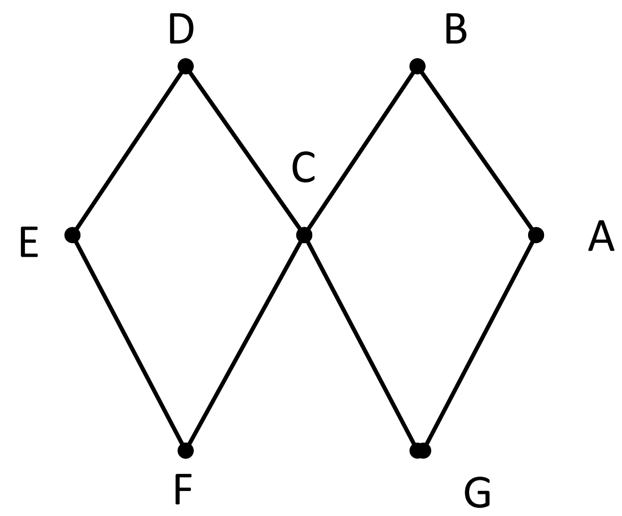
Unit 7 Day 4
Section 4.5
& Practice 4.1-4.5

Hamiltonian Circuits and Paths

Warm Up ~ Day 4

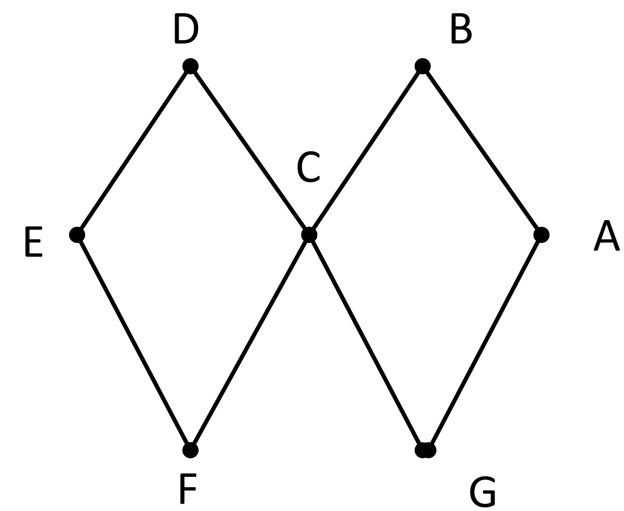


- **Is the following graph…**

1) **Connected?**
2) **Complete?**
3) **An Euler Circuit? If so, write the circuit. If not, explain why not.**
4) **An Euler Path? If so, write the path. If not, explain why not.**

5) **What is the degree of vertex C?**

Warm Up ~ Day 4
ANSWERS



- Is the following graph…

1) Connected? Yes!
2) Complete? No!
3) An Euler Circuit? If so, write the circuit. If not, explain why not. Yes! E,D,C,B,A,G,C,F,E
4) An Euler Path? If so, write the path. If not, explain why not. No, its an Euler circuit! They are mutually exclusive!
5) What is the degree of vertex C? deg(C)=4

Section 4.3 Answers

Packet p. 3

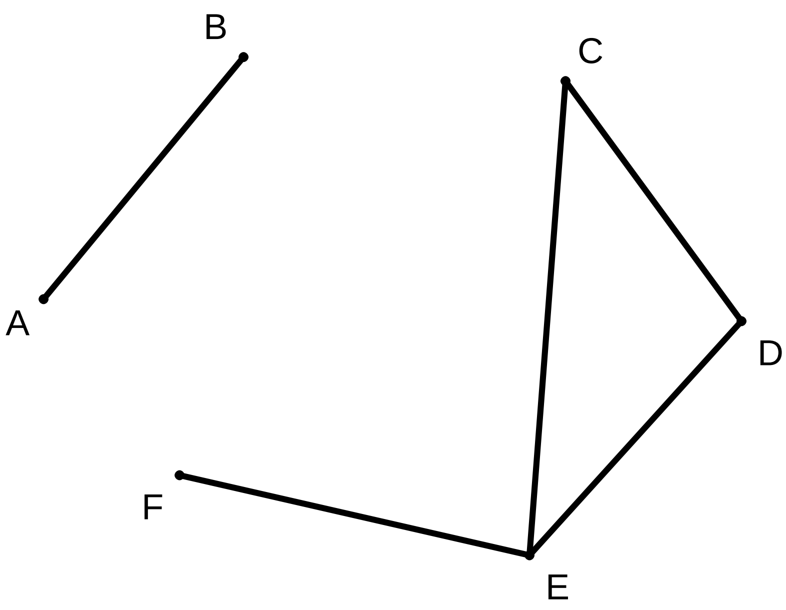
3. Draw a graph with vertices = {A, B, C, D, E, F} and edges = {AB, CD, DE, EC, EF}.

a. Name two vertices that are not adjacent.

b. F, E, C is one possible path from F to C. This path has a length of 2, since two edges were traveled to get from F to C. Name a path from F to C with a length of 3.

c. Is this graph connected? Explain why or why not?

d. Is this graph complete? Explain why or why not?



a. Sample Answer: F, D

b. F, E, D, C

c. No. There is no path from A to F

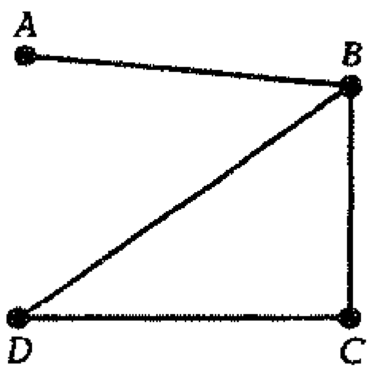
d. No. To be complete, every vertex must be connected to every other vertex.

Packet p. 3

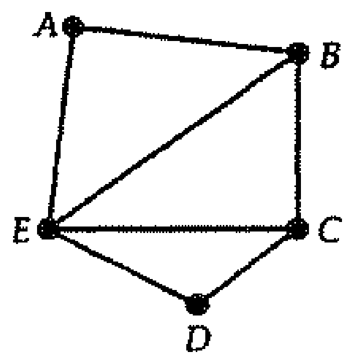
5. Construct a graph for each adjacency matrix. Label the vertices A, B, C, ...

a. $\begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix}$ b. $\begin{bmatrix} 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 0 \end{bmatrix}$ c. $\begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 \end{bmatrix}$

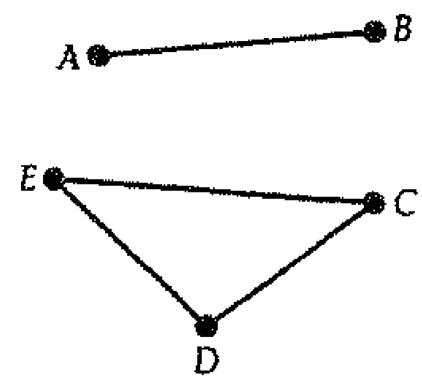
a.



b.



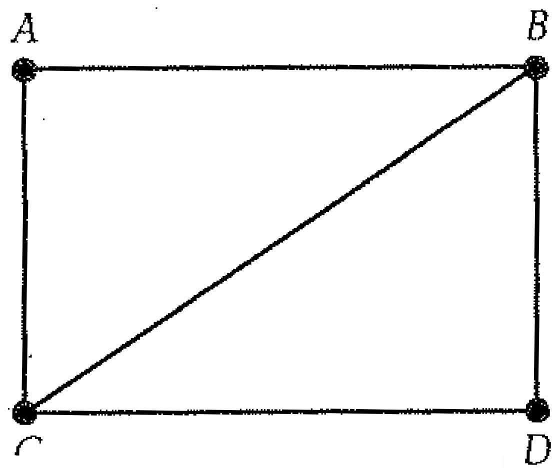
c.



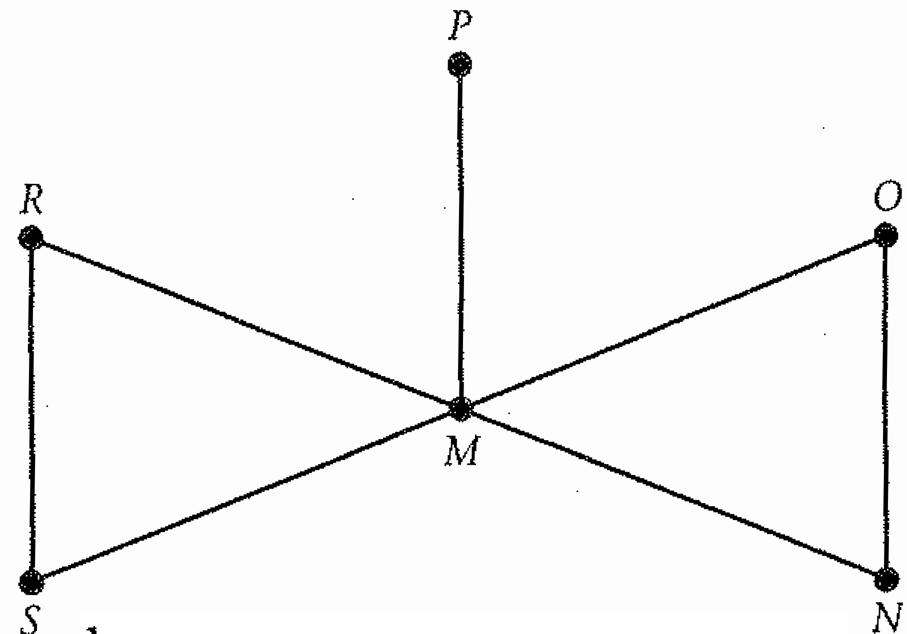
Packet p. 3

6. Create an adjacency matrix for each of the following graphs:

a.



b.



a.

$$\begin{array}{c} \\ A \\ B \\ C \\ D \end{array} \begin{array}{c} \begin{array}{cccc} A & B & C & D \end{array} \\ \begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix} \end{array}$$

b.

$$\begin{array}{c} \\ M \\ N \\ O \\ P \\ R \\ S \end{array} \begin{array}{c} \begin{array}{cccccc} M & N & O & P & R & S \end{array} \\ \begin{bmatrix} 0 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

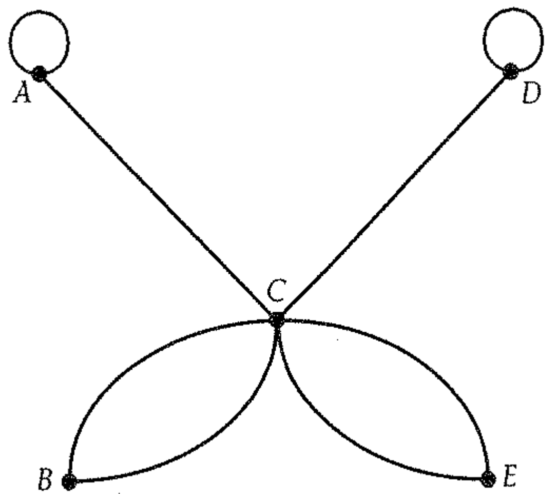
Packet p. 3-4

7. The number of vertices that are adjacent to a given vertex; the degree of each vertex.

8. deg(V)=3, deg(W)=4, deg(X)=2, deg(Y)=2, deg(Z)=1

Packet p. 4

9. An edge that connects a vertex to itself is called a **loop**. If a graph contains a loop or **multiple edges** (more than one edge between two vertices), the graph is known as a **multigraph**. When finding the degree of a vertex on which there is a loop, the loop is counted twice. For example, deg(*A*) = 3.



* b.

$$\begin{array}{c} \\ A \\ B \\ C \\ D \\ E \end{array} \begin{array}{c} \begin{array}{ccccc} A & B & C & D & E \end{array} \\ \begin{bmatrix} 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 \\ 1 & 2 & 0 & 1 & 2 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 2 & 0 & 0 \end{bmatrix} \end{array}$$

a. deg(B)=2, deg(C)=6, deg(D)=3, deg(E)=2

a. Find the degree of vertices *B*, *C*, *D*, and *E*.

* b. Give the adjacency matrix for the above multigraph.

c. Compare an adjacency matrix for a graph and one for a multigraph. Without seeing the graph, can you tell which belongs to the graph and which belongs to the multigraph? Explain how you know.

c. Yes, you can tell the difference. A graph will have 0s on the main diagonal and 1s and 0s elsewhere. A multigraph is characterized by numbers greater than 1 in the graph (indicating multiple edges) and/or numbers other than 0s on the main diagonal (indicating loops).

Packet p. 4-5

10. a) Euler Circuit – all even degree vertices
    b) Neither – more than 2 odd degree vertices
    c) Euler Path – 2 odd degree vertices, rest even
    d) Euler Circuit – all even degree vertices

11. Answers may vary – one possible:
    e,d,f,h,d,c,h,b,c,g,a,h,g,f,e

Packet p. 5

12. Only complete graphs with odd number of vertices will have Euler circuits: K_3 K_5 K_7 …K_{2n-1}

9. a) Yes b) No c) Yes

10. a) No, not the same number of indegrees as outdegrees.

b) Yes! b, e, f, g, c, b, a, c, d, f

Start at mismatched in and out degrees.

Tonight's Homework

- Finish Review Packet p. 6-8
- Packet p. 9

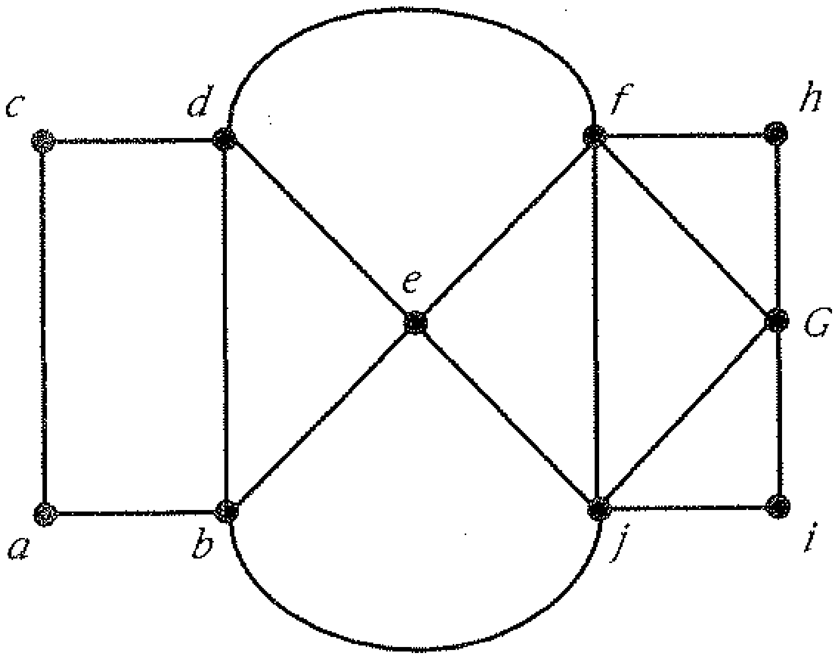
Arrival

- Get out Notebook Paper – we'll start with Notes
- Open Packet to p. 4 and 5 for HW Check later
- Update your Outline for Post Snow

Wed 12/12	Day 4	3 Hour Delay Sec 4.5: Hamiltonian Circuits & Paths Packet p. 6-8 = Practice 4.1-4.5	Packet p. 9 *Finish Practice p. 6-8*
Thurs 12/13	Day 5	**Quiz Sections 4.1-4.5** Sec 4.6: Graph Coloring	Packet p. 10
Fri 12/14	Day 6	Sec 5.1: Planarity & Coloring	Packet p. 11-12
Mon 12/17	Day 7	Sec 5.2: Traveling Salesman Problem Sec 5.3: Finding the Shortest Route	Packet p. 13-14
Tues 12/18	Day 8	Sec 5.4: Trees & Their Properties Sec 5.5: Spanning Trees	Packet p. 15-18
Wed 12/19	Day 9	Sec 5.6: Binary Trees, Expression Trees, & Traversals	Packet p. 19
Thurs 12/20	Day 10	Test Review + Half of Test (must finish this part that day)	Finish Review *STUDY FOR THE TEST*
Fri 12/21 Early Release	Day 11	**Unit 7 Test** other Half of Test	Enjoy your evening!

Notes Day 4
Hamiltonian Circuits and Paths
Section 4.5

The last section was about Euler Circuits that visit each <u>EDGE</u> only once. This section is about visiting each <u>VERTEX</u> only once.



Suppose you are a city inspector, but instead of inspecting the streets, you must inspect the fire hydrants at each street intersection.

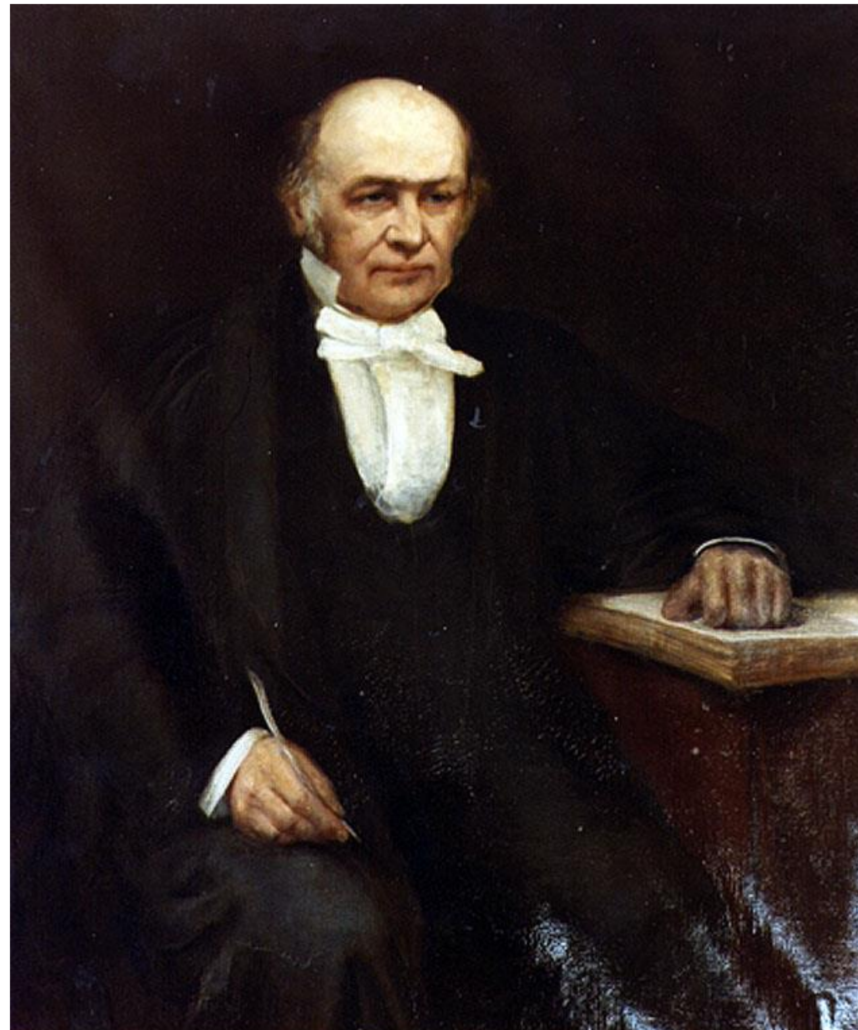
Can you start at the Garage (G), visit each intersection only once and return to the Garage?
Try it.

A graph that visits each <u>vertex only once</u> is known as a **HAMILTONIAN PATH**

If that Hamilton path can <u>end at the starting vertex,</u> it is called a **HAMILTONIAN CIRCUIT**

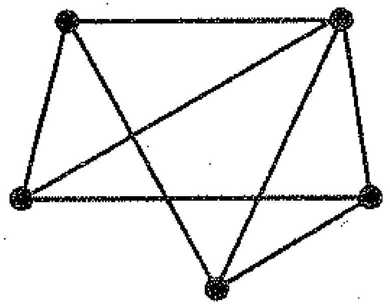
Sir William Rowan Hamilton (1805-1865)

- How old was he when he died?
- Irish mathematician, appointed astronomer and knighted at age 30.
- Born in Dublin, Ireland and the 4^{th} of 9 children!
- He carries the title of discovering Algebra.

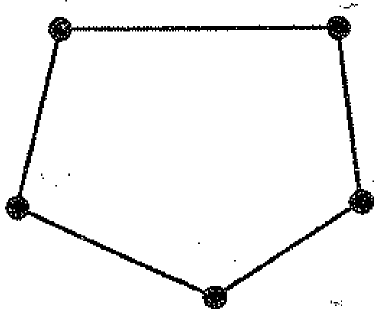


Hamiltonian Circuits & Paths

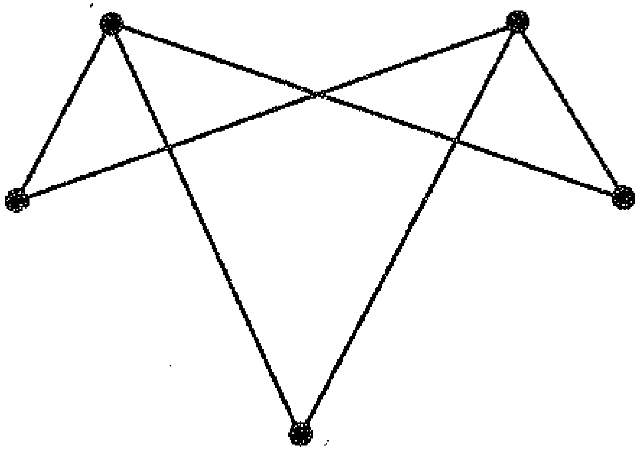
Try to find a Hamiltonian Circuit for each of the graphs.



a.



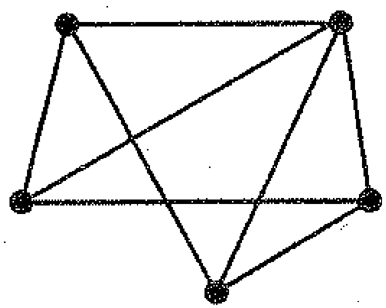
b.



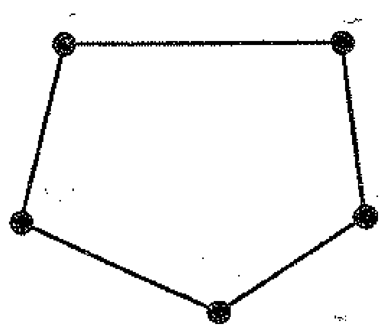
c.

Hamiltonian Circuits & Paths ANSWERS

Try to find a Hamiltonian Circuit for each of the graphs.



a.



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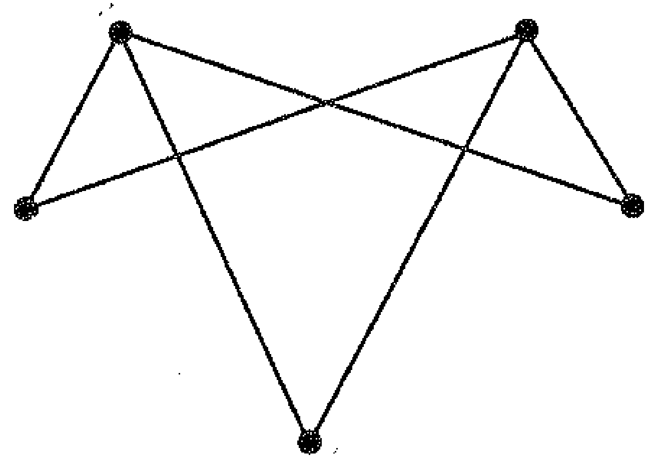


c.

Yes it does have a Hamilton Circuit

Yes, it has a Hamilton Circuit

Does NOT have a Hamilton Circuit

Hamiltonian Circuits & Paths

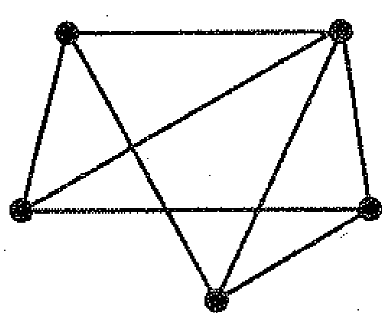
A simple test for determining whether a graph has a Hamiltonian circuit has not been found. It may be impossible.

There is a test to <u>guarantee the existence</u> of a Hamiltonian circuit. If the graph fails the test, however, there still may be a Hamiltonian circuit.

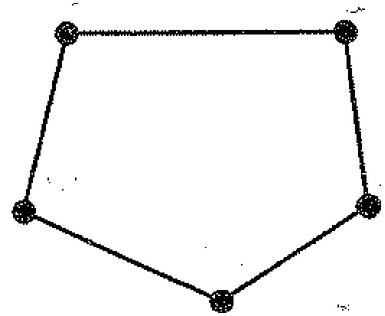
> If a connected graph has n vertices, where $n > 2$ and each vertex has degree of a least $n/2$, then the graph has a Hamiltonian circuit.

HW refers to this "theorem."

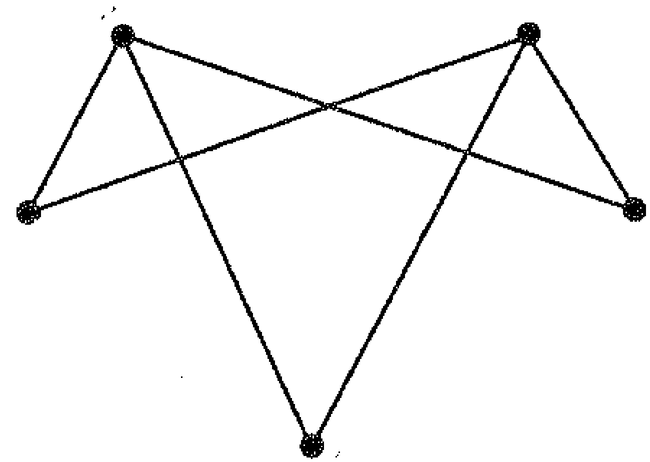
Try it with each graph from before.



a.

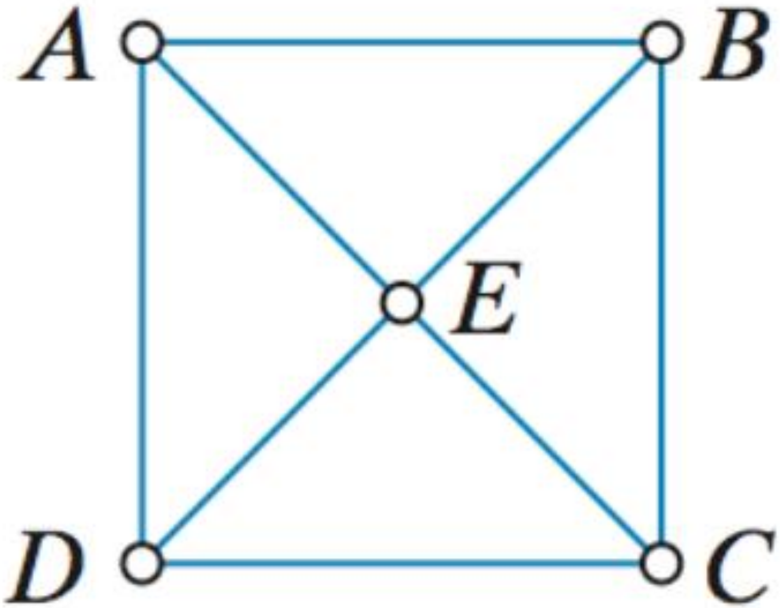


b.

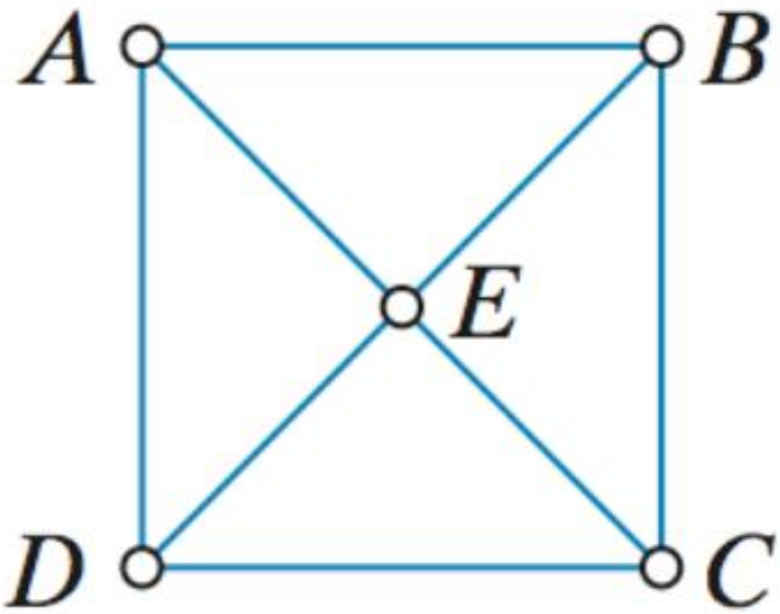


c.

Find all circuits and paths – Euler or Hamilton…



Find all circuits and paths – Euler or Hamilton… ANSWERS



No Euler Circuit or Path

Yes Hamilton Circuit: A,B,C,D,E,A

Yes Hamilton Path: A,B,C,D,E

Drawing connections

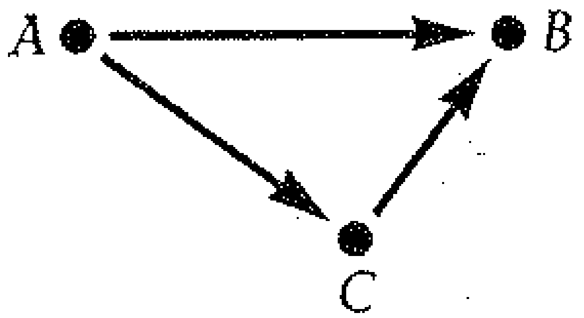
- **Note that if a graph has a Hamilton circuit, then it automatically has a Hamilton path – (the Hamilton circuit can always be truncated into a Hamilton path by dropping the last vertex of the circuit.)**

- **Contrast this with the mutually exclusive relationship between Euler circuits and paths: If a graph has an Euler circuit it <u>CANNOT</u> have an Euler path and vice versa.**

Now try: Packet p. 9 #1 in your HW!

Tournaments

A Tournament is a Complete Digraph.



Remember:
Digraphs MUST have directional arrows

This digraph shows
A beat B
A beat C
C beat B

One interesting characteristic of a complete digraph is that every tournament contains at least one Hamiltonian Path.

If there is exactly one Hamiltonian Path, it can be used to rank the teams in order from winner to loser.

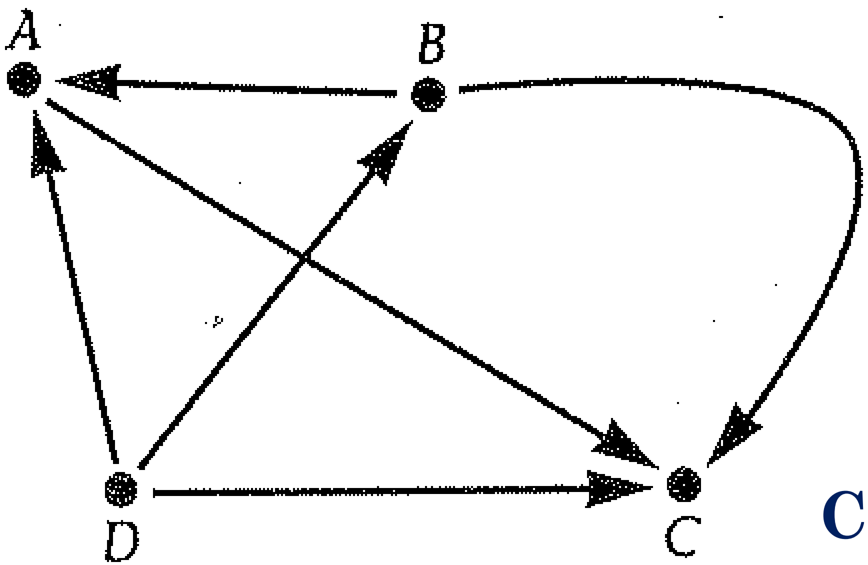
Hamiltonian Path: A C B shows A is 1st, C is 2nd, B is 3rd

Tournaments

Suppose four teams play in the school soccer round-robin tournament. The results of the competition follow:

Game	AB	AC	AD	BC	BD	CD
Winner	B	A	D	B	D	D

Draw a digraph to represent the tournament. Find a Hamiltonian path and use it to rank the participants from winner to loser.



This is a complete digraph.

Is there only one Hamiltonian Path?

Rank the teams from first place to last.

Construct an adjacency matrix.

(Directed edge from B to A means that B beat A.)

Tournaments ANSWERS

Suppose four teams play in the school soccer round-robin tournament. The results of the competition follow:

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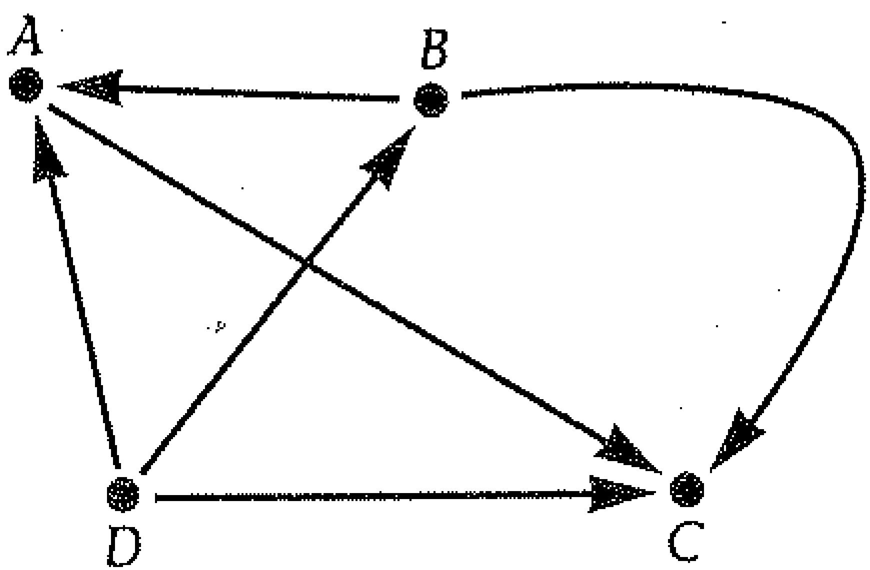


(Directed edge from B to A means that B beat A.)

This is a complete digraph.

Is there only one Hamiltonian Path? Yes!

Rank the teams from first place to last. D, B, A, C

Tip: go from highest to lowest outdegree

Construct an adjacency matrix.

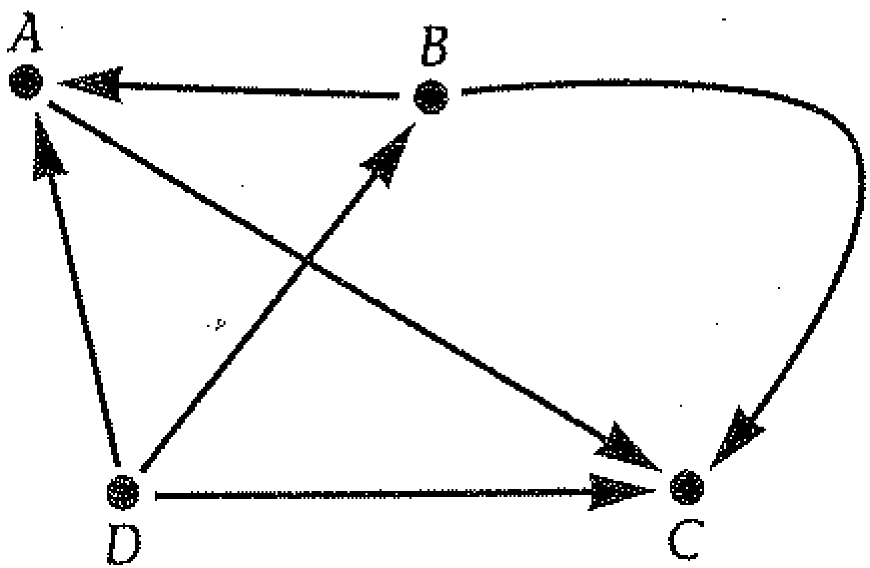
Tournaments ANSWERS

Suppose four teams play in the school soccer round-robin tournament. The results of the competition follow:

Game	AB	AC	AD	BC	BD	CD
Winner	B	A	D	B	D	D

Draw a digraph to represent the tournament. Find a Hamiltonian path and use it to rank the participants from winner to loser.

Construct an adjacency matrix.

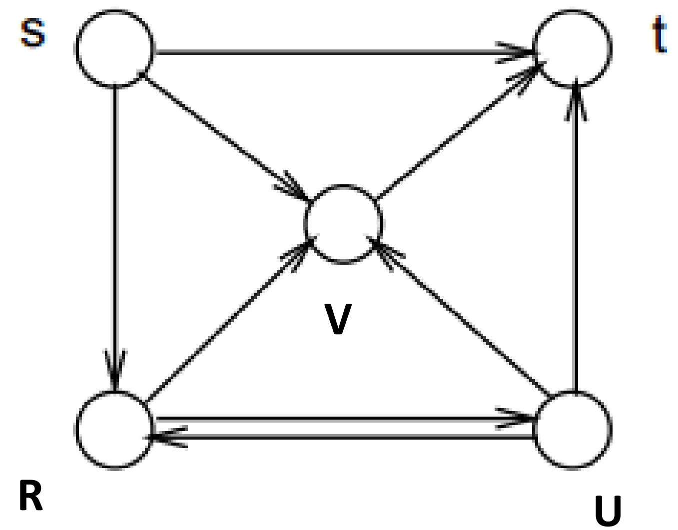


$$\begin{array}{c} \\ A \\ B \\ C \\ D \end{array} \begin{array}{c} \begin{array}{cccc} A & B & C & D \end{array} \\ \begin{bmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 \end{bmatrix} \end{array}$$

In matrix for digraph, 1 shows a win and 0 shows a loss

Can we rank RSTUV?

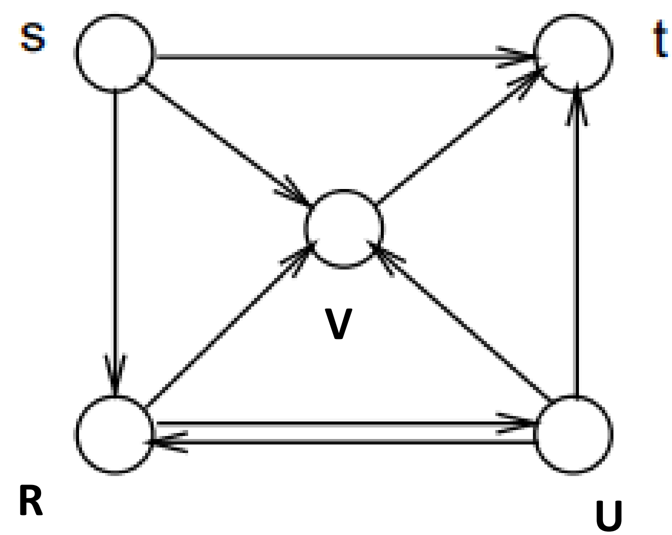
- Does this have exactly one Hamilton path?
- Is this complete?



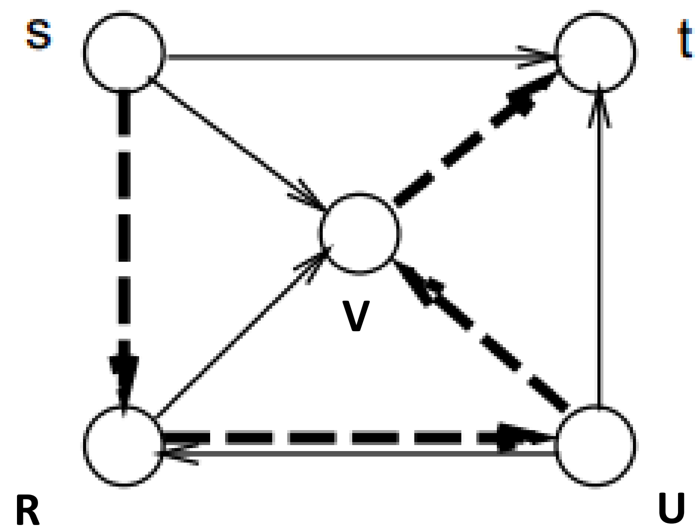
Now, Try Packet p. 7 # 6 (part of HW)

Can we rank RSTUV? ANSWERS

- **Does this have exactly one Hamilton path?**
- **Is this complete?**



- **NO! No ranking**
- **Not complete because all vertices are not adjacent (S and U; R and T)**



Now, Try Packet p. 7 # 6 (part of HW)

Packet p. 6-8

= Review & Practice of 4.1-4.5

Homework

- Finish Review Packet p. 6-8
- Packet p. 9