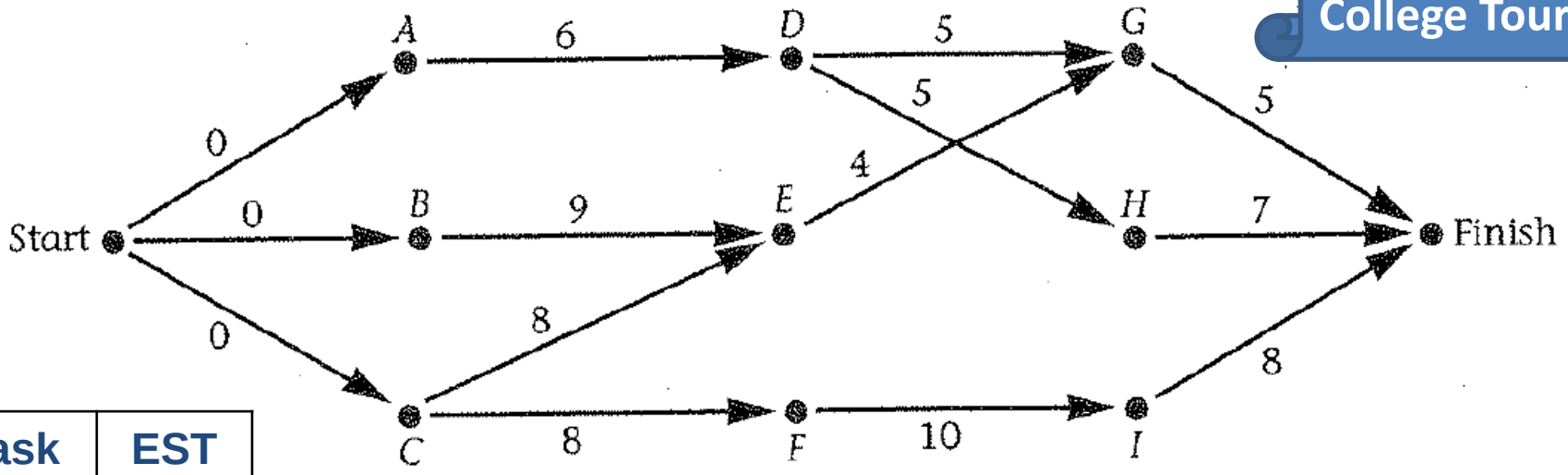
Unit 7 Day 2
Section 4.3-4.4

- Vocabulary & Graphical Representations

Warm Up ~ Day 2

See board for directions to turn in your College Tour.

List the tasks and earliest start times in a table.
Determine the minimum project time and all the critical paths.



Task	EST
A	
B	
C	
D	
E	
F	
G	
H	
I	

Minimum Project Time =

Critical Path(s) =

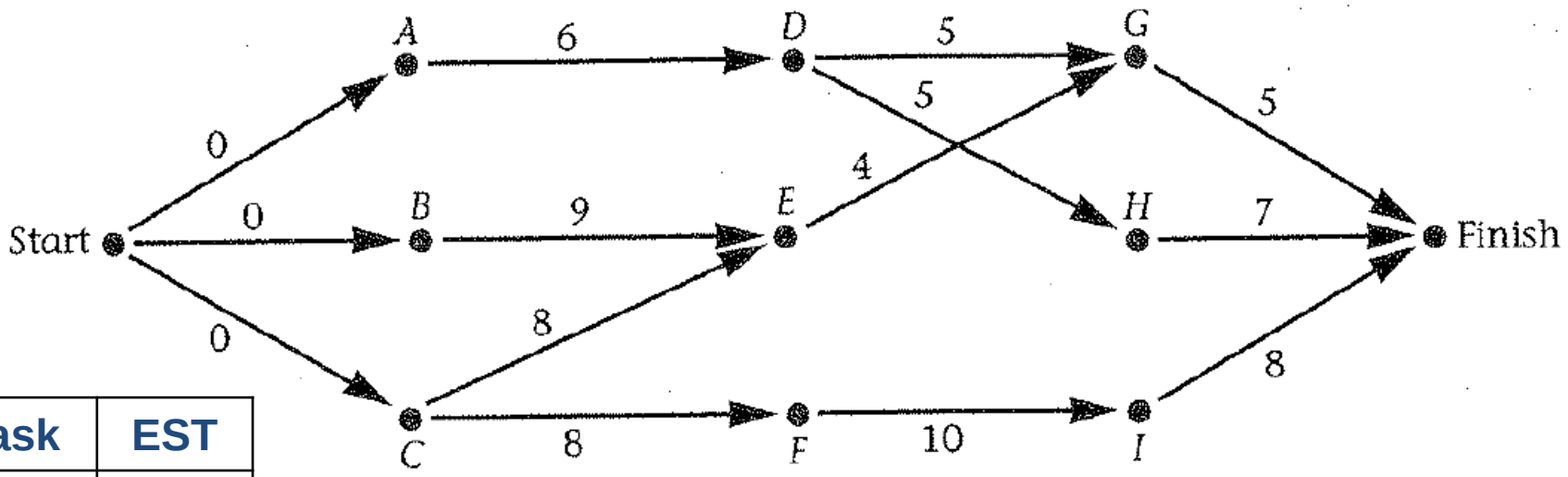
What is the Latest Start time for D?

What is the Latest Start time for F?

Warm Up ~ Day 2

Name: _____ Per: __

List the earliest start times in a table AND on the graph. Determine the minimum project time and all the critical paths. Find the LSTs requested – <u>show work for LSTs</u>!



Task	EST
A	
B	
C	
D	
E	
F	
G	
H	
I	

Minimum Project Time = ___

Critical Path(s) = _____

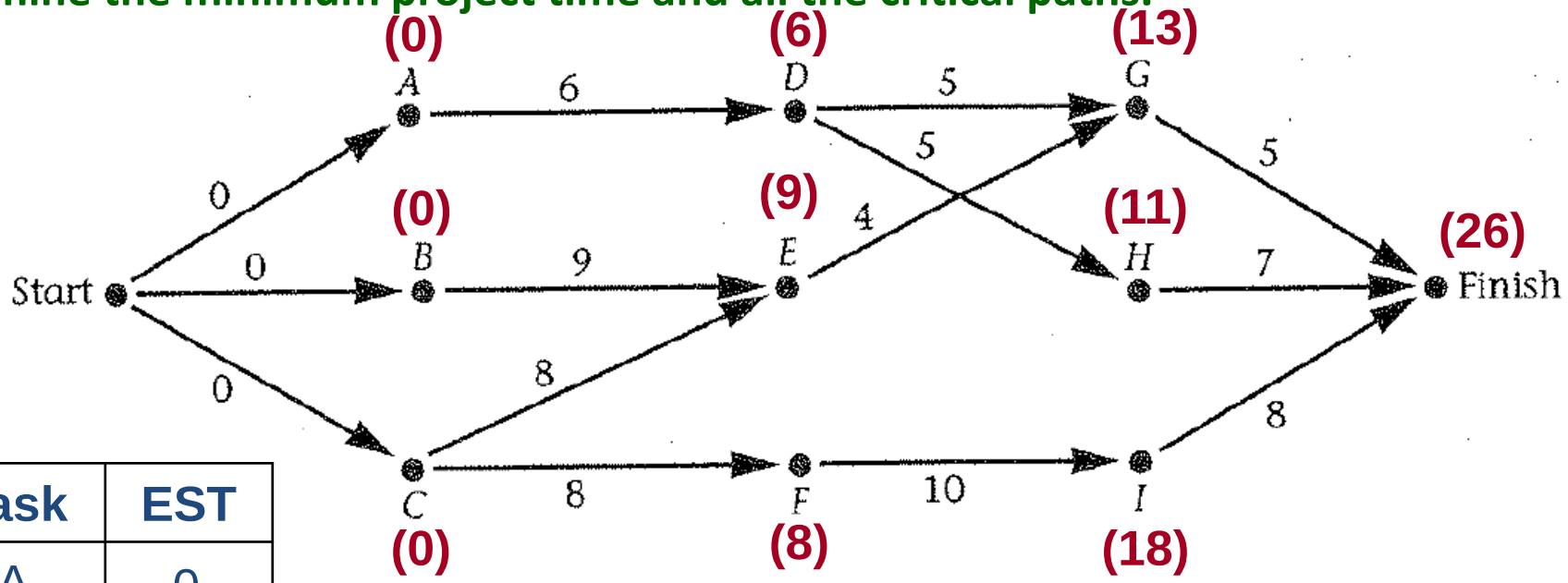
What is the Latest Start time for D? ____

What is the Latest Start time for F? ____

Show work for LSTs!

Warm Up ~ Day 2 ANSWERS

List the tasks and earliest start times in a table, as in exercise #1.
Determine the minimum project time and all the critical paths.



Task	EST
A	0
B	0
C	0
D	6
E	9
F	8
G	13
H	11
I	18

Minimum Project Time = **26**

Critical Path(s) = **Start-CFI-Finish**

What is the Latest Start time for F? **8**

What is the Latest Start time for D? **14**

Section 4.2 – Exercise #9,10
LATEST START TIME Algorithm ANSWERS ☺

The general algorithm for finding the LST for each task in a graph is :

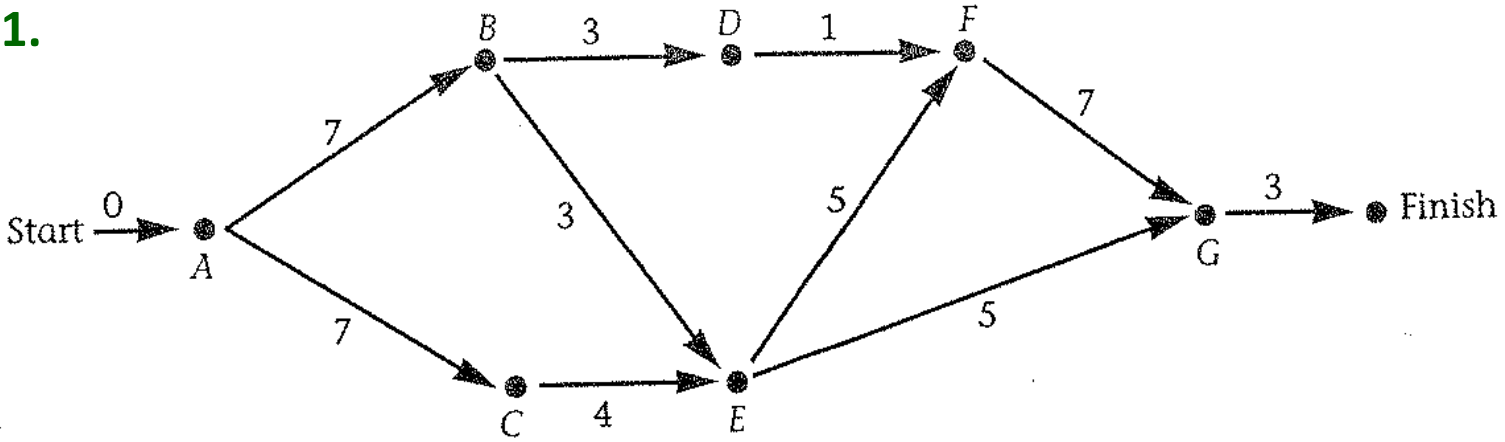
1. Begin with Finish and use the minimum project time.

2. Subtract from the minimum project time the time that it takes to complete the task(s) preceding it. This is the LST for the preceding tasks. Label the vertex with its LST.

3. If the time of a preceding task is dependent on more than one edge, choose the earliest time.

4. Continue until all vertices are labeled.

Try the algorithm on the graph from exercise #1.



Key

__ = EST

__ = LST

Section 4.2 – Exercise #9,10
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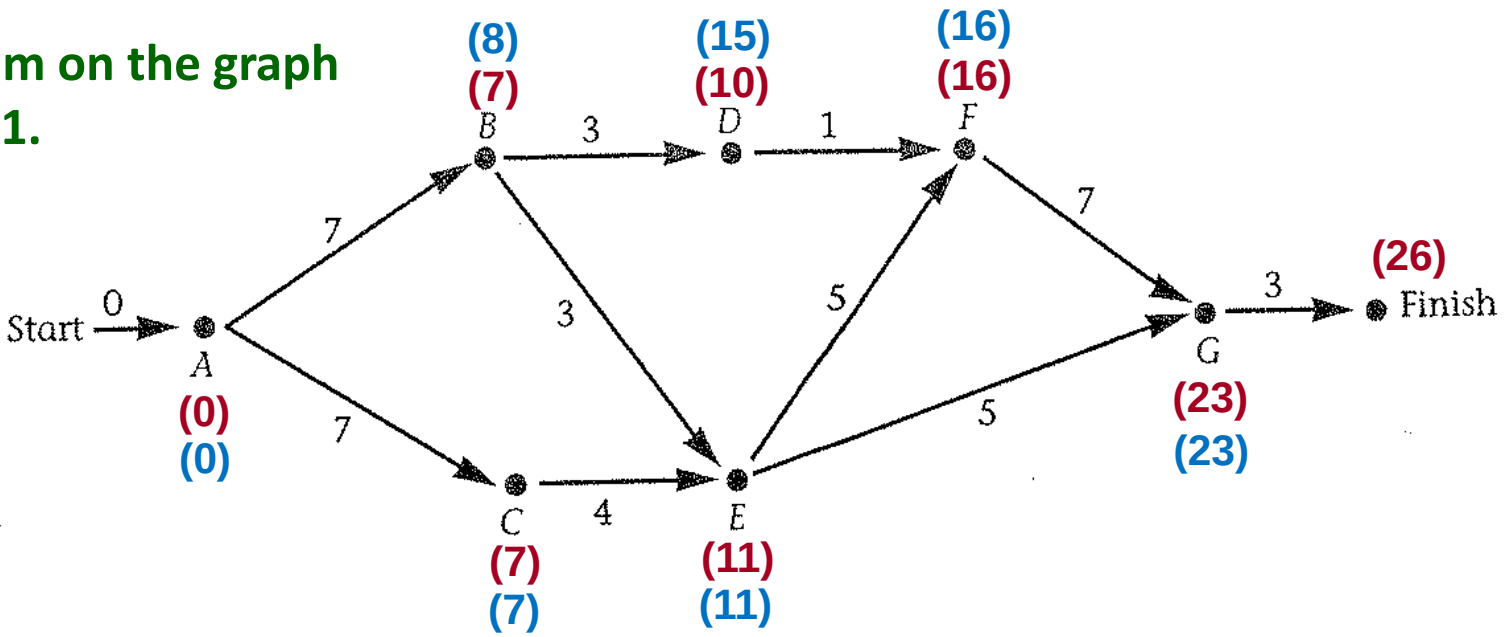


Key

*** = EST**

*** = LST**

4.1 HW ANSWERS

Section 4.1 – Exercise #5

To help organize the task of completing the family dinner, Mrs. Shu listed the following tasks.

Task	Time (min.)	Prerequisite Task
Start	0	---
A. Wash hands		
B. Defrost hamburger		
C. Shape meat into patties	ANSWERS VARY	ANSWERS VARY
D. Cook hamburgers		
E. Peel and slice potatoes		
F. Fry potatoes		
G. Make salad		
H. Set table		
I. Serve food		

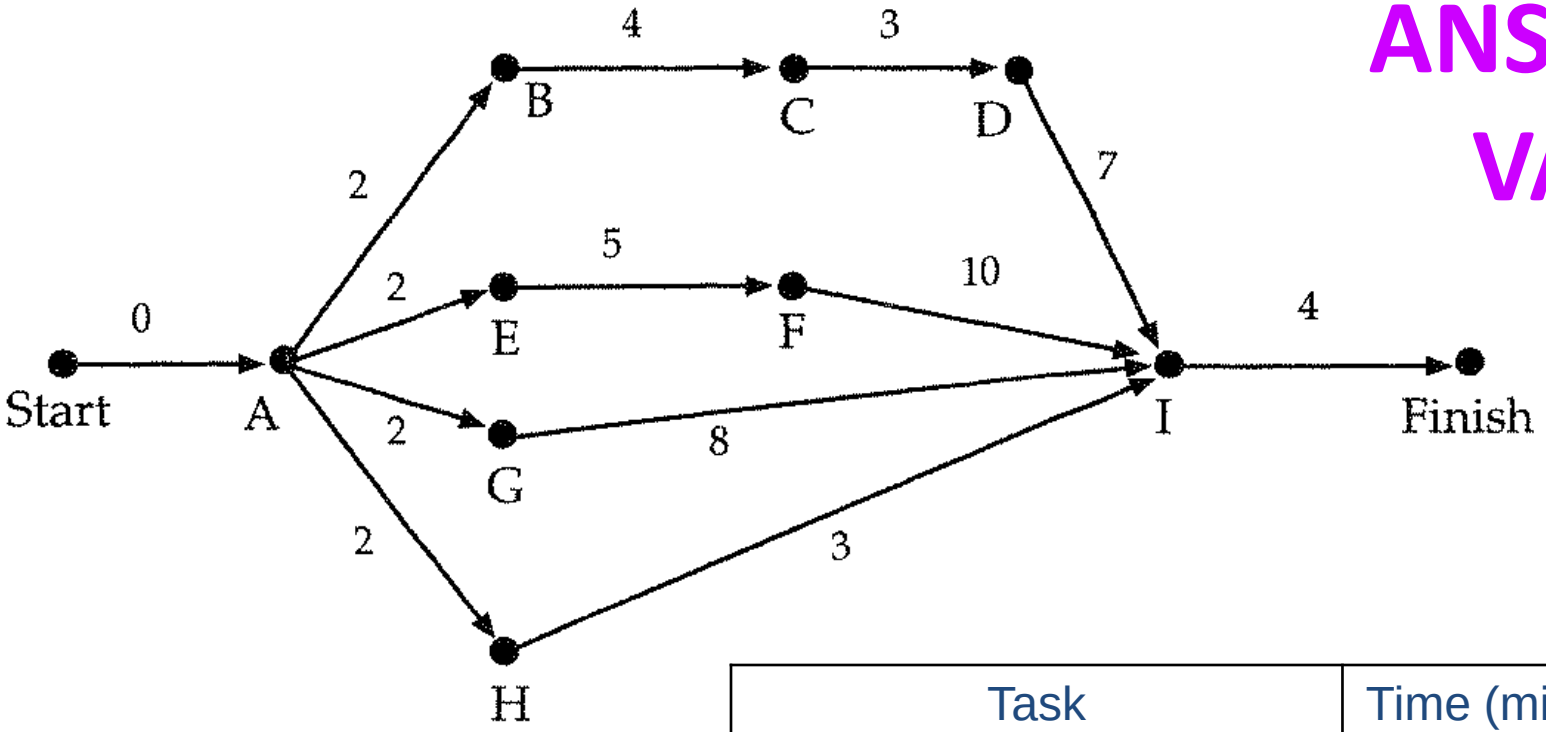
a) **Complete the table by making reasonable time estimates in minutes for each of these tasks and indicating the prerequisites.**

b) **Construct a graph using the information from your table.**

c) **Using this information what do you think is the least amount of time needed to prepare dinner?**

Possible answer for exercise #5

ANSWERS VARY

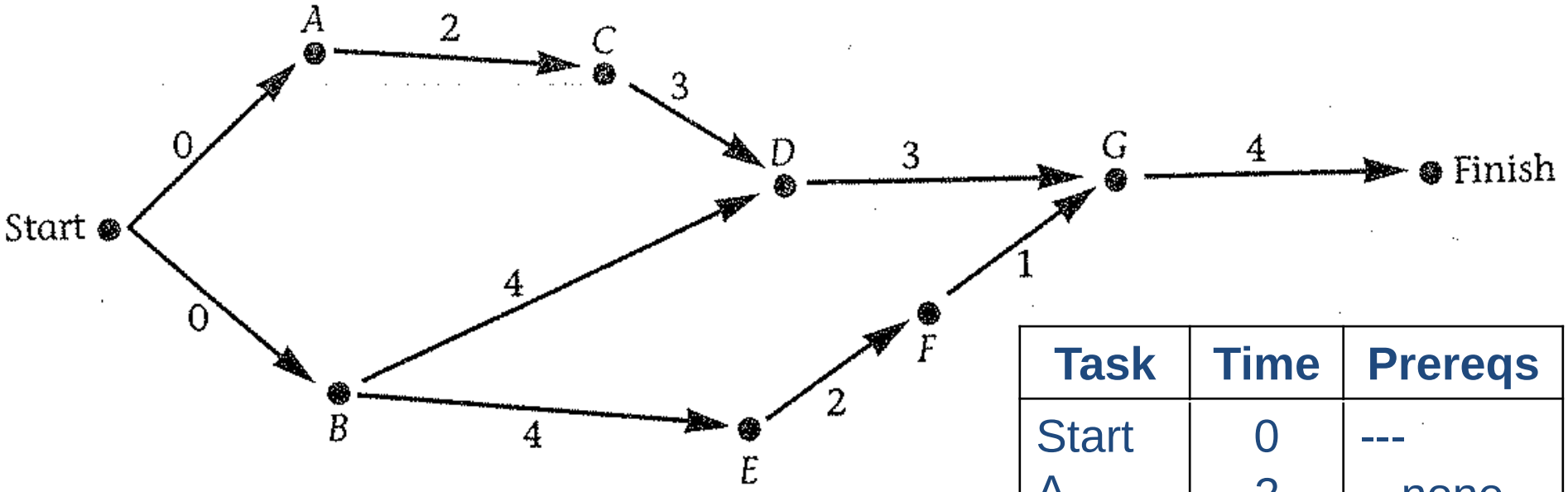


Task	Time (min.)	Prereqs
Start	0	---
A. Wash hands	2	None
B. Defrost hamburger	4	A
C. Shape meat into patties	3	B
D. Cook hamburgers	7	C
E. Peel and slice potatoes	5	A
F. Fry potatoes	10	E
G. Make salad	8	A
H. Set table	3	A
I. Serve food	4	D, F, G, H

In this case, the least amount of time would be 21 minutes.

Section 4.1 – Exercise #7

Complete the task table for this graph.



Task	Time	Prereqs
Start	0	---
A	2	none
B	4	none
C	3	A
D	3	B, C
E	2	B
F	1	E
G	4	D, F
Finish		

Task D can't begin before both A and C are complete and 5 minutes have elapsed.

G can't start before 8 minutes have elapsed.

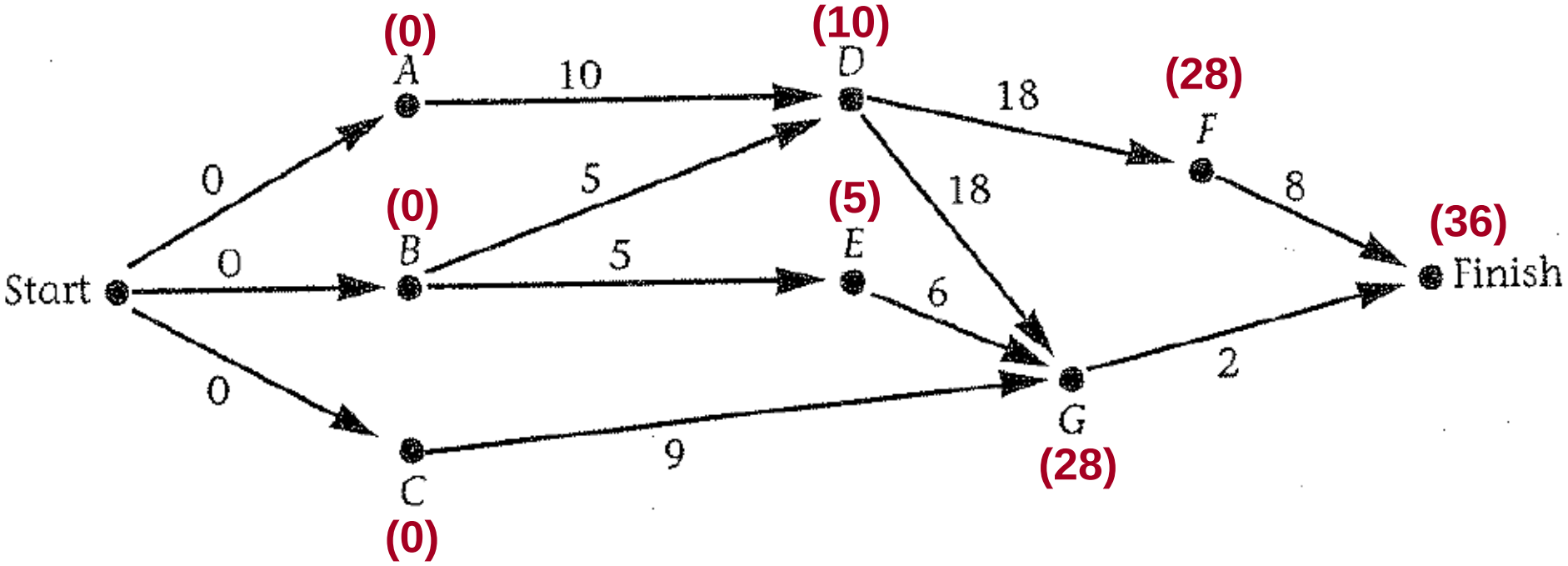
Therefore, the project can't be completed faster than 12 minutes.

Notice, the path from Start-A-C-D-G-Finish is the longest path in the graph.

4.2 HW Answers

Section 4.2 – Exercise #7

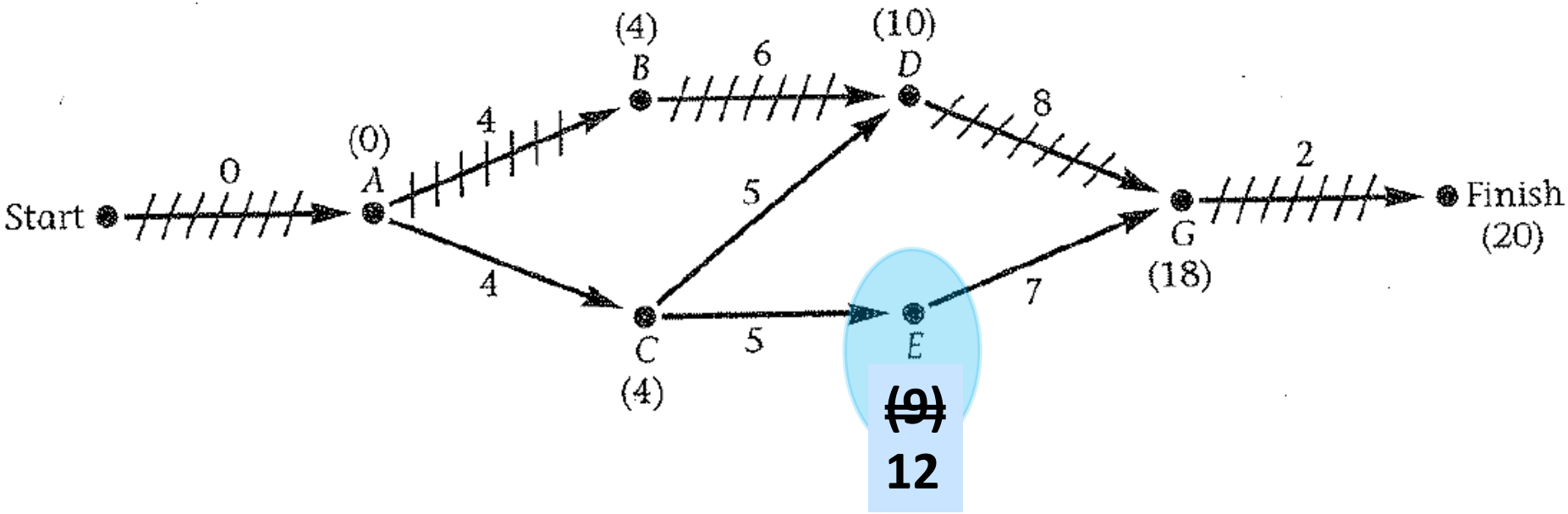
Determine the minimum project time and the critical path for the following graph.



Minimum Project Time = 36

Critical Path(s) = Start-ADF-Finish

Section 4.2 – Exercise #8
LATEST START TIME



Task E can begin as early as day 9. If E begins on day 9, when will it be completed? **Day 16**

The entire project will still be completed on DAY 20 → If E begins on day 10? **Day 17**

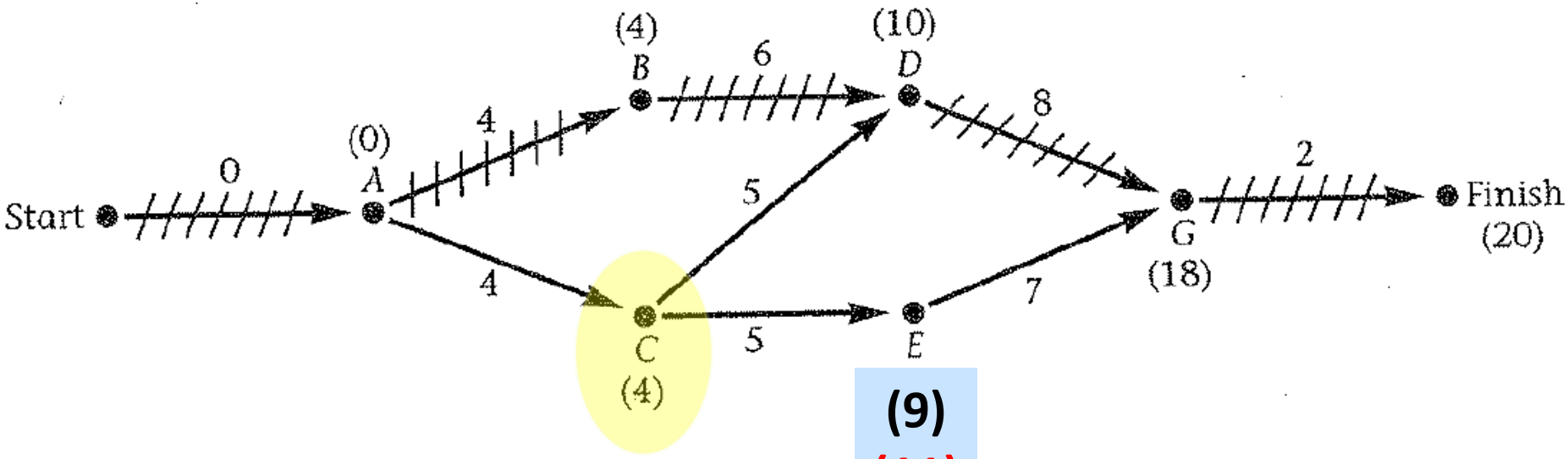
If E begins on day 11? **Day 18**

If E begins on day 12? **Task G will be delayed and the whole project will be delayed.**

What is the latest day on which task E can begin if task G is to begin on day 18? **Day 11**

If an activity is not on the critical path, it is possible for it to start later than its earliest start time and not delay the project. The latest a task can begin with out delaying the project's minimum completion time is known as the ***LATEST START TIME (LST)*** for the task.
In this example, the ***latest start time for task E is day 11***.

Section 4.2 – Exercise #8
LATEST START TIME



What about the LATEST START TIME for task C?

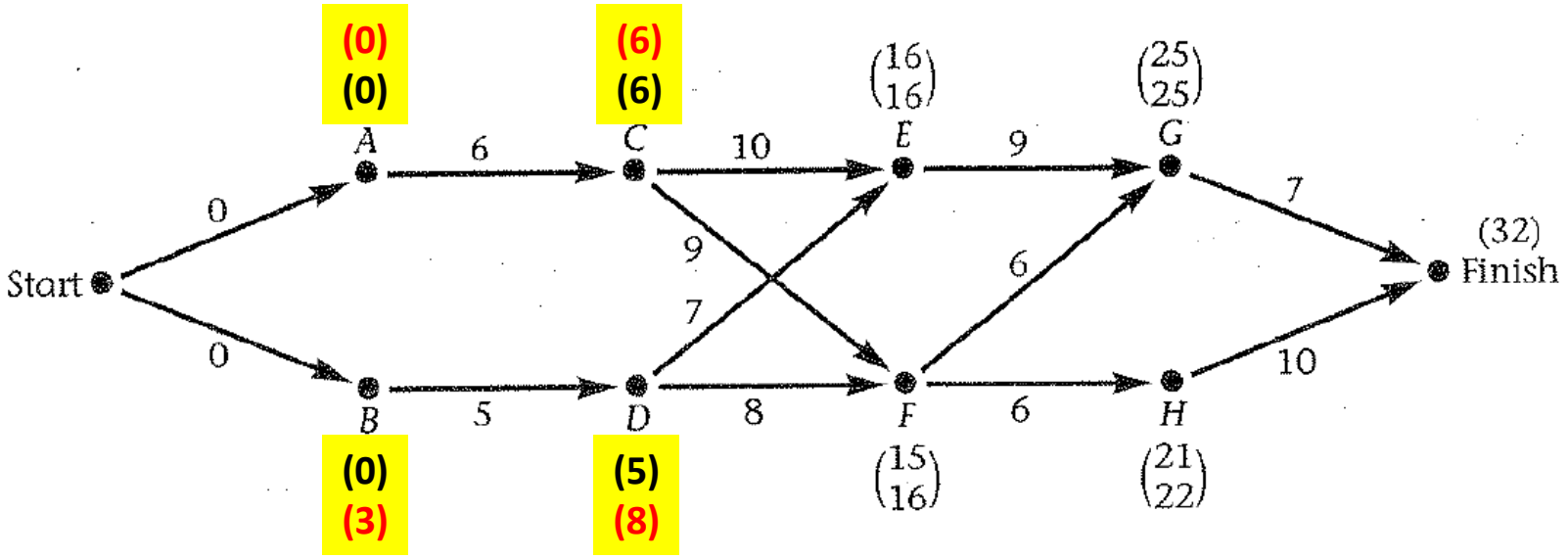
Task C effects the start times of both task D and E.

Task D is on the critical path, so the latest it can start is Day 10.

We determined already that the latest start time for task E is Day 11

So, what is the latest task C can begin without delaying the project? **Day 5**

Section 4.2 – Exercise #9
LATEST START TIME



To find the LST for each task, begin at the Finish and use the Minimum Project Time. Work through the graph in reverse order.

Subtract from the Minimum Project Time the time it takes to complete the task preceding it.

For example: **32 – 10 is 22, the LST for task H.**

32 – 7 is 25, the LST for task G. **That's the same as its EST since G is on the critical path.**

For task F use 22 – 6 = 16, the LST for task F.

What about the LST for task D? It's a prerequisite for both task E and F.

Going to E (16 – 7 = 9), Going to F (16 – 8 = 8) → Choose the smallest time.

Section 4.2 – Exercise #9,10
LATEST START TIME Algorithm ☺

The general algorithm for finding the LST for each task in a graph is :

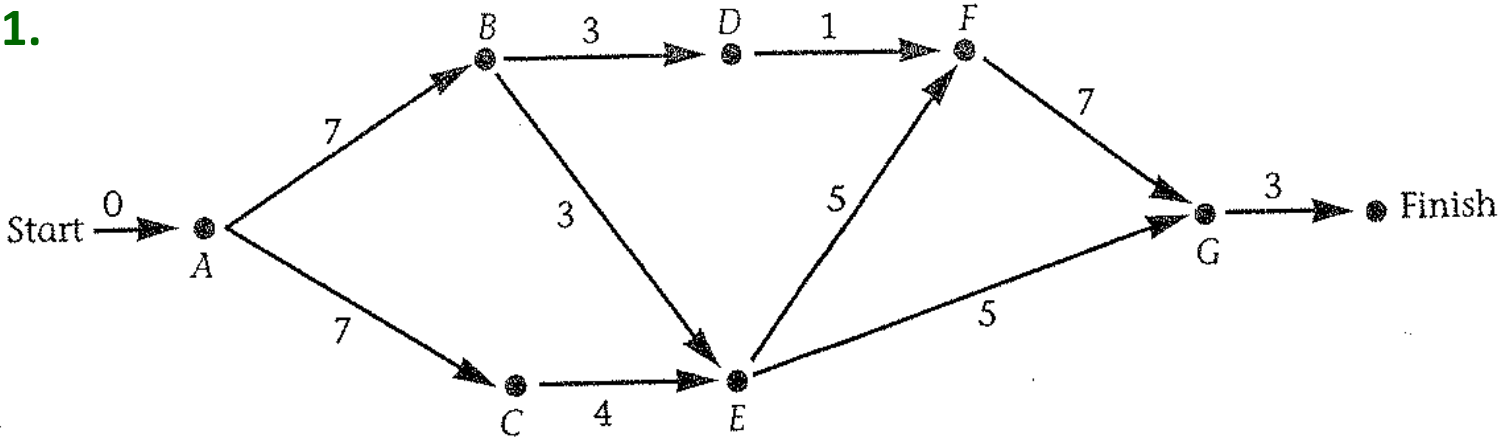
1. Begin with Finish and use the minimum project time.

2. Subtract from the minimum project time the time that it takes to complete the task(s) preceding it. This is the LST for the preceding tasks. Label the vertex with its LST.

3. If the time of a preceding task is dependent on more than one edge, choose the earliest time.

4. Continue until all vertices are labeled.

Try the algorithm on the graph from exercise #1.



Key

___ = EST

___ = LST

Section 4.2 – Exercise #9,10
LATEST START TIME Algorithm ANSWERS ☺

The general algorithm for finding the LST for each task in a graph is :

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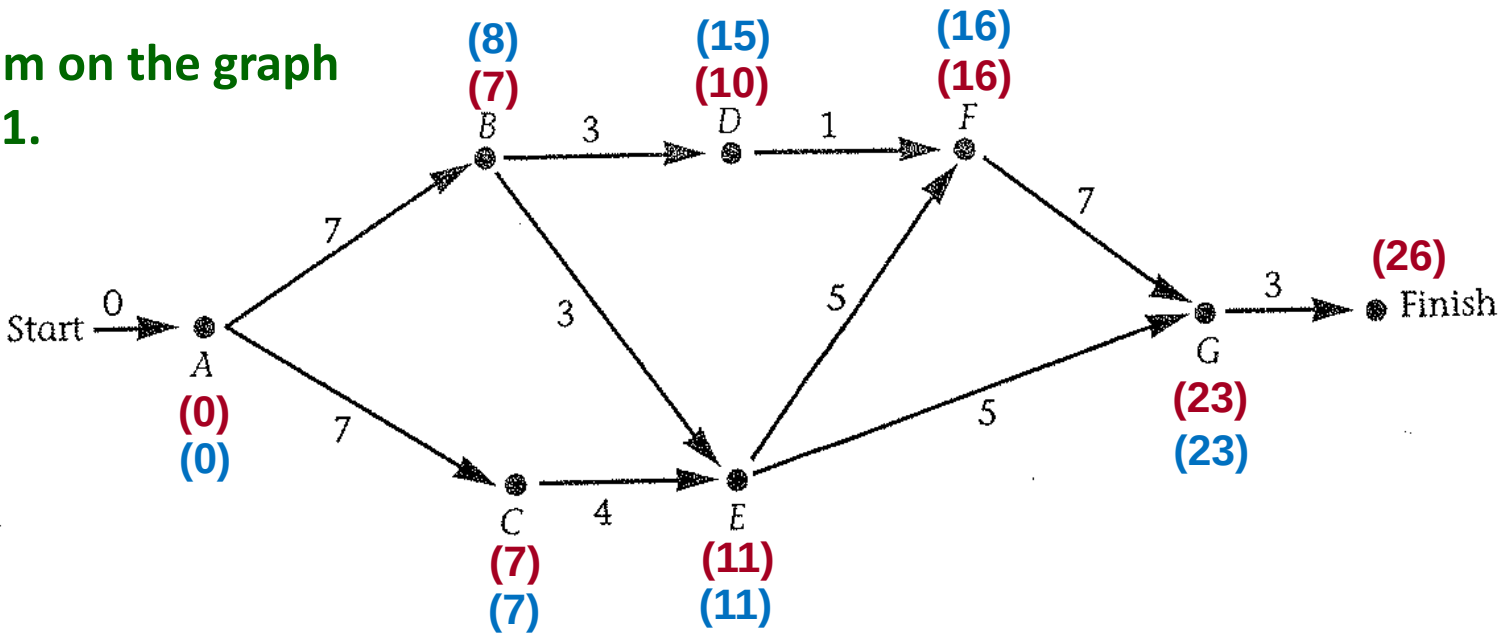


Key

*** = EST**

*** = LST**

Tonight's HW

In your HW packet…

Packet p. 3

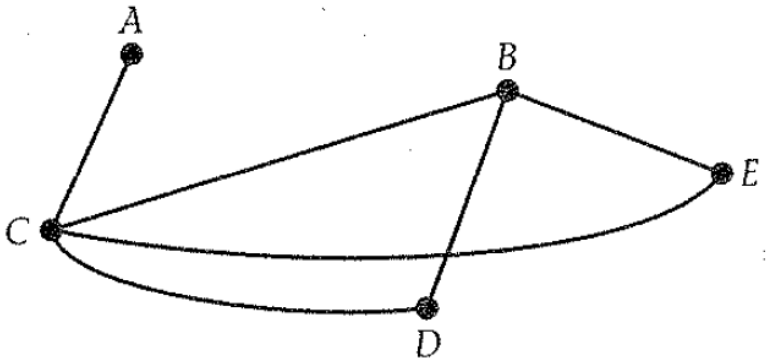
and Packet p. 4 #8, 9

The Vocabulary and Representations of Graphs

Section 4.3

Vocabulary for Graphs

- **Graphs show relationships between objects.**
  - **Vertices are the objects**
  - **Edges are the relationships**

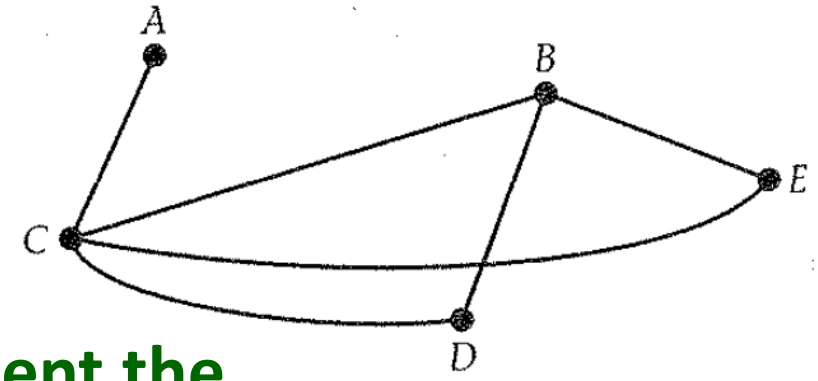


The vertices in this graph represent the starting five players on high school basketball team.

1. **Which player has only one friend?**
2. **How many friends does E have? Who are they?**
3. **Redraw the graph so that A has no friends.**

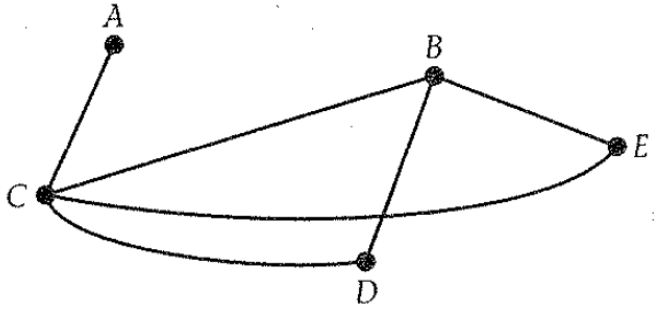
Vocabulary for Graphs ANSWERS

- Graphs show relationships between objects.
  - Vertices are the objects
  - Edges are the relationships



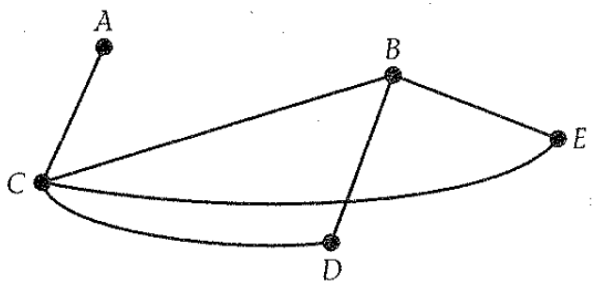
The vertices in this graph represent the starting five players on high school basketball team.

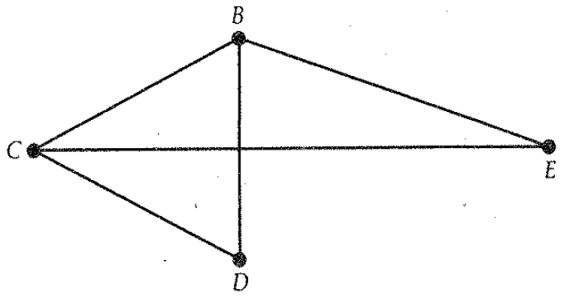
1. Which player has only one friend? **A**
2. How many friends does E have? **2**
   Who are they? **B, C**
3. Redraw the graph so that A has no friends.

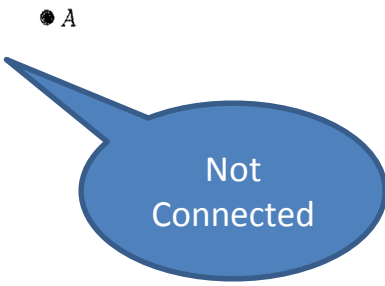


The Vocabulary & Representations of Graphs

When two vertices are connected with an edge, they are said to be **Adjacent**.

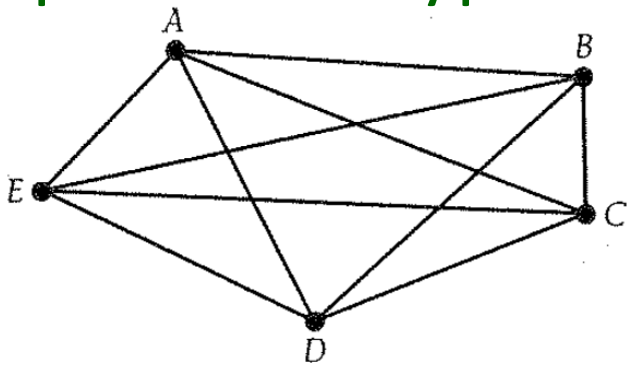


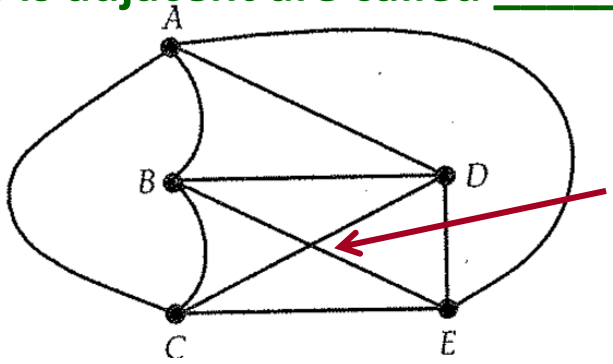




A graph is **Connected** if there is a *path* between each pair of vertices.
Every vertex is *reachable* from any other vertex.

Graphs in which every pair of vertices is adjacent are called **Complete**.





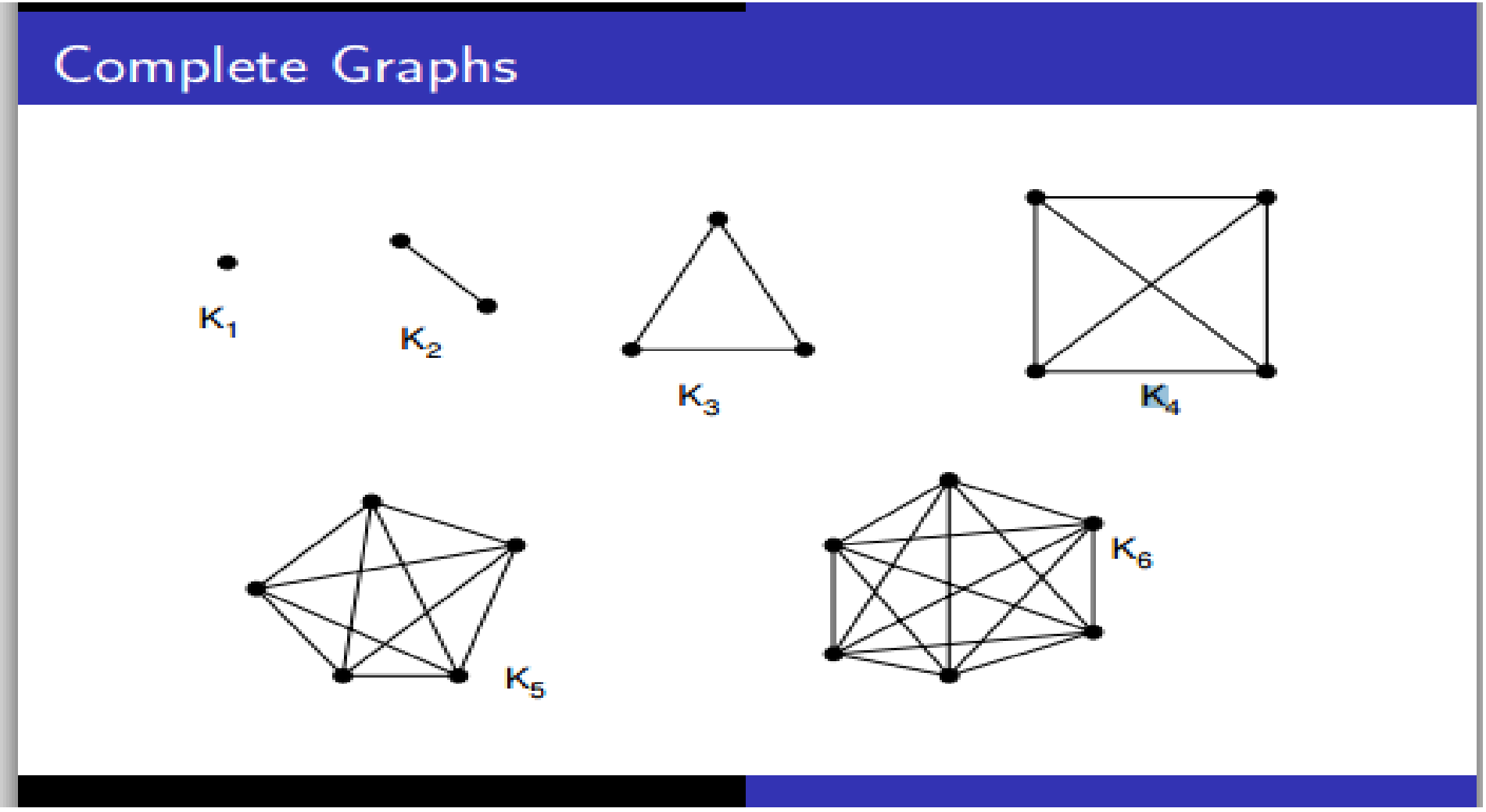
Notice, this is not a vertex even though the edges cross.

These two complete graphs have the same representation. Each vertex is adjacent to every other.

Complete graphs are denoted by K_n, where n is the number of vertices in the graph.

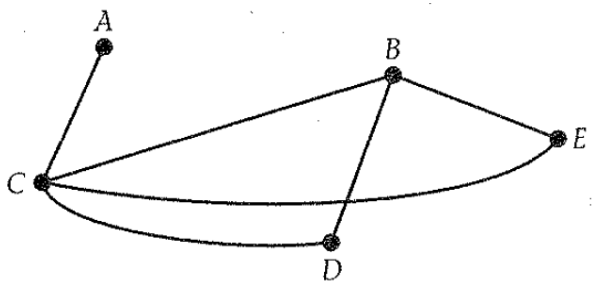
The two complete graphs above can be denoted as K_5.

Complete Graphs - Every vertex is connected to all other vertices.



Alternate Representations of Graphs

Graphs can be represented in several different ways. The diagram we have used so far is just one of those ways.



We can also represent a graph by listing the Set of Vertices and the Set of Edges.

The graph above can be represented like this:

Vertices = { A, B, C, D, E } Edges = { AC, CB, CE, CD, BD, BE }

A third way to represent this information is with an Adjacency Matrix.

$$\begin{array}{c} \\ A \\ B \\ C \\ D \\ E \end{array} \begin{array}{c} \begin{array}{ccccc} A & B & C & D & E \end{array} \\ \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \end{bmatrix} \end{array}$$

The entry in row 2, column 4 is a 1. That indicates that vertices B and D are adjacent. An edge exists between them.

You will be expected to represent graphs as:

a Diagram

Sets of Vertices and Edges

an Adjacency Matrix

Representations of Graphs

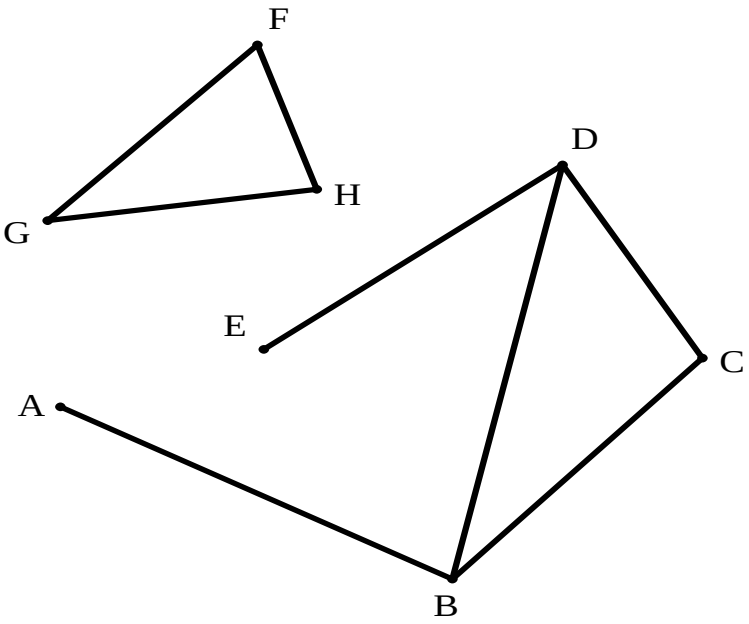
1. Is this graph connected?

2. Is this graph complete?

3. Name two vertices that are adjacent to C.

4. Name a path from A to C of length 3.

5. How many vertices are adjacent to D?
   (also known as the DEGREE of the vertex)



6. Represent this graph as sets of vertices and edges.

7. Represent this graph as an adjacency matrix.

	A	B	C	D	E	F	G	H
A								
B								
C								
D								
E								
F								
G								
H								

Representations of Graphs ANSWERS

1. Is this graph connected?

   No

2. Is this graph complete?

   No

3. Name two vertices that are adjacent to C.

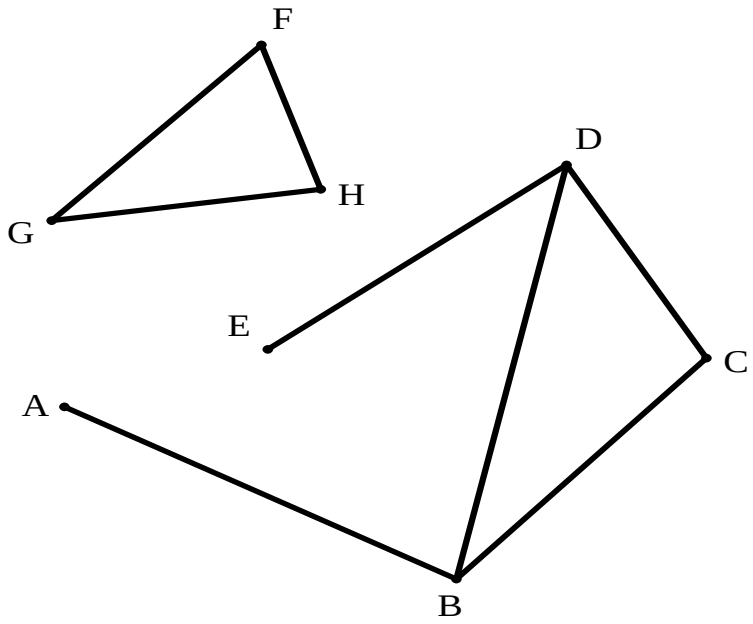
   D & B

4. Name a path from A to C of length 3.

   A, B, D, C

5. How many vertices are adjacent to D?
   (also known as the DEGREE of the vertex)

   3 – E,B,C



6. Represent this graph as sets of vertices and edges.

   Vertices = { A, B, C, D, E, F, G, H }
   Edges = { AB, ED, DC, DB, CB, FG, FH, GH}

7. Represent this graph as an adjacency matrix.

	A	B	C	D	E	F	G	H
A	0	1	0	0	0	0	0	0
B	1	0	1	1	0	0	0	0
C	0	1	0	1	0	0	0	0
D	0	1	1	0	1	0	0	0
E	0	0	0	1	0	0	0	0
F	0	0	0	0	0	0	1	1
G	0	0	0	0	0	1	0	1
H	0	0	0	0	0	1	1	0

Representations of Graphs

Construct a diagram from this adjacency matrix.

$$\begin{array}{c} \\ A \\ B \\ C \\ D \\ E \end{array}\begin{array}{c} \begin{array}{ccccc} A & B & C & D & E \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 1 & 0 \end{bmatrix} \end{array}$$

1. **Is this graph connected?**

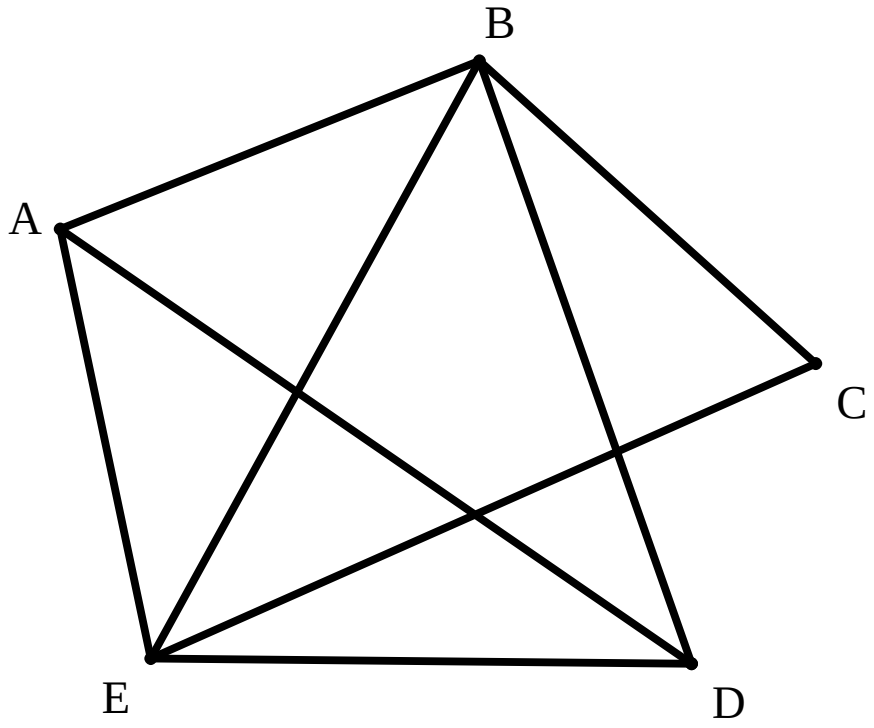
2. **Is this graph complete?**

3. **How many vertices are adjacent to E? (What is the degree of E?)**

Representations of Graphs ANSWERS

Construct a diagram from this adjacency matrix.

	A	B	C	D	E
A	0	1	0	1	1
B	1	0	1	1	1
C	0	1	0	0	1
D	1	1	0	0	1
E	1	1	1	1	0



1. **Is this graph connected?**

   Yes

2. **Is this graph complete?**

   No

3. **How many vertices are adjacent to E? (What is the degree of E?)**

   4 – A,B,C,D

Representations of Graphs

Consider these countries in South America and their borders.

Argentina (A) borders Uruguay (U), Paraguay (Pa), Bolivia (B), and Chile (Ch).

Paraguay (Pa) borders Bolivia (B).

Bolivia (B) borders Chile (Ch) and Peru (Pe).

Ecuador (E) borders Peru (Pe) and Columbia (Co).

Venezuela (V) borders Columbia (Co).

Construct a graph representing these border relationships.

a) Is it a complete graph?

b) Is it a connected graph?

Representations of Graphs ANSWERS

Consider these countries in South America and their borders.

Argentina (A) borders Uruguay (U), Paraguay (Pa), Bolivia (B), and Chile (Ch).

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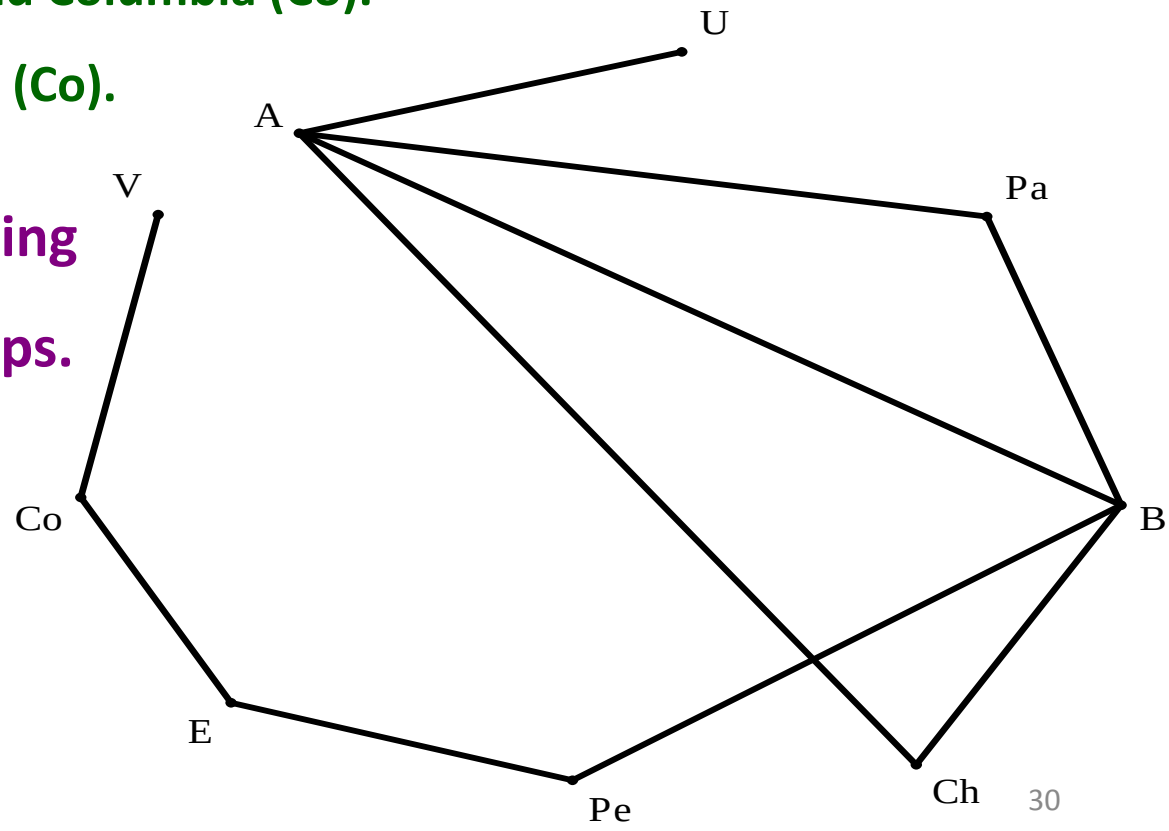


a) Is it a complete graph?

No

b) Is it a connected graph?

Yes

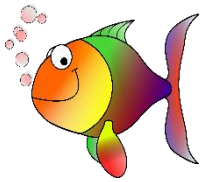
Do this:

1. Mr. Butler bought six different types of fish. Some of the fish can live in the same aquarium, but others cannot. Guppies can live with Mollies, Swordtails can live with Guppies, Plecostomi can live with both Mollies and Guppies, Gold Rams can live only with Plecostomi, and Piranhas cannot live with any of the other ifsh. Draw a graph to illustrate this.

Gold Rams

Plecostomi

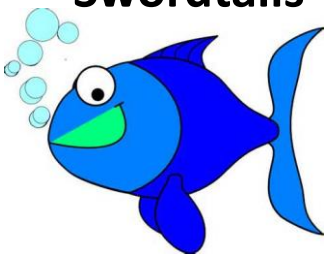
Mollies



Piranhas

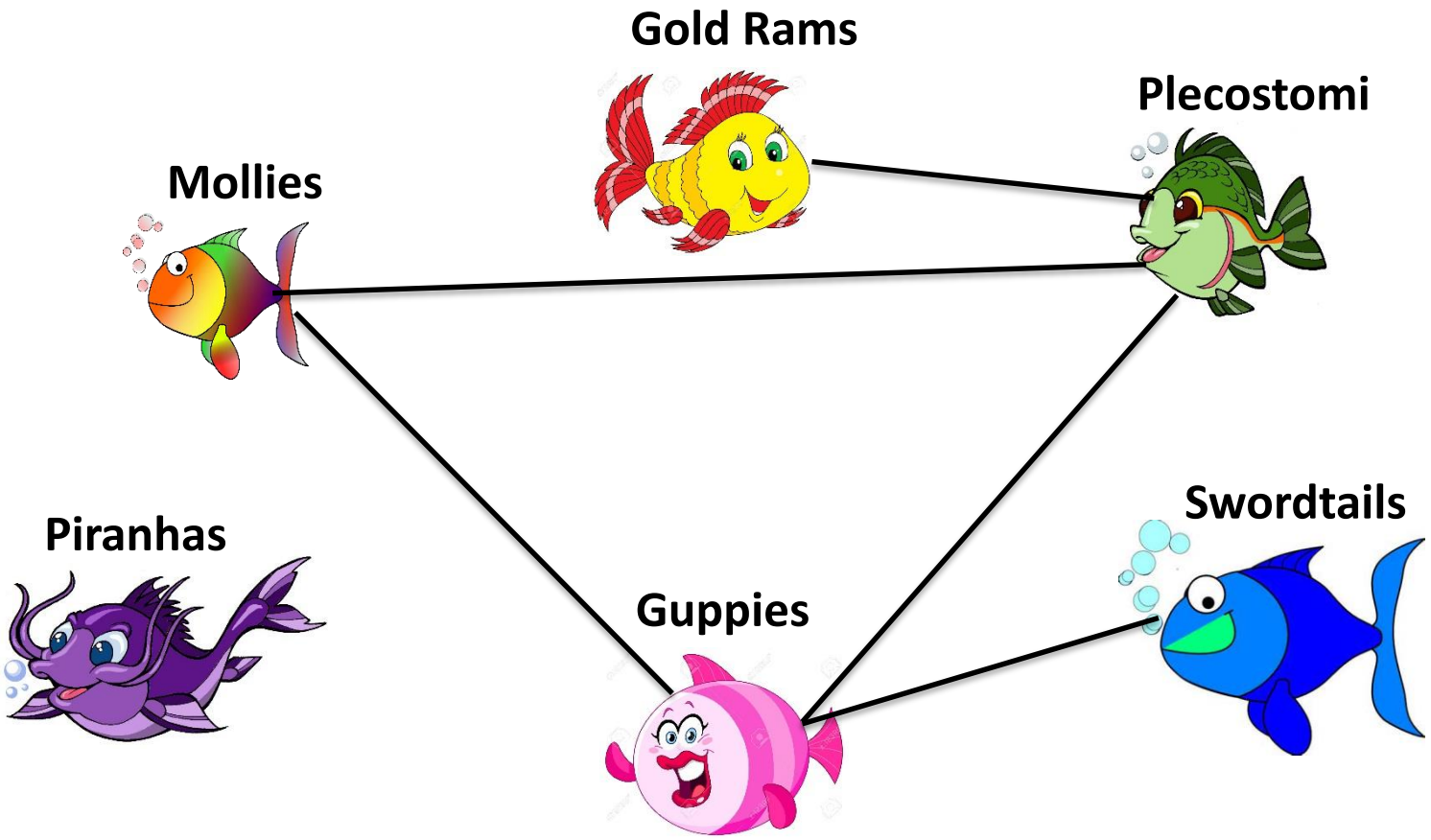
Guppies

Swordtails



Do this ANSWERS:

1. Mr. Butler bought six different types of fish. Some of the fish can live in the same aquarium, but others cannot. Guppies can live with Mollies, Swordtails can live with Guppies, Plecostomi can live with both Mollies and Guppies, Gold Rams can live only with Plecostomi, and Piranhas cannot live with any of the other ifsh. Draw a graph to illustrate this.



Another…

2. Construct a graph for each of the following sets of vertices and edges. Which of the graphs are connected? Which are complete?

a. $V = \{A, B, C, D, E\}$
$E = \{AB, AC, AD, AE, BE\}$.

b. $V = \{M, N, O, P, Q, R, S\}$
$E = \{MN, SR, QS, SP, OP\}$.

c. $V = \{E, F, G, J, K, M\}$
$E = \{EF, KM, FG, JM, EG, KJ\}$.

d. $V = \{W, X, Y, Z\}$
$E = \{WX, XZ, YZ, XY, WZ, WY\}$.

Another… ANSWERS

2. Construct a graph for each of the following sets of vertices and edges. Which of the graphs are connected? Which are complete?

a. $V = \{A, B, C, D, E\}$
$E = \{AB, AC, AD, AE, BE\}$.

a. Connected but NOT complete

b. $V = \{M, N, O, P, Q, R, S\}$
$E = \{MN, SR, QS, SP, OP\}$.

b. NOT connected and NOT complete

c. $V = \{E, F, G, J, K, M\}$
$E = \{EF, KM, FG, JM, EG, KJ\}$.

c. NOT connected and NOT complete

d. $V = \{W, X, Y, Z\}$
$E = \{WX, XZ, YZ, XY, WZ, WY\}$.

d. Connected and complete

Homework

In your HW packet…

Packet p. 3

and Packet p. 4 #8, 9