TSP Methods

In Exercises 1 through 4,

a. Construct a tree diagram showing all possible circuits that begin at *S*, visit each vertex of the graph exactly once, and end at *S*.

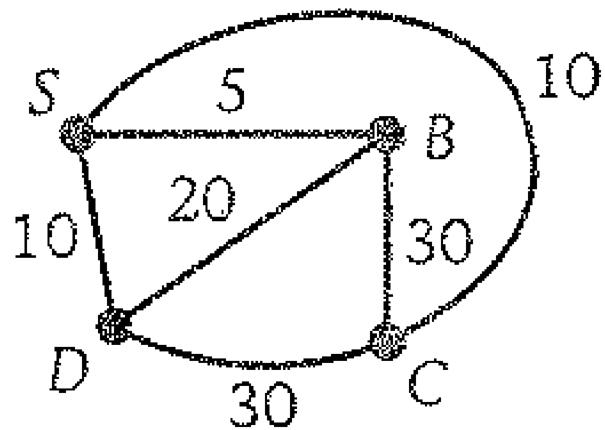
b. Find the total weight of each route.

c. Identify the shortest circuit.

d. Use the nearest-neighbor algorithm to find the shortest circuit.

e. Does the nearest-neighbor algorithm produce the optimal solution?

3.



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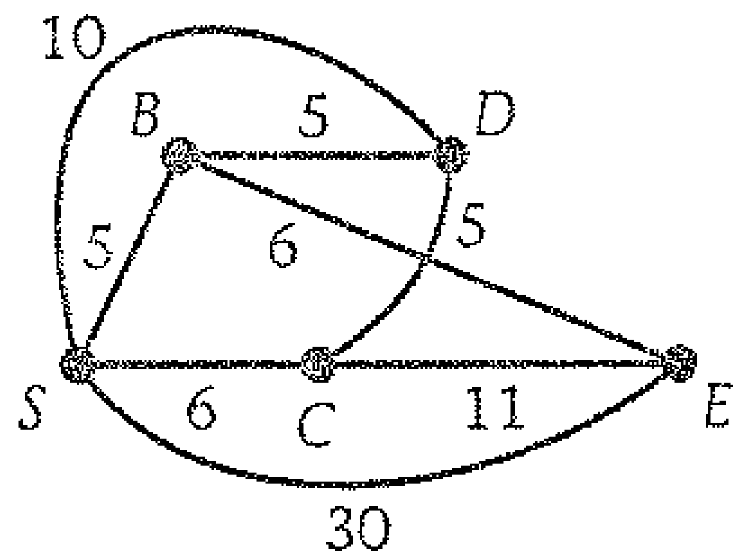

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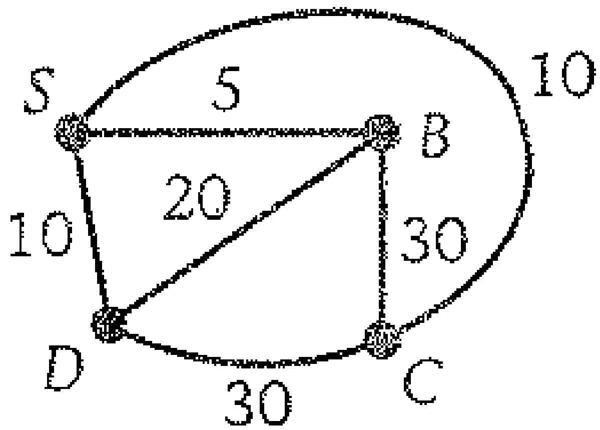

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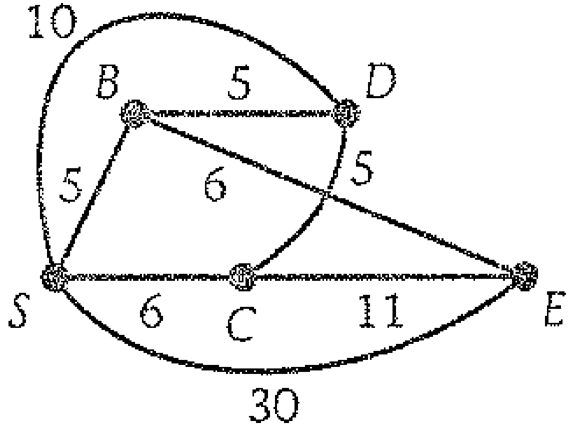


Notes Section 5.3
Finding the Shortest Route

Shortest Route

In this section, we are interested in finding the shortest route between two vertices. This is not a Hamiltonian path like the TSP. We don't need to travel to every vertex in the graph and return to the starting point. Instead, we only need to get from one vertex in the graph to another with minimal cost.

There is a specific method for this and it is attributed to E. W. Dijkstra, one of the original theorists in modern computer science.

Shortest Route – Dijkstra's Algorithm

- **Label the starting vertex S and circle it. Examine all edges that have S as an endpoint. Darken the edge with the shortest length and circle the vertex at the other endpoint of the darkened edge.**
- **Examine all un-circled vertices that are adjacent to the critical vertices in the graph.**
- **Using only circled vertices and darkened edges between the vertices that are circled, find the lengths of all paths from S to each vertex being examined. Choose the vertex and the edge that yield the shortest path. Circle this vertex and darken this edge. Ties are broken arbitrarily.**
- **Repeat steps 2 and 3 until all vertices are circled. The darkened edges of the graph from the shortest routes from S to every other vertex in the graph.**

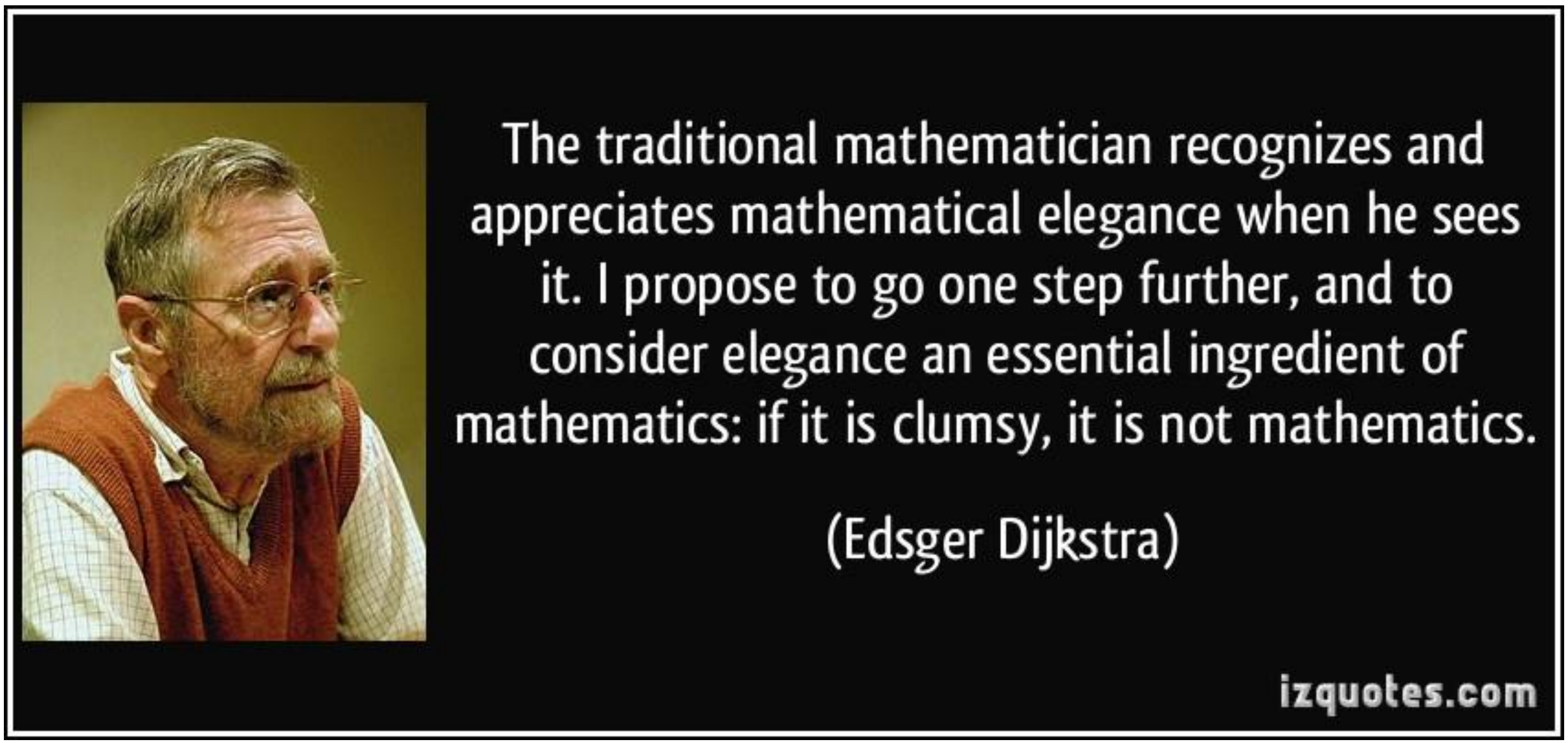
This can be a tricky algorithm, so we will go through two examples in detail.

This is a video from Patrick JMT of Dijkstra’s shortest route algorithm.

https://youtu.be/eFZCPlZCyIM

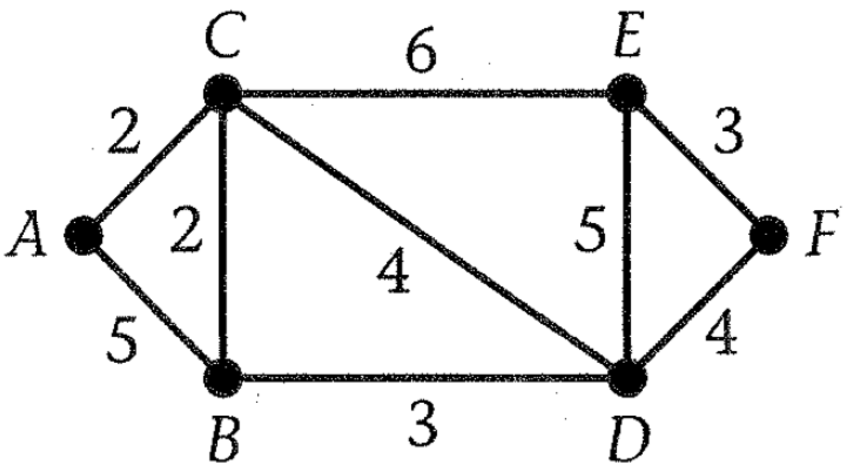
This is a nice online animation of Dijkstra’s shortest route algorithm.

http://optlab-server.sce.carleton.ca/POAnimations2007/DijkstrasAlgo.html



Shortest Route –

Find the shortest path and shortest time from *A* to *F* in the graph.



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- Repeat steps 2 and 3 until all vertices are circled. The darkened edges of the graph from the shortest routes from S to every other vertex in the graph.

Copy the graph into your notebook, follow along, and write down the steps as we go.

Shortest Route – ANSWERS

Find the shortest path and shortest time from *A* to *F* in the graph.

- Label the starting vertex S and circle it. Examine all edges that have S as an endpoint. Darken the edge with the shortest length and circle the vertex at the other endpoint of the darkened edge.
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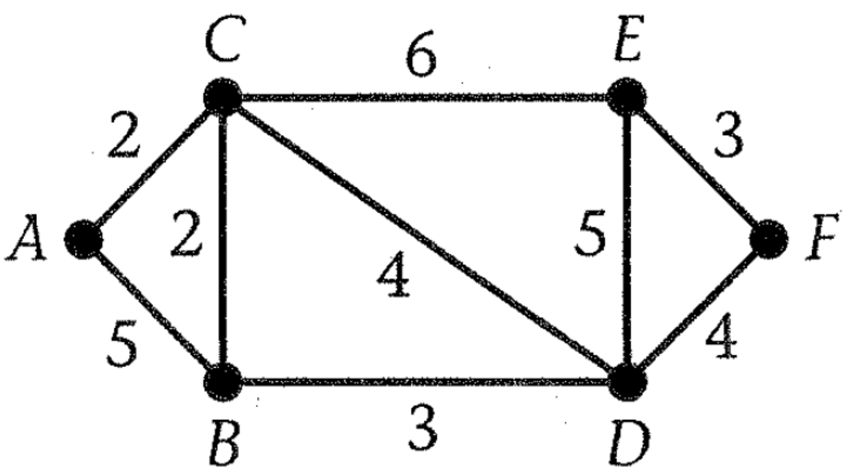

Copy the graph into your notebook, follow along, and write down the steps as we go.

Shortest Route: ACDF

Shortest time: 10

Shortest Route –

Find the shortest path from *A* to *G* in the graph.

Copy the graph into your notebook, follow along, and write down the steps as we go.

- Label the starting vertex S and circle it. Examine all edges that have S as an endpoint. Darken the edge with the shortest length and circle the vertex at the other endpoint of the darkened edge.
- Examine all un-circled vertices that are adjacent to the critical vertices in the graph.
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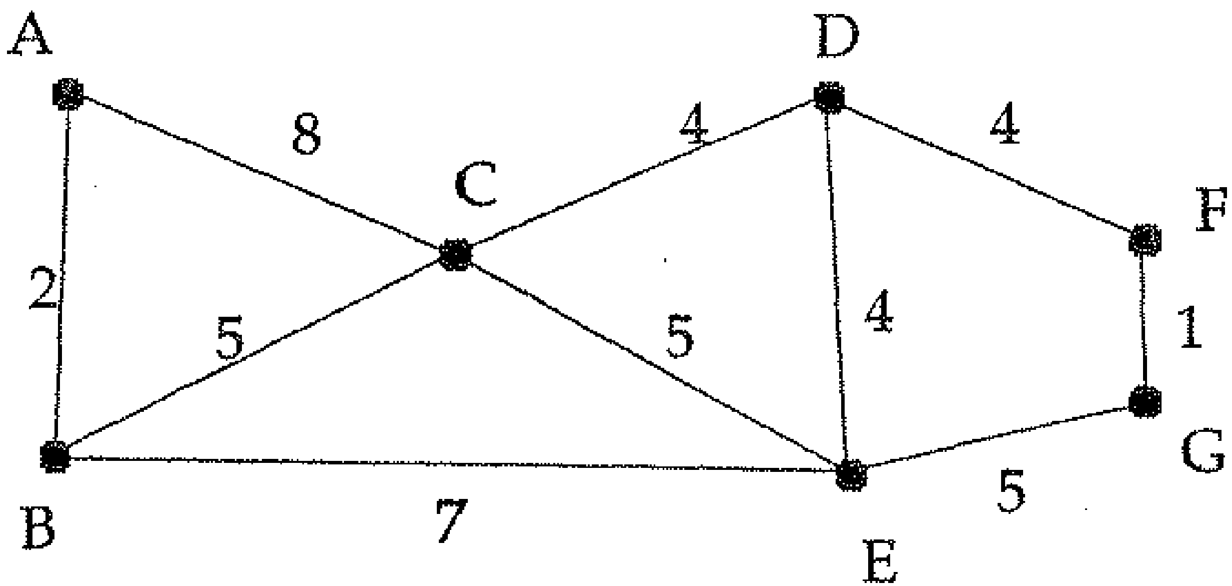
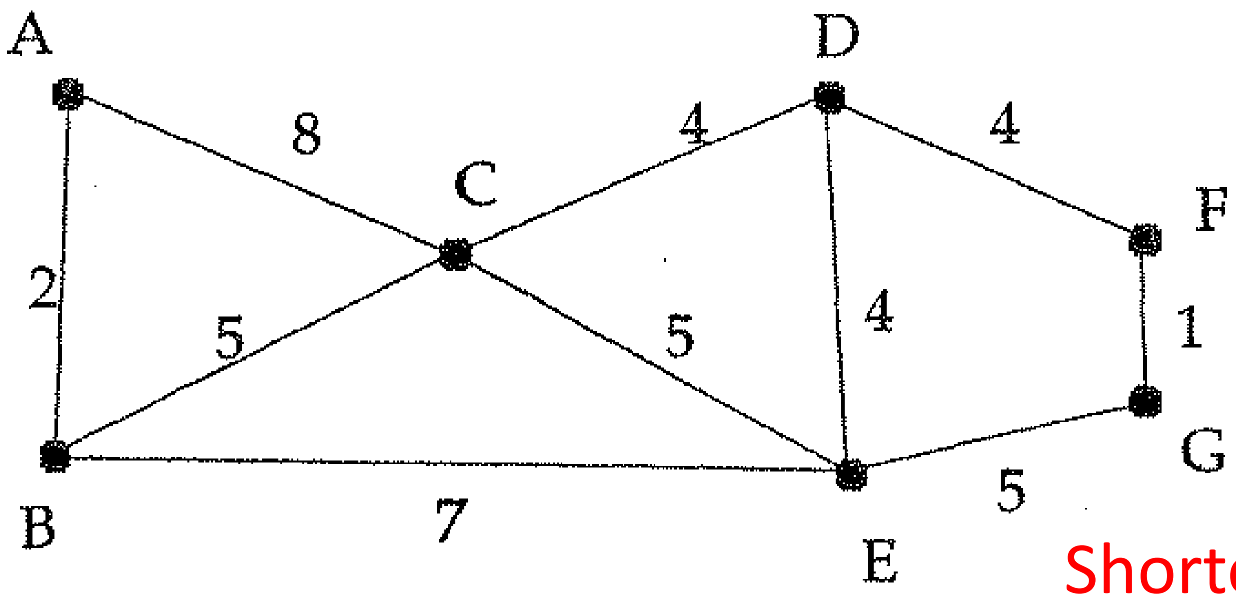


Shortest Route – ANSWERS

Find the shortest path from A to G in the graph.

Copy the graph into your notebook, follow along, and write down the steps as we go.

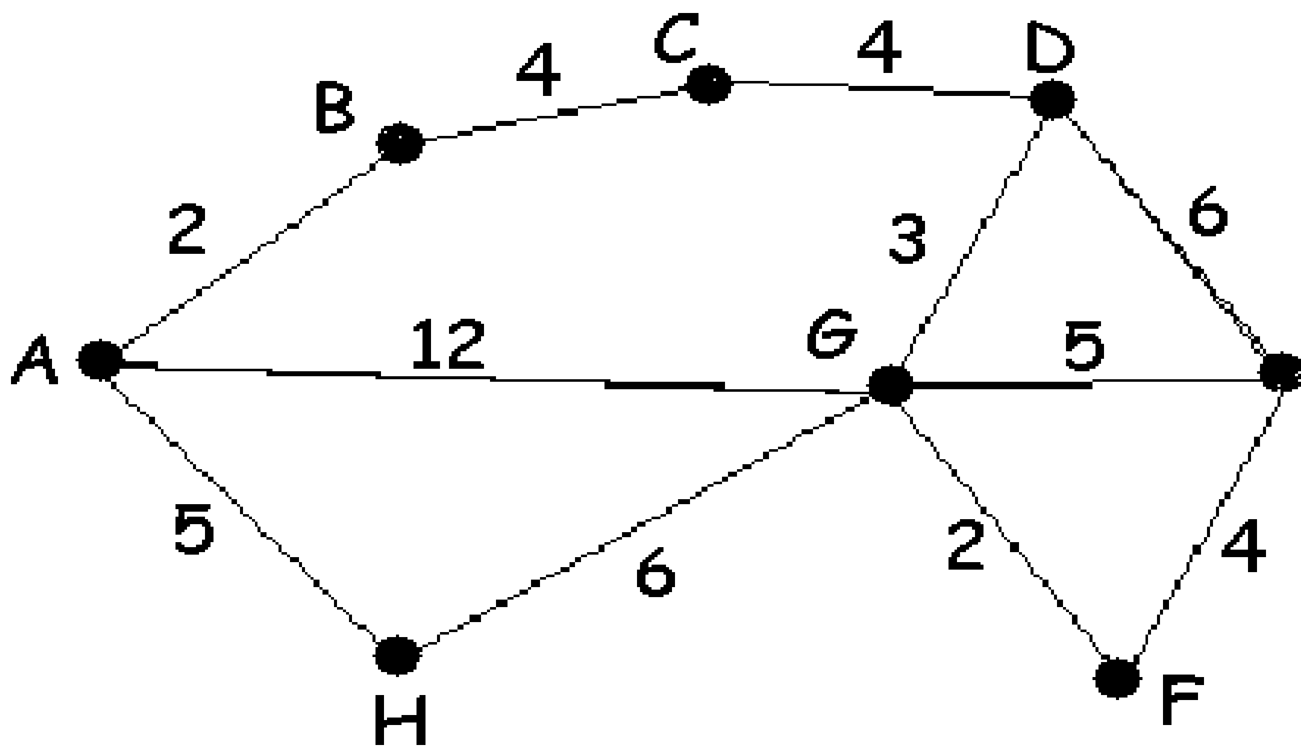
- Label the starting vertex S and circle it. Examine all edges that have S as an endpoint. Darken the edge with the shortest length and circle the vertex at the other endpoint of the darkened edge.
- Examine all un-circled vertices that are adjacent to the critical vertices in the graph.
- Using only circled vertices and darkened edges between the vertices that are circled find the lengths of all paths from S to each vertex being examined. Choose the vertex and the edge that yield the shortest path. Circle this vertex and darken this edge. Ties are broken arbitrarily.
- Repeat steps 2 and 3 until all vertices are circled. The darkened edges of the graph from the shortest routes from S to every other vertex in the graph.



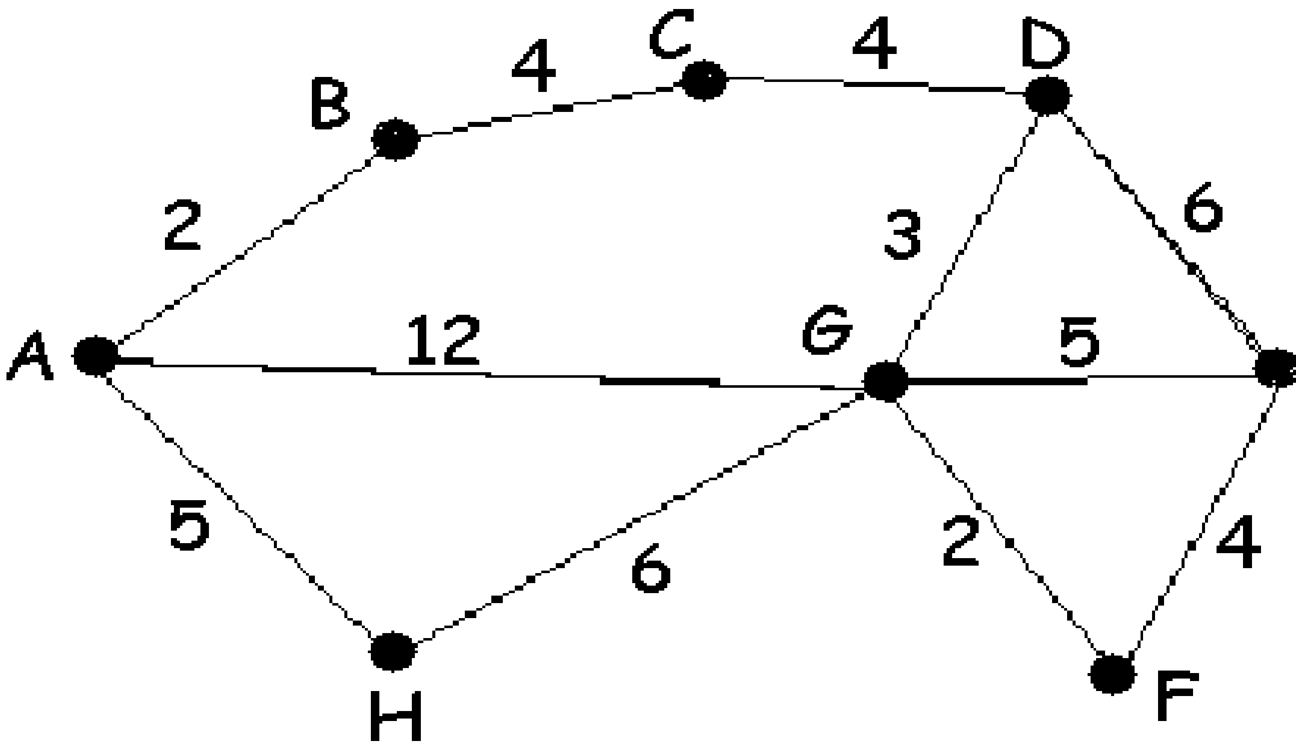
Shortest Route: ABEG

Shortest time: 14

Example #1
Find the shortest path from A to E.



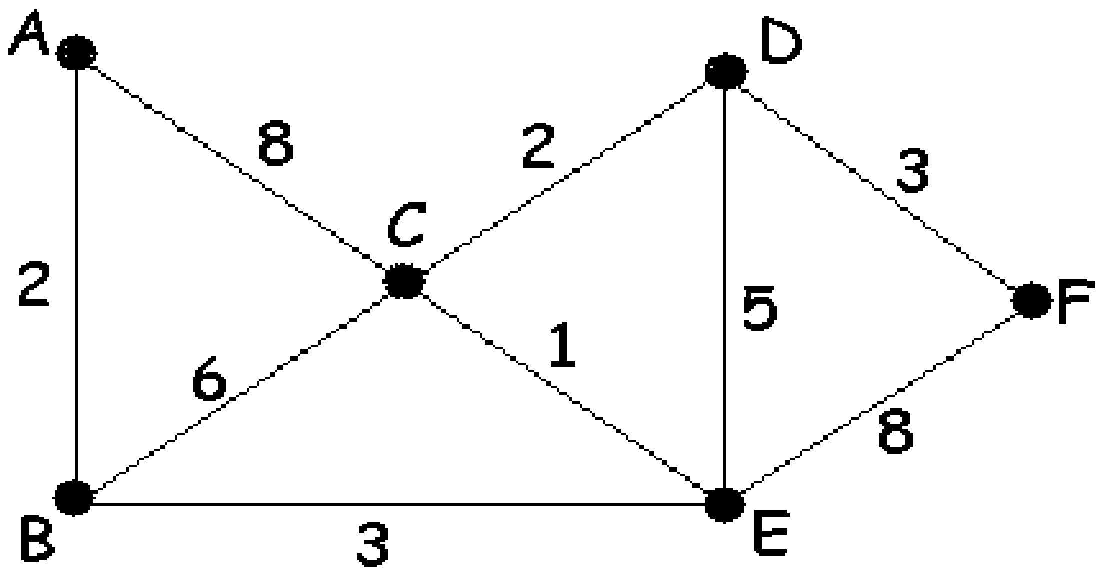
Example #1 ANSWERS
Find the shortest path from A to E.



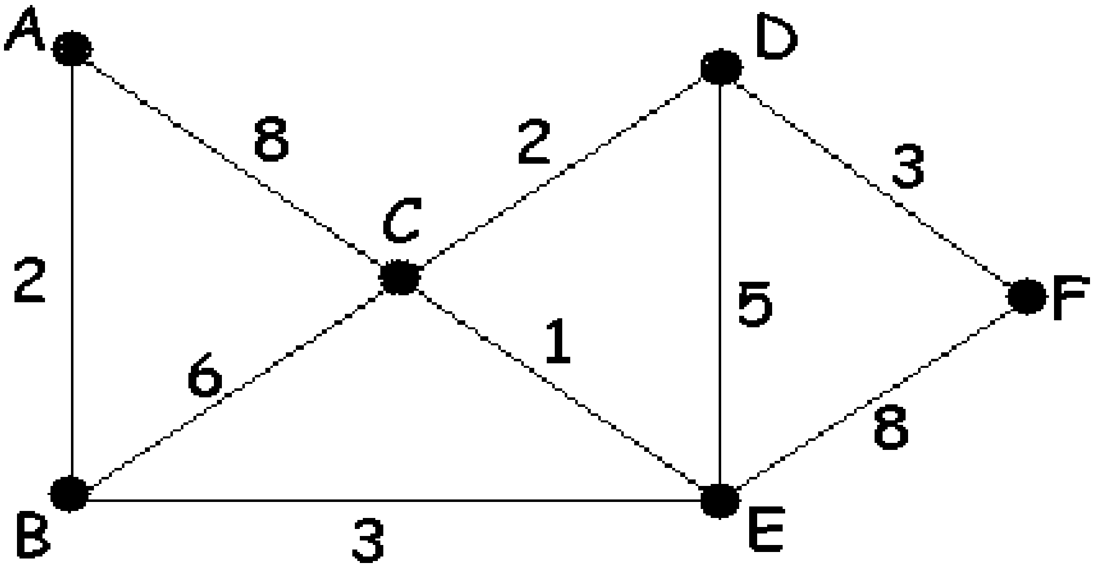
If there is a tie, arbitrarily choose a path!

Shortest Path: ABCDE or AHGE
Shortest Time: 16

Example #2
Find the shortest path from A to F.



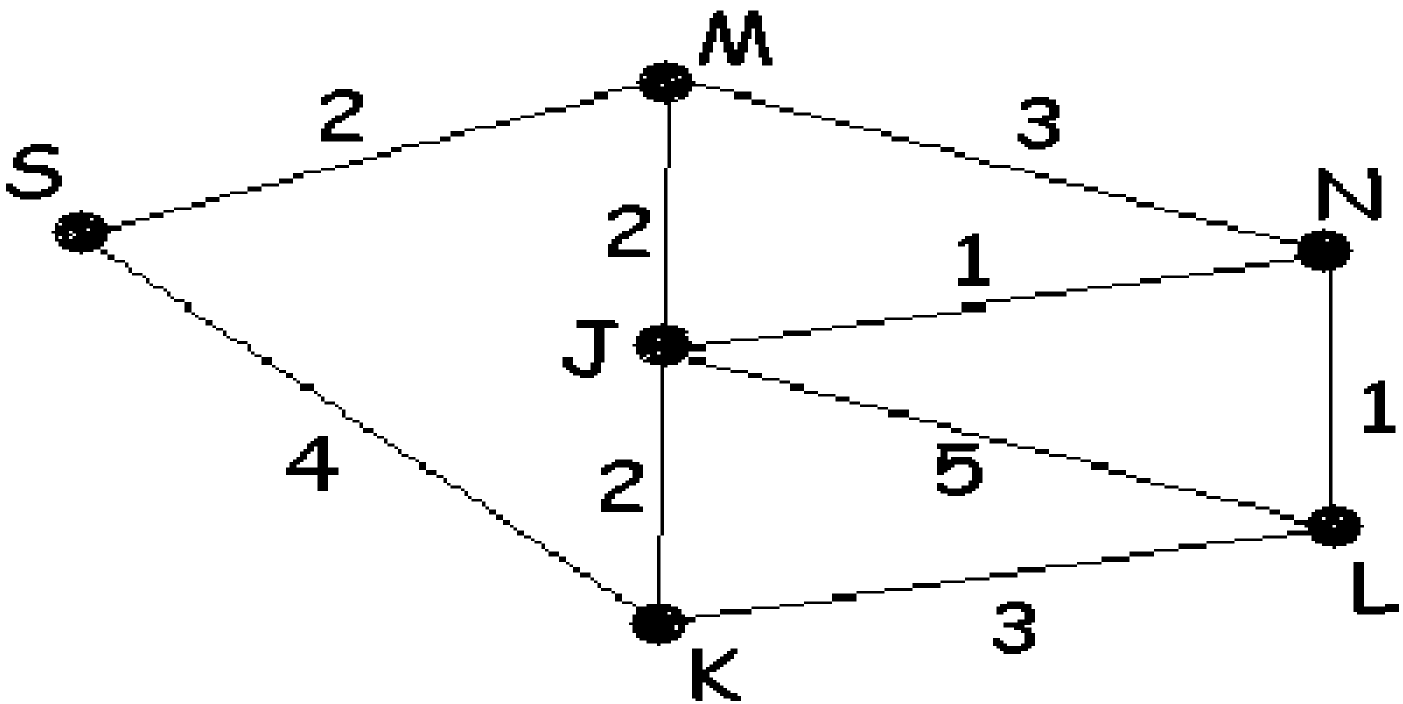
Example #2 ANSWERS
Find the shortest path from A to F.



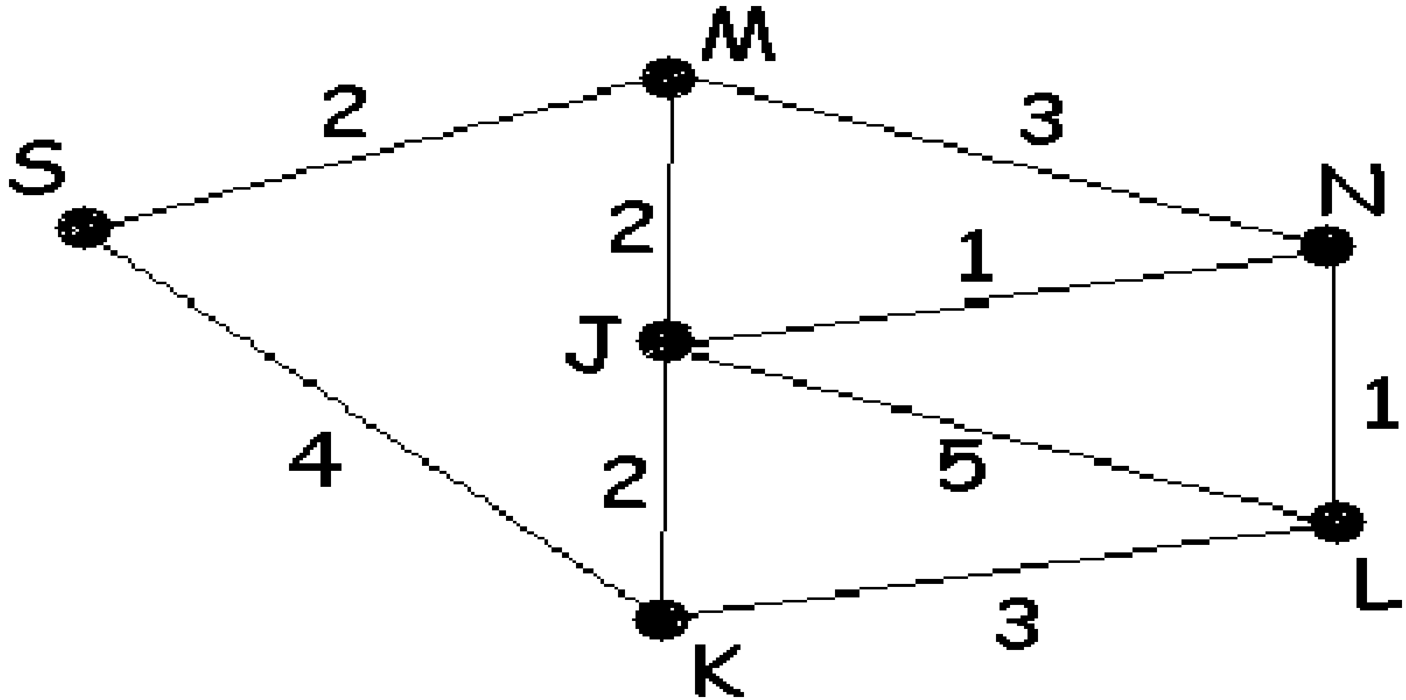
Shortest Path: ABECDF

Shortest Time: 11

Example #3
Find the shortest path from S to L.



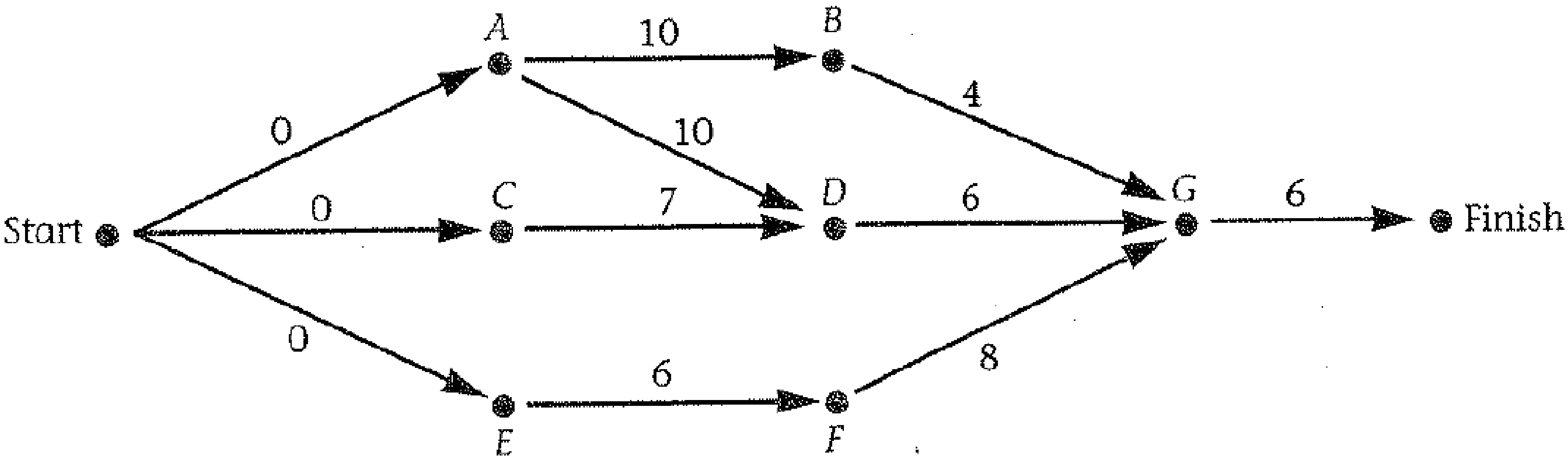
Example #3 ANSWERS
Find the shortest path from S to L.



Shortest Path: SMNL or SMJNL

Shortest Time: 6

REVIEW: And, finally, this:



a) Label each vertex with its earliest-start time.

b) What is the earliest completion time for this project?

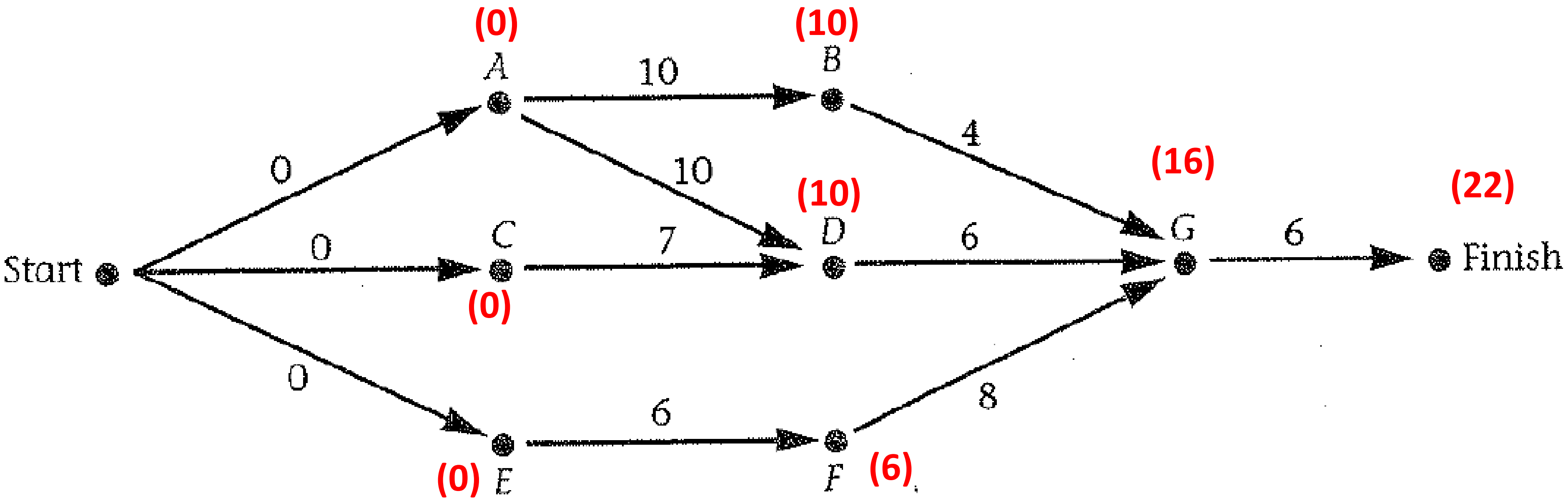
c) What is the critical path?

d) What happens to the minimum project time if task A's time is reduced to 9 days? To 8 days?

e) Will the project time continue to be affected by reducing the time of task A? Explain why or why not.

f) What is the Latest Start Time for D? B?

And, finally, this: ANSWERS



a) Label each vertex with its earliest-start time.

b) What is the earliest completion time for this project? 22

c) What is the critical path? Start ADG Finish

d) What happens to the minimum project time if task A's time is reduced to 9 days? To 8 days?
21 20

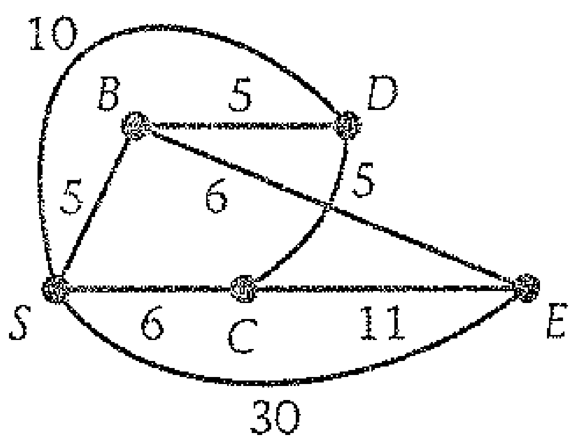
e) Will the project time continue to be affected by reducing the time of task A? Explain why or why not.
Yes, the critical path will be affected.

f) What is the Latest Start Time for D? B?

10, 12

Extra Practice

a. Construct a tree diagram showing all possible circuits that begin at S, visit each vertex of the graph exactly once, and end at S.
b. Find the total weight of each route.
c. Identify the shortest circuit.
d. Use the nearest-neighbor algorithm to find the shortest circuit.
e. Does the nearest-neighbor algorithm produce the optimal solution?



Extra Practice ANSWERS

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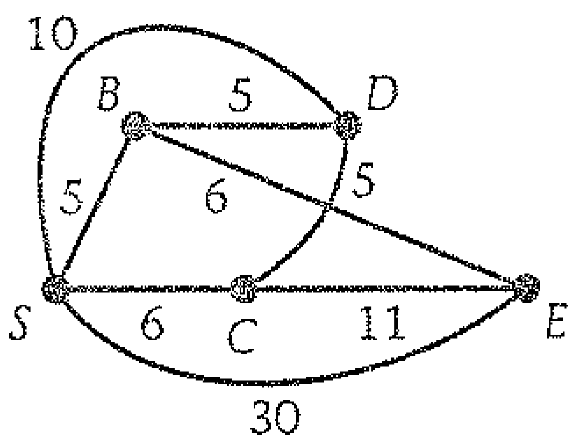

ANSWERS

b. SBDCES = 56 SDBECS = 38
SBECDS = 37 SDCEBS = 37
SCEBDS = 38 SECDBS = 56
SCDBES = 52 SEBDCS = 52

c. SBECDS, SDCEBS = 37
d. SBDCES = 56
e. No, not an optimal solution.

Homework Day 6

Packet p. 13-14

Some heads-up….

- #5 -> all you need to do is divide 9! by 1 million and do the same for each scenario

- #6 refers to the algorithm we discussed in class. Just describe your method of finding the shortest path by explaining each step.