Unit 7 Day 3
Section 4.4

- Euler Circuits and Paths

Warm Up ~ Day 3

Set up a Unit 7 Warm Up Sheet in the Table format on a NEW, full sheet of Notebook paper. Title it Unit 7 Warm-Ups.

1. Create a Task Graph based on the Task Table shown.
2. List earliest start times in a table AND mark them on your graph.

Activity	Time	Depends on
A	3	----
B	5	----
C	2	A
D	3	A
E	3	B, D
F	5	C, E
G	1	C
H	2	F, G

Task	EST
A	
B	
C	
D	
E	
F	
G	
H	

3. Minimum Project Time =

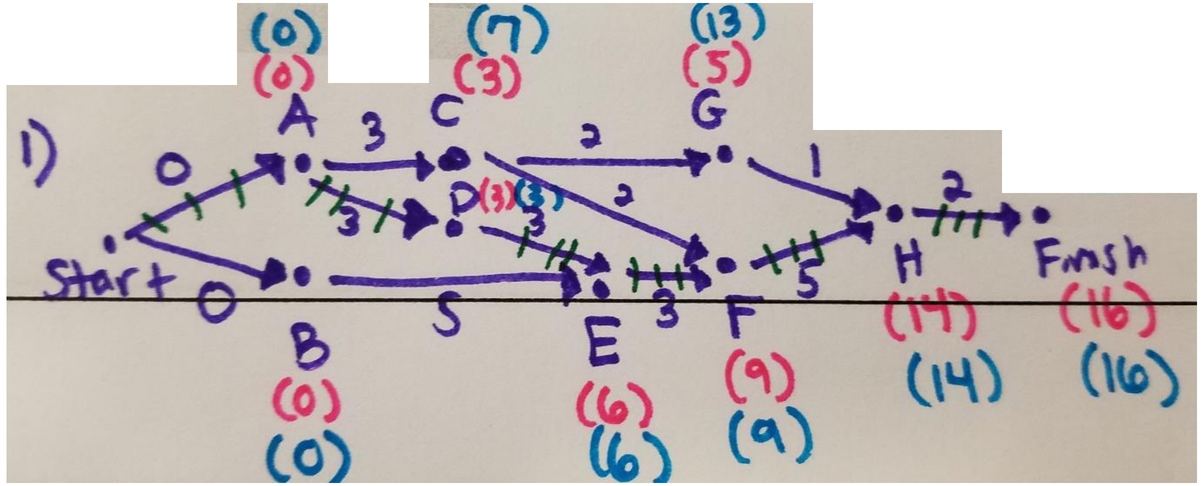
4. Critical Path(s) =

5. What is the Latest Start time for D?

6. What is the Latest Start time for C?

Warm Up ~ Day 3 ANSWERS

1. Create a Task Graph based on the Task Table shown.



2. List earliest start times in a table & mark them on your graph. EST = pink above

Activity	Time	Depends on
A	3	----
B	5	----
C	2	A
D	3	A
E	3	B, D
F	5	C, E
G	1	C
H	2	F, G

Task	EST
A	0
B	0
C	3
D	3
E	6
F	9
G	5
H	14

3. Minimum Project Time =
16

4. Critical Path(s) =
Start – ADEFH – Finish

5. What is the Latest Start Time (LST) for D? **3**

6. What is the Latest Start Time (LST) for C? **7**

Tonight's Homework

In your HW packet…

Packet p. 4 #10

and Packet p. 5

Euler Circuits and Paths

Section 4.4

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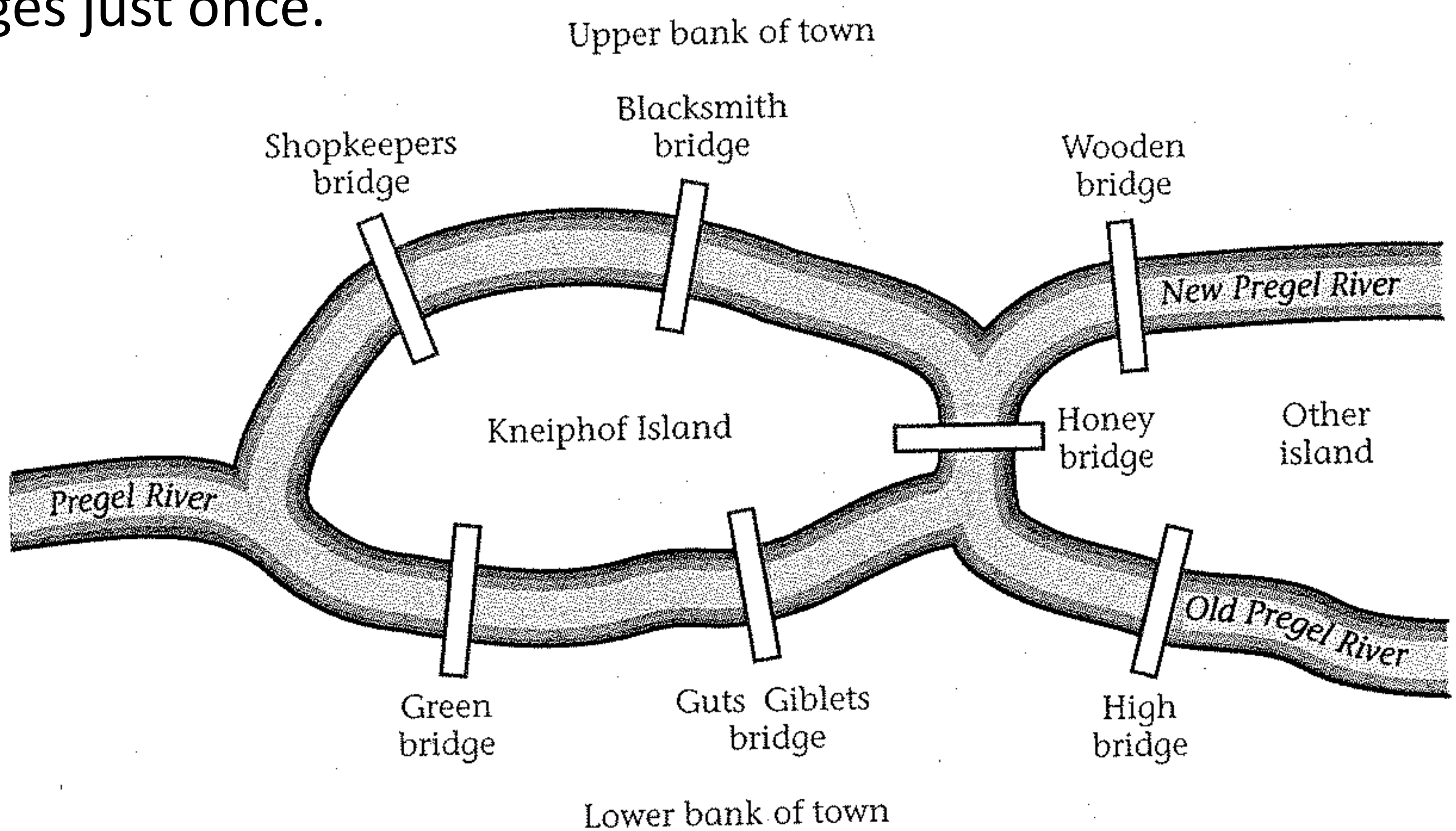
The Seven Bridges of Königsberg

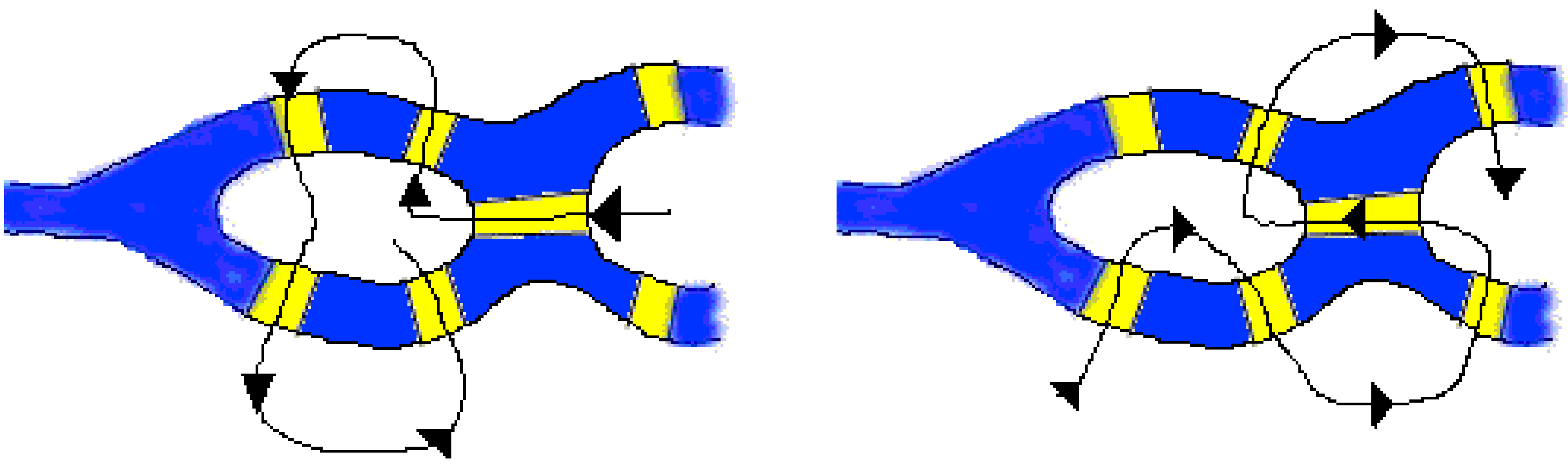
The medieval town of Königsberg has a river running through it. There is an island and a fork in the river that together divide the city into four separate land areas. At the time, seven bridges connected the four land areas. The puzzle asked whether it was possible for a stroller to take a walk around the town, crossing each of the seven bridges just once.



The Seven Bridges of Königsberg Solution

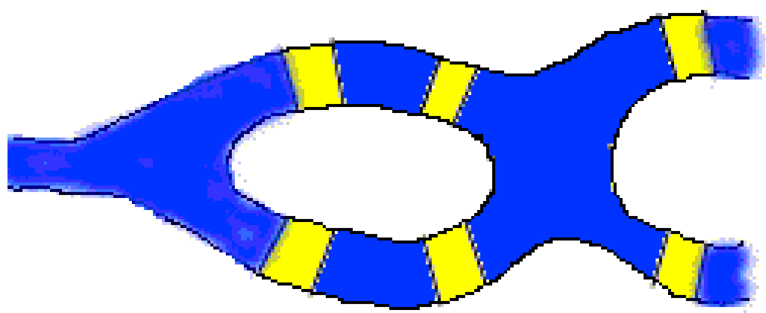
- Having trouble? That's okay, so did Euler. It doesn't seem possible to cross every bridge exactly once. In fact it isn't.

Failed Attempts ☺



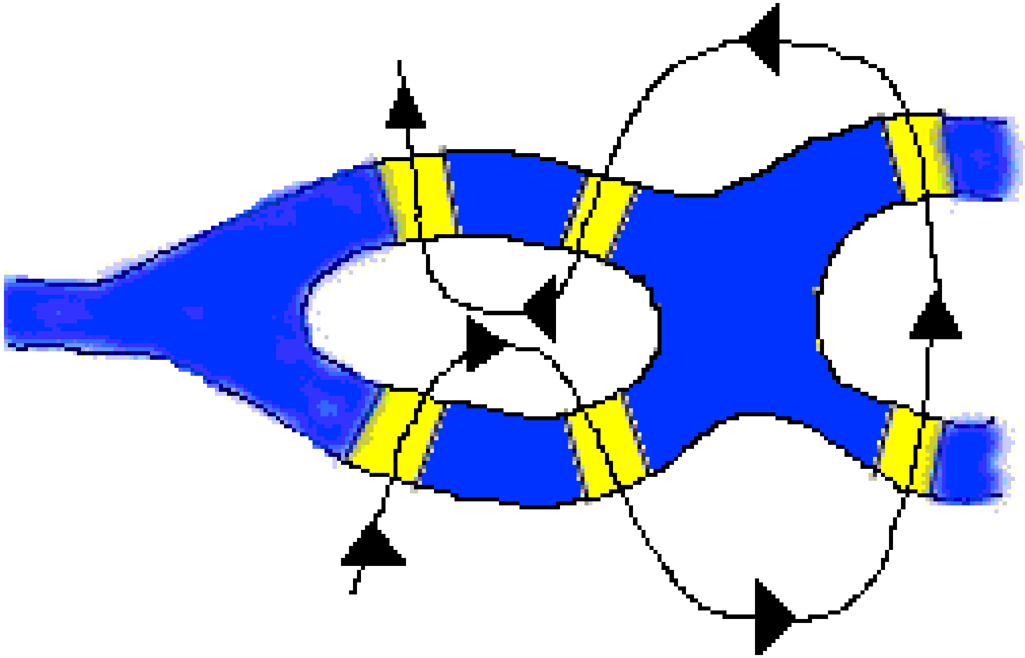
Königsberg Problem #2

- Suppose they had decided to build one fewer bridge in Konigsberg, so that the map looked like this:

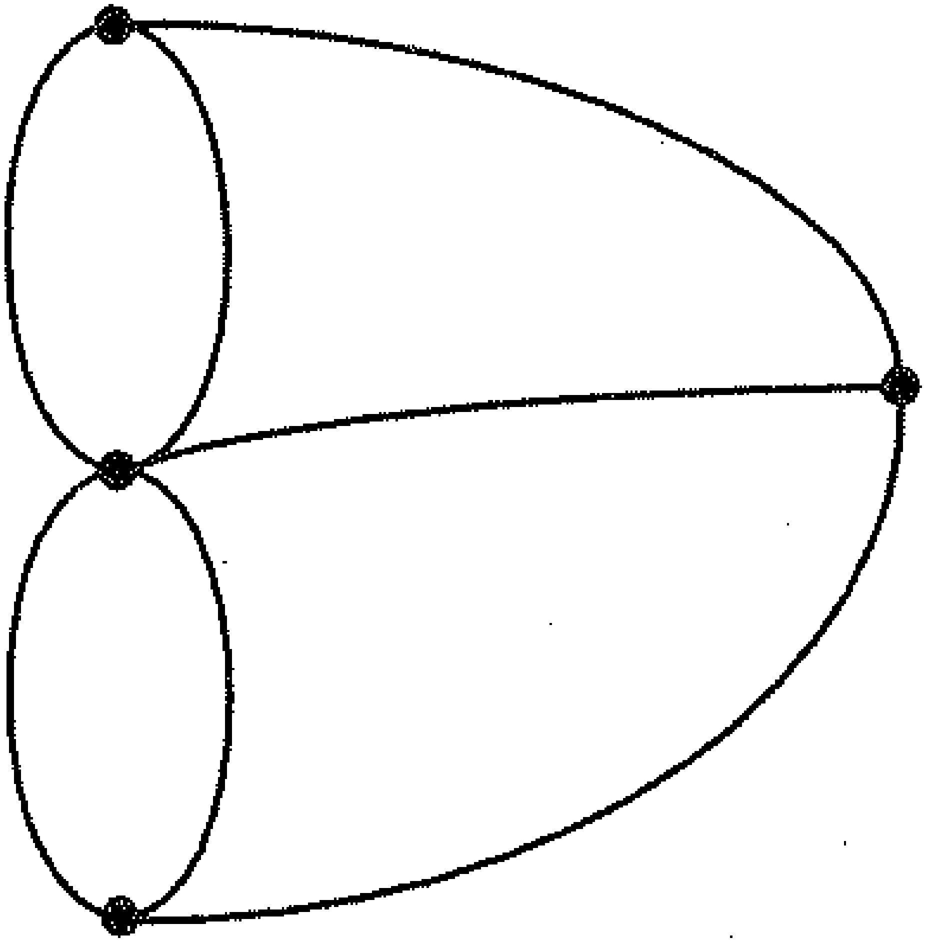


Problem #2 **Solution**

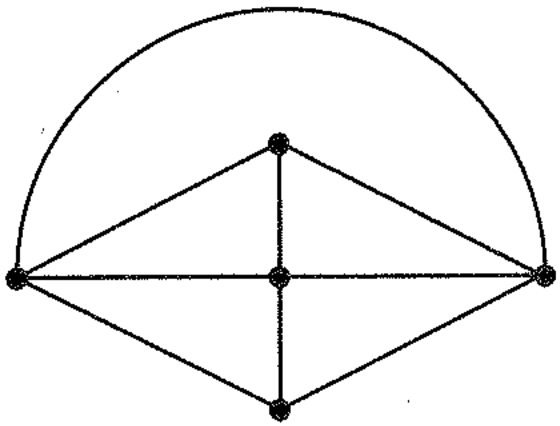
- What makes this one different from the 'real' Konigsberg problem? (Hint: How many bridges lead to each piece of land? Why is having an odd number of bridges leading to a single piece of land problematic?)



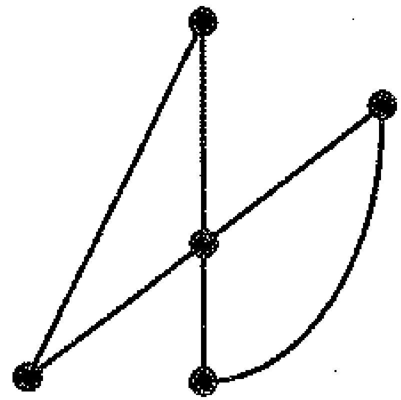
Attempt to draw this figure without lifting your pencil from the page and without tracing any of the lines more than once.



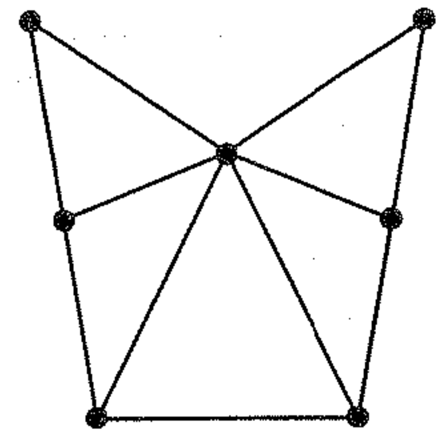
Try to reproduce the following figures without lifting your pencil or tracing the lines more than once. Also, write the degree of each vertex (number of vertices adjacent to it).



a.



b.



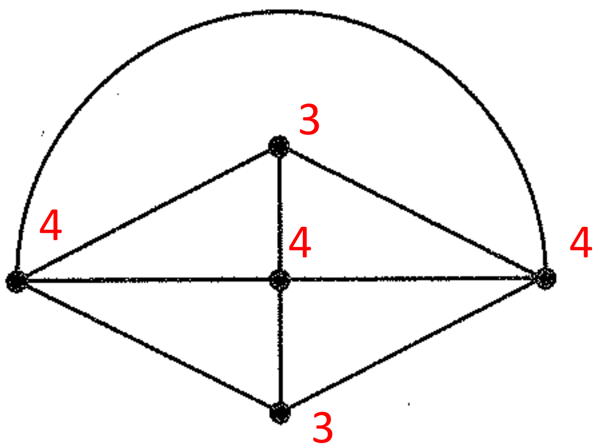
c.

1. When can you draw the figures without retracing any edges and still end up at your starting point?

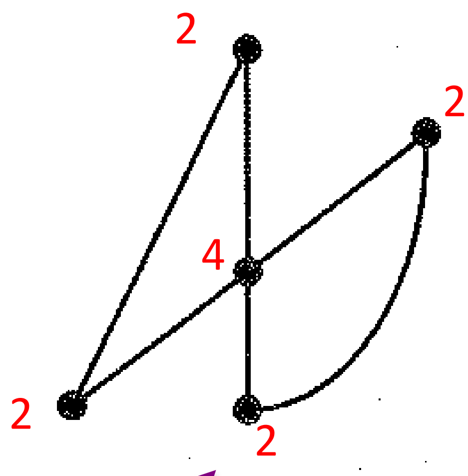
2. When can you draw the figure without retracing and end up at a point other than the one from which you began?

3. When can you not draw the figure without retracing?

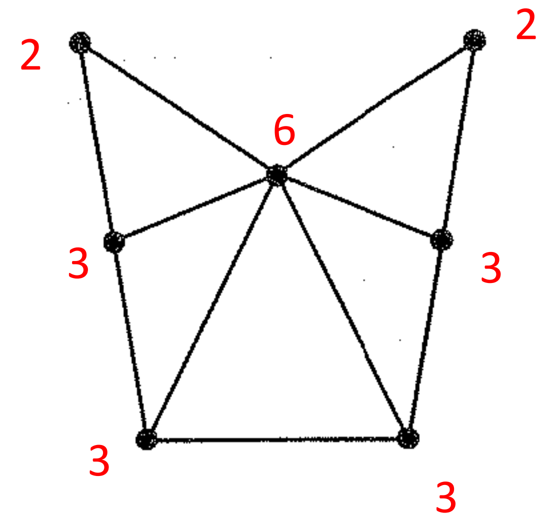
ANSWERS Try to reproduce the following figures without lifting your pencil or tracing the lines more than once. Also, write the degree of each vertex (number of vertices adjacent to it).



a.



b.



c.

1. When can you draw the figures without retracing any edges and still end up at your starting point?

2. When can you draw the figure without retracing and end up at a point other than the one from which you began?

3. When can you not draw the figure without retracing?

Our Findings (Hopefully)…

*Write this down!!

You can draw it and you end up where you started:

ALL DEGREES ARE EVEN

This is an

EULER CIRCUIT

You can draw it, but you don’t end where you started:

EXACTLY TWO OF THE VERTICES HAVE AN ODD DEGREE

This is an

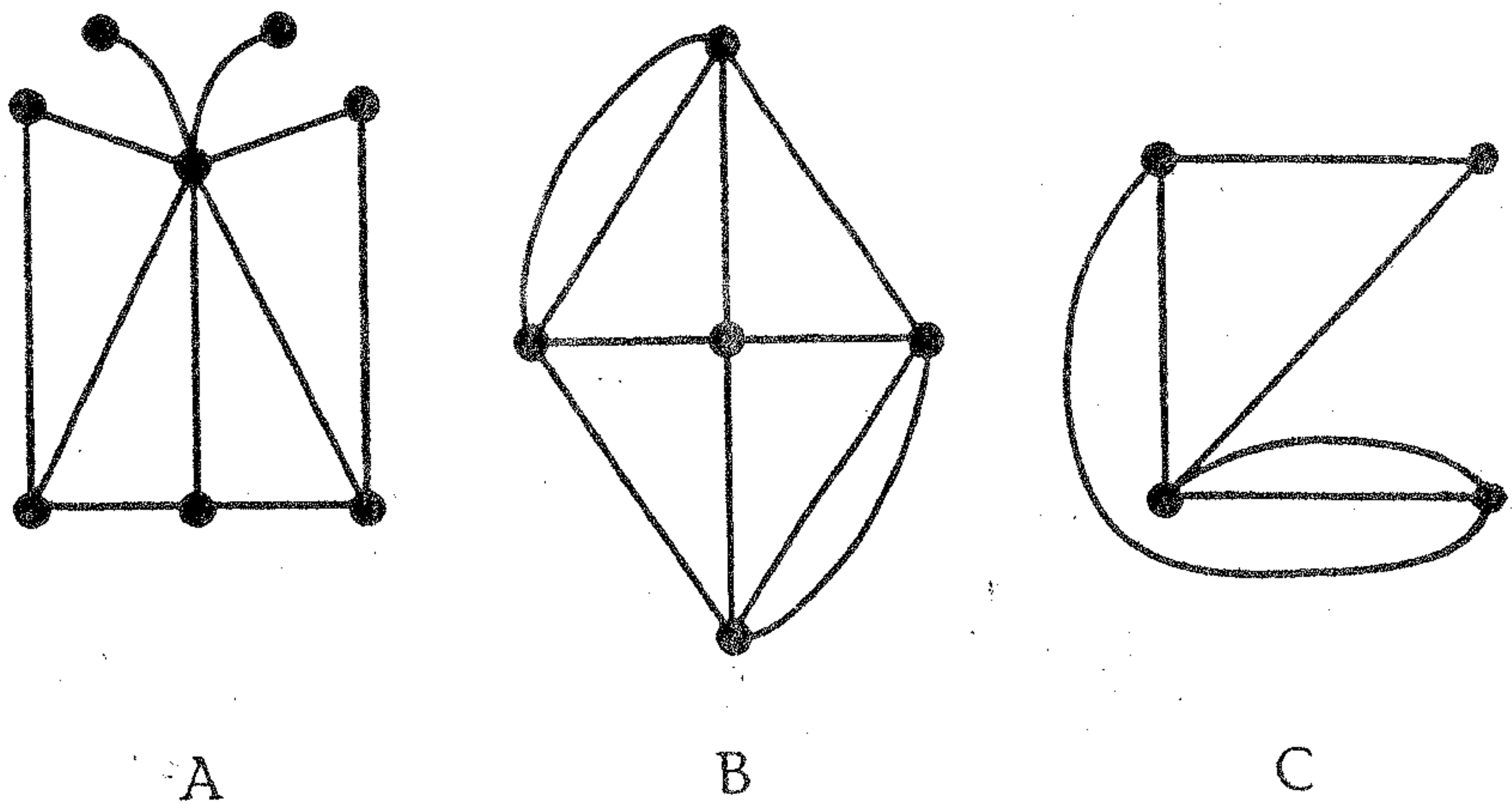
EULER PATH

Can’t draw it:

NEITHER OF THE PREVIOUS CASES

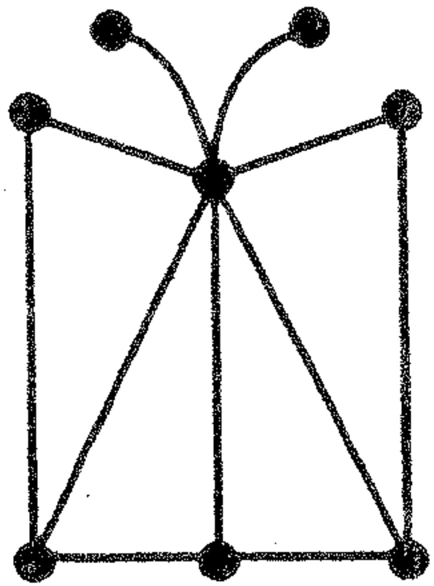
Not Possible

Determine if there is an Euler Circuit, Euler Path, or neither.



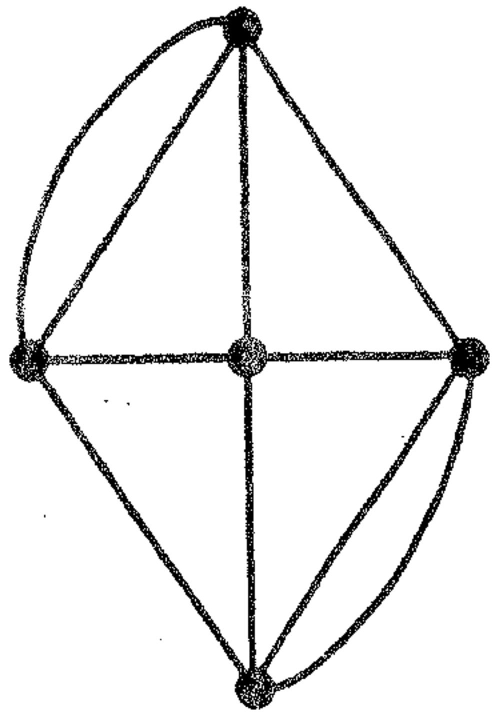
Euler Circuit: all degrees are even
Euler Path: exactly 2 vertices have an odd degree

Determine if there is an Euler Circuit, Euler Path, or neither.



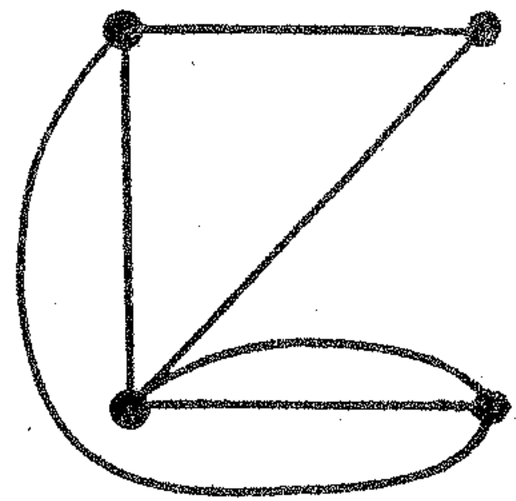
Neither

A



Euler Circuit

B



Euler Path

C

Euler Circuit: all degrees are even

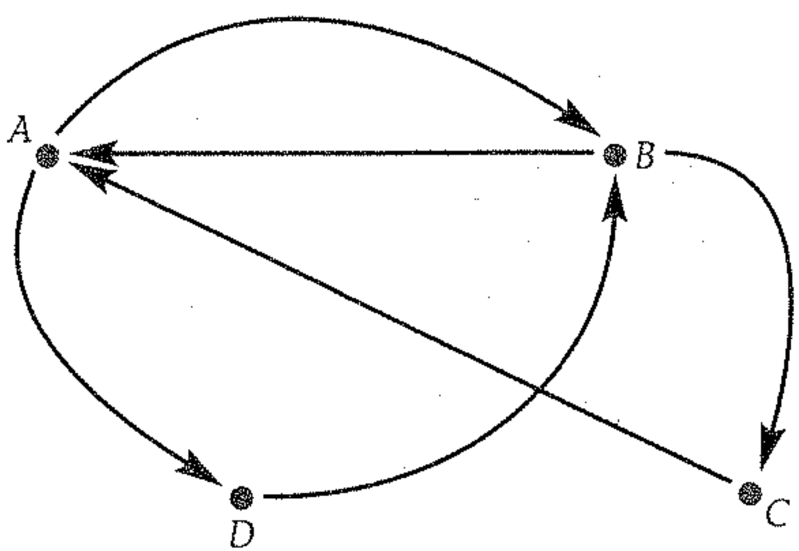
Euler Path: exactly 2 vertices have an odd degree

A "tidbit" on Digraphs

Directional Graphs **or DIGRAPHS are graphs with edges that have direction.**

Many applications of graphs require that the edges have direction.

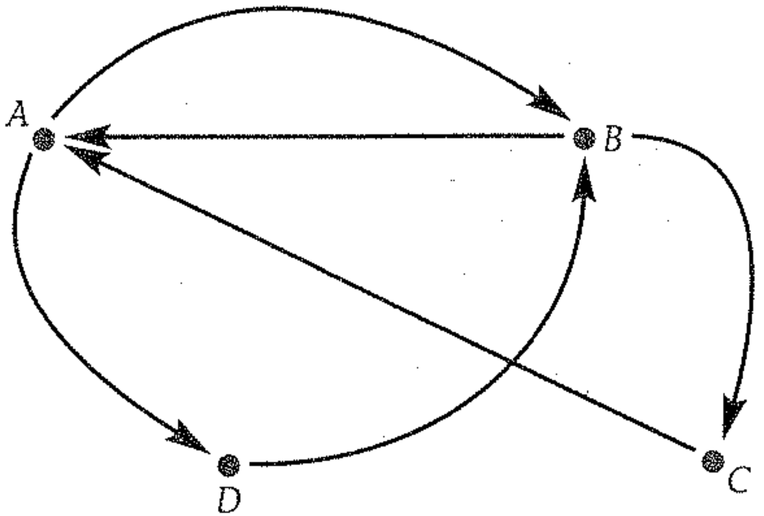
- **A city with one-way streets**
- **A business model with buyers and sellers**
- **Water flow through a filtration system**



Indegree – # of edges coming into a vertex.

Outdegree – # of edges going out of a vertex.

A few more “tidbits” on Digraphs



Indegree – # of edges coming into a vertex.

Outdegree – # of edges going out of a vertex.

Vertices – { A, B, C, D }

Ordered Edges – { AB, BA, BC, CA, DB, AD }

Notice how the edges are ordered.

If the Indegree and Outdegree are equal at every vertex then the Digraph has an Euler Circuit (“Directed” Euler Circuit)

If this represented a competition, who would win, A vs C? C

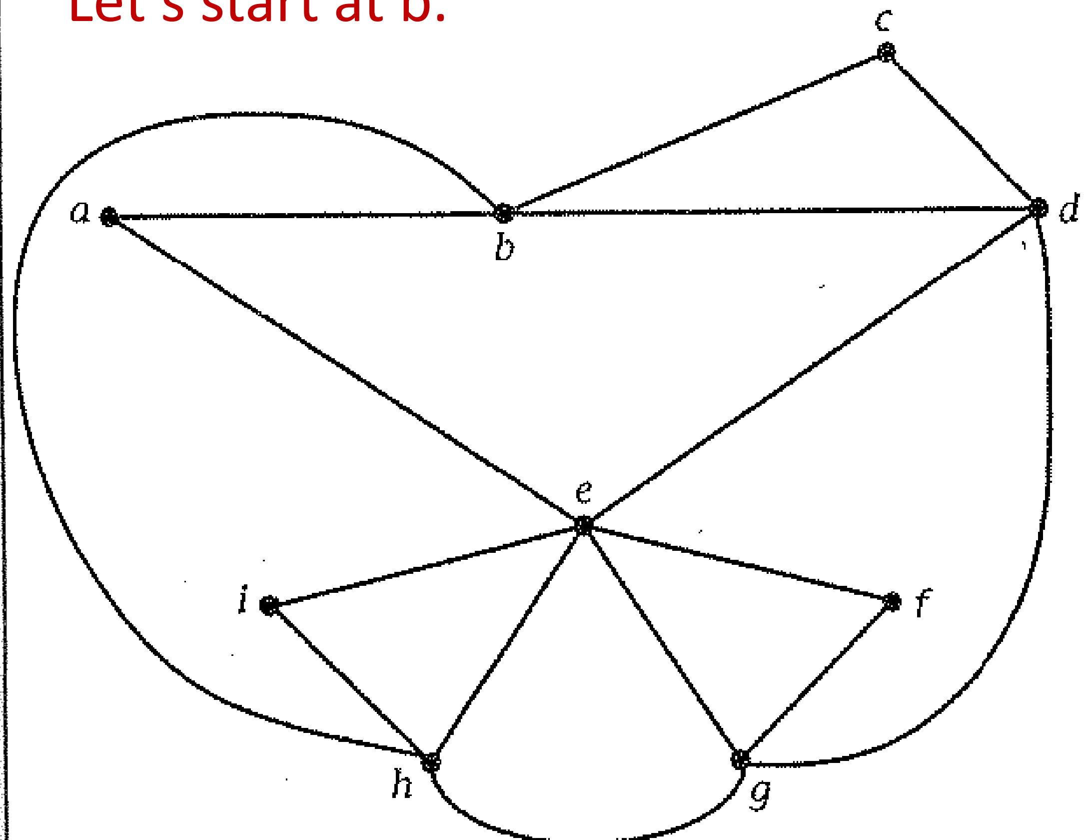
A vs D? A

When a graph is small, it's easy to find an Euler Circuit by trial & error, but when the graph is bigger you need an algorithm.

Euler Circuit Algorithm

1. Pick any vertex, and label it S.
2. Construct a circuit, C, that begins and ends at S.
3. If C is a circuit that includes all edges of the graph, go to step 8.
4. Choose a vertex, V, that is in C and has an edge that is not in C.
5. Construct a circuit C' that starts and ends at V using edges not in C.
6. Combine C and C' to form a new circuit. Call this new circuit C.
7. Go to step 3.
8. Stop. C is an Euler circuit for the graph.

Let's start at b.



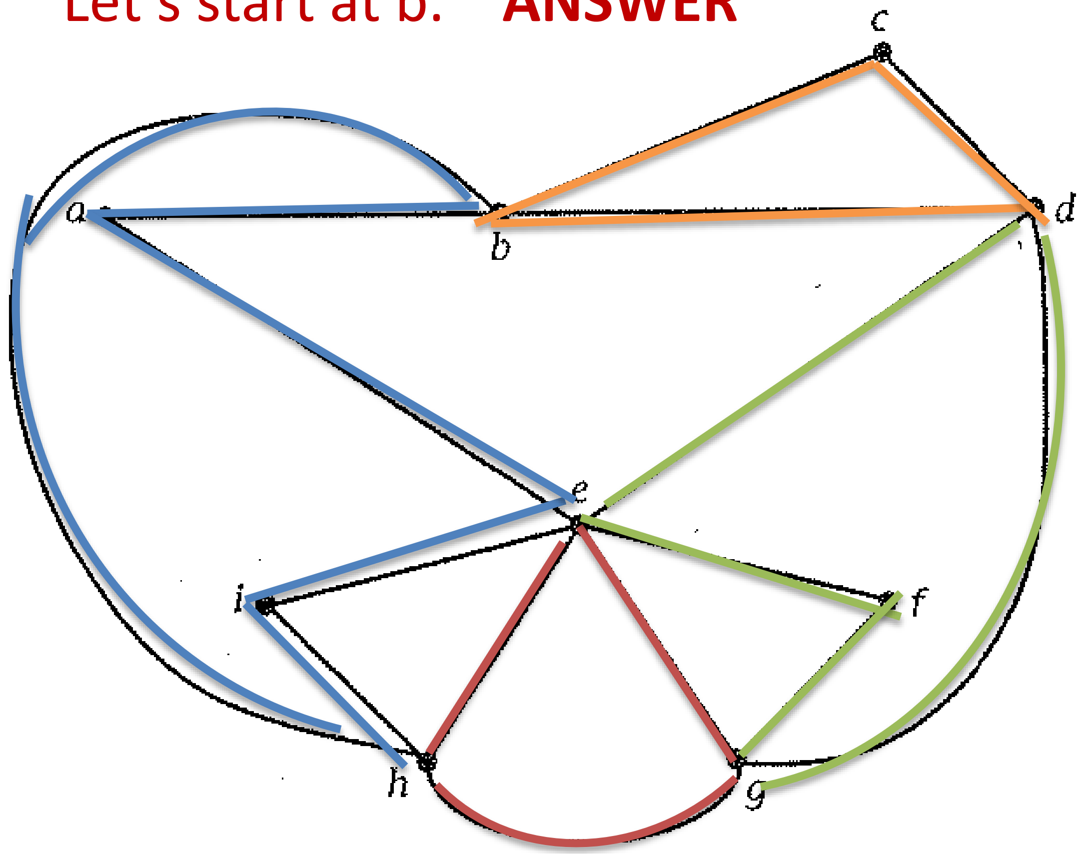
<u>Circuit</u> - cycle with no repetitions of vertices or edges, other than the repetition of the starting and ending vertex

When a graph is small, it's easy to find an Euler Circuit by trial & error, but when the graph is bigger you need an algorithm.

Euler Circuit Algorithm

1. Pick any vertex, and label it *S*.
2. Construct a circuit, *C*, that begins and ends at *S*.
3. If *C* is a circuit that includes all edges of the graph, go to step 8.
4. Choose a vertex, *V*, that is in *C* and has an edge that is not in *C*.
5. Construct a circuit *C*′ that starts and ends at *V* using edges not in *C*.
6. Combine *C* and *C*′ to form a new circuit. Call this new circuit *C*.
7. Go to step 3.
8. Stop. *C* is an Euler circuit for the graph.

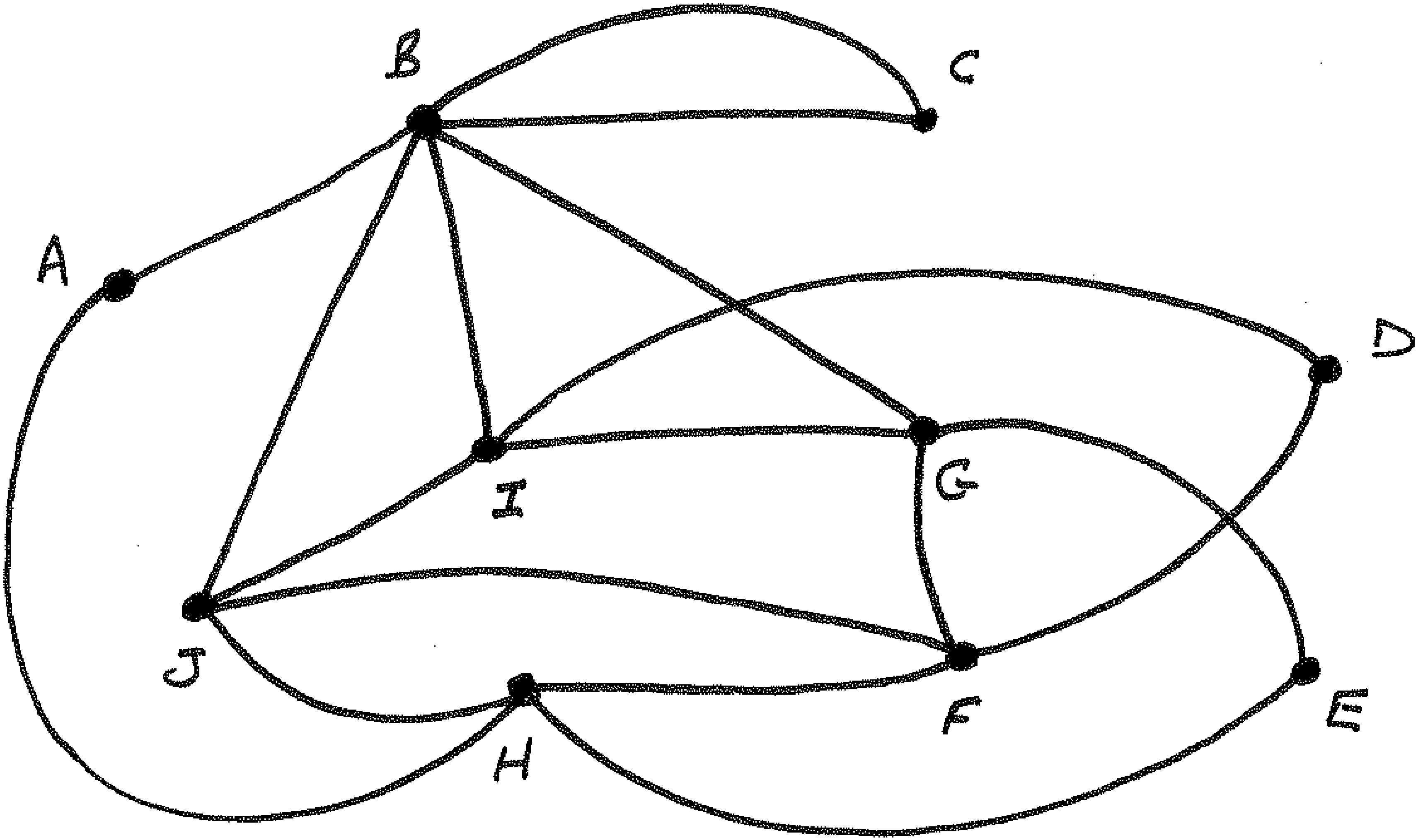
Let's start at b. **ANSWER**



<u>Circuit</u> - cycle with no repetitions of vertices or edges, other than the repetition of the starting and ending vertex

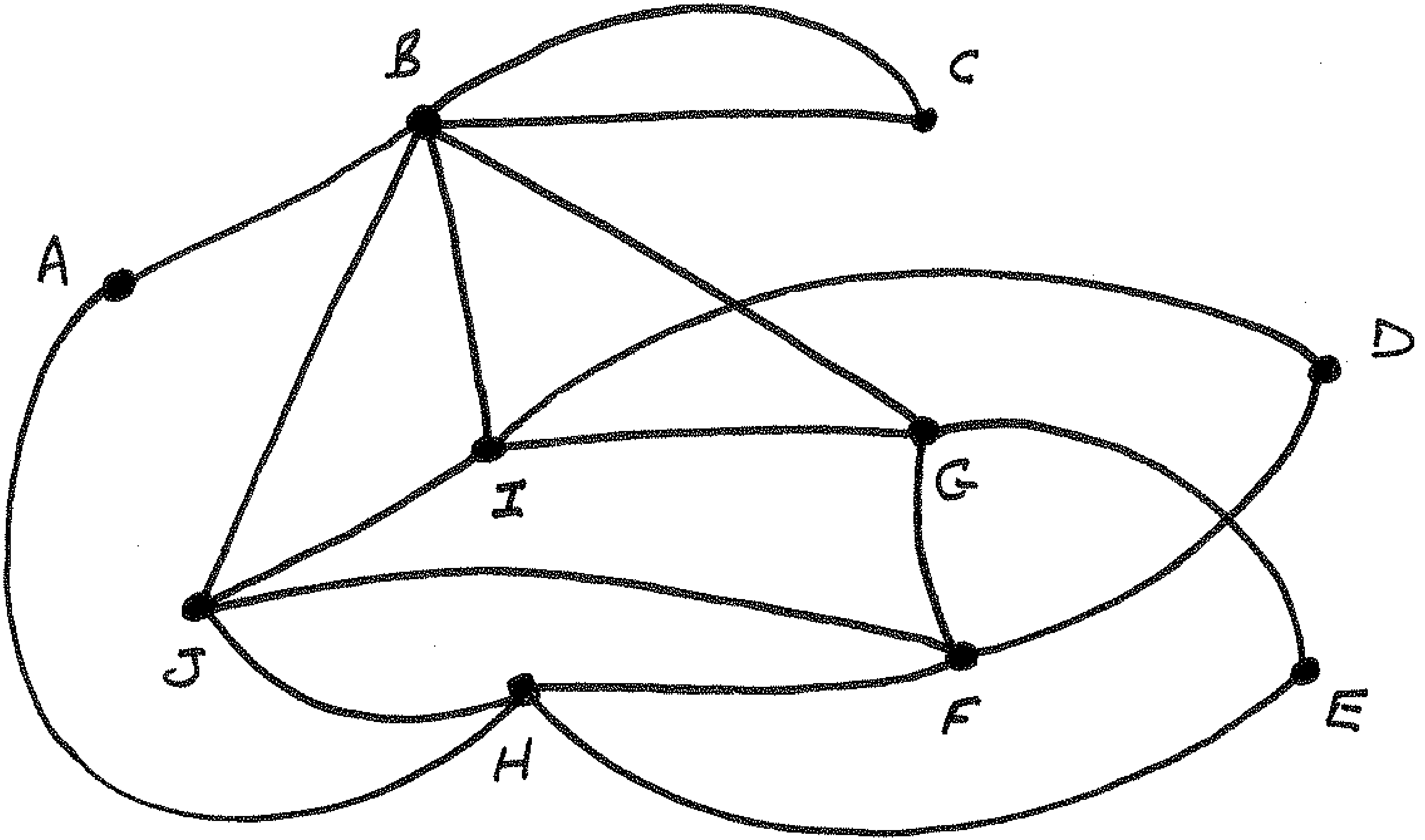
Euler Circuits Example

Identify an Euler Circuit by the algorithm.



Euler Circuits Example ANSWER

Identify an Euler Circuit by the algorithm.



Homework

In your HW packet…

Packet p. 4 #10

and Packet p. 5

Next slide skipped

- For Fall ‘18, split old Day 2 into two days, so HW was changed.

Homework

- In your HW packet… p. 3-5