Unit 7 Day 4
Notes: graph coloring,
Graph theory review
& Quiz
Warm-Up

• Phones OFF & in Blue Pockets!

• Get out paper for notes!

• Agenda
  – Notes first,
  – Then do practice and HW questions
  – Quiz at the end
Notes: Graph Coloring
Section 4.6
Explore This

Here is a table of clubs at Central High School and students who hold offices in these clubs.

<table>
<thead>
<tr>
<th></th>
<th>Math Club</th>
<th>Honor Club</th>
<th>Science Club</th>
<th>Art Club</th>
<th>Pep Club</th>
<th>Spanish Club</th>
</tr>
</thead>
<tbody>
<tr>
<td>Matt</td>
<td>x</td>
<td>x</td>
<td>x</td>
<td>—</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>Marty</td>
<td>x</td>
<td>—</td>
<td>—</td>
<td>x</td>
<td>x</td>
<td>—</td>
</tr>
<tr>
<td>Kim</td>
<td>—</td>
<td>x</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>x</td>
</tr>
<tr>
<td>Lois</td>
<td>x</td>
<td>—</td>
<td>x</td>
<td>—</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>Dot</td>
<td>x</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>x</td>
<td>—</td>
</tr>
</tbody>
</table>

Each club at Central High wants to meet once a week. Since several students hold offices in more than one organization, it is necessary to arrange the meeting days so that no students are scheduled for more than one meeting on the same day. Is it possible to create such a schedule? What is the minimum number of days needed?
1. Construct a graph in which the vertices represent the clubs and the edges represent CONFLICTS. Ex: Since Matt is in 3 clubs, those 3 clubs have conflicts with each other.

2. Then assign the vertices(clubs) days of the week to meet. Use a different color to represent different days.

3. What is the fewest number of days we can use without having a conflict? Remember, adjacent clubs must have different days, because they are in conflict.

<table>
<thead>
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<td>—</td>
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<td>—</td>
<td>X</td>
<td>—</td>
</tr>
</tbody>
</table>

Do this after the quiz!
Monday
Math
Honor
Science
Spanish
Art
Tuesday
Wednesday
Pep
Wednesday
Tuesday
Monday
Graph Coloring Example

Color the map using **four or fewer** colors. Each region must not touch the same color.

- Represent the map with a graph in which each vertex represents a region of the map.

- Draw edges between vertices if the regions on the map have a common border.

- Label the graph using a minimum number of colors. This number is the graph's **chromatic number.**
Practice

Find the chromatic number for each of the following graphs.

a. 4

b. 3

c. 2
Practice

2. A) Draw a graph that has four vertices and a chromatic number of 3.

B) Draw a graph that has four vertices and a chromatic number of 1.
Practice

3. As the number of vertices in a graph increases, a systematic method of labeling (coloring) the vertices becomes necessary. One way to do this is to create a coloring algorithm.

A) It is possible to begin the coloring process in several different ways, but one way is to color first the vertices with the conflict. How can the vertices be ranked from those with the most conflict to those with the least?

B) After having colored the vertex with the most conflict, which other vertices can receive that same color?

C) Which vertex would then get the second color? Which other vertices could get that same second color?

D) When would the coloring process be complete?

E) Refer back to parts A and D of this exercise and create an algorithm that colors a graph.

This is “Exercise 3” that your HW will refer to.
Up Next...Graph Theory Practice

• Review for Quiz 😊
Determine if the below exists. Write the path or circuit, if it exists, or explain why it doesn’t exist.

- Euler Path?
  Yes! ZPYZWYXWVX
- Euler Circuit?
  No, there is an Euler path and these events are mutually exclusive
- Hamiltonian Path?
  Yes! WVXYPZ
- Hamiltonian Circuit?
  Yes! WVXYPZW
Draw a directed graph to represent the tasks below...

<table>
<thead>
<tr>
<th>TASK</th>
<th>TIME (in days)</th>
<th>PREREQUISITE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Start</td>
<td>0</td>
<td>none</td>
</tr>
<tr>
<td>A</td>
<td>2</td>
<td>none</td>
</tr>
<tr>
<td>B</td>
<td>3</td>
<td>none</td>
</tr>
<tr>
<td>C</td>
<td>3</td>
<td>A,B</td>
</tr>
<tr>
<td>D</td>
<td>2</td>
<td>C</td>
</tr>
<tr>
<td>F</td>
<td>4</td>
<td>B,C</td>
</tr>
<tr>
<td>G</td>
<td>6</td>
<td>D,F</td>
</tr>
<tr>
<td>H</td>
<td>3</td>
<td>G</td>
</tr>
</tbody>
</table>

- Determine the Minimum Project Time. 19 days
- State the Critical Path. Start – BCFGH-Finish
- What is the Latest Start Time for D 8 days
Does this have exactly one Hamilton Path? Is this complete?

If so, rank the Teams.

1\textsuperscript{st}: NCSU
2\textsuperscript{nd}: App State
3\textsuperscript{rd}: Duke
4\textsuperscript{th}: UNC
Construct a graph for the adjacency matrix...

\[
A = \begin{bmatrix}
0 & 1 & 1 & 1 \\
1 & 0 & 1 & 0 \\
1 & 1 & 0 & 0 \\
1 & 0 & 0 & 0
\end{bmatrix}
\]

- What is the degree of vertex A? vertex C? \( \text{degA}=3, \text{degC}=2 \)
- Is the graph complete? Explain using def.
- Is the graph connected? Explain using def.

This graph is NOT complete since every vertex does not connect to each other vertex

This graph is connected since there is a path to all vertices.
Homework Day 4

Packet p. 10
Quiz Time