

# **Unit 7 Day 1**

**4.1 and 4.2**

**Task Graphs and  
Critical Paths**

**Remember:**

**College Tour Activity is due  
at start of class tomorrow!!**

**\*Paper Clip Rubric on Top**

**\*Staple the other pages**

**\*Be sure to answer ALL  
questions AND show work for  
your calculations**

# Homework

- Packet p. 1-2 (Day 1)
- Finish College Tour Activity

# Notes Day 1

## Task Graphs and Critical Paths

# GRAPH APPLICATIONS

- How does a building contractor organize all of the jobs needed to complete a project?
- How do your parents manage to get all the food ready for Thanksgiving dinner at the same time?

## PLANNING!!

- Everybody plans. Planning your day-to-day activity seems like second nature, but it's not formal or organized.
- For people who work in the business world, this haphazard method of planning is not an efficient or cost effective way to complete a job.
- A scientific, more organized method must be used.

# GRAPH APPLICATIONS

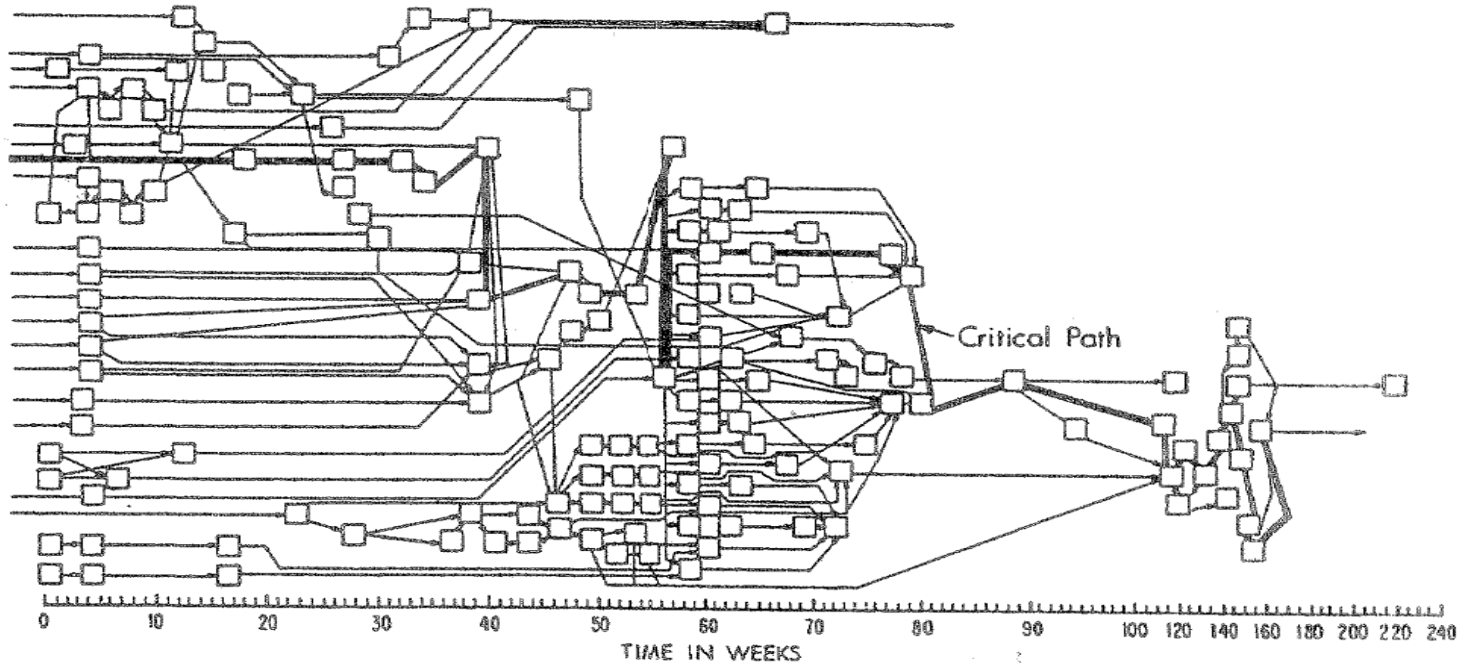
**What do you know about graphs ?**

- 1. Line graphs**
- 2. Linear equations, Quadratic equations**
- 3. Bar graphs**
- 4. Circle graphs, Pie charts**

**The graphs we will examine look different from graphs you are used to thinking of.**

**But, they still provide a way of displaying and organizing information.**

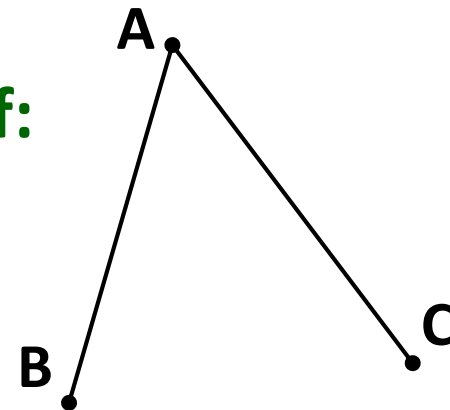
# GRAPH APPLICATIONS



System Flow Plan - Propulsion Component.

This new kind of graph is made up of:

- Vertices (points)
- Edges (connecting lines)



This graph has 3 vertices & 2 edges

The Central High yearbook staff has only **16** days left before the deadline for completing their book. They are running behind schedule and still have several tasks to finish. The remaining tasks and times that it takes to complete each task are listed in the following table.

Is it possible to complete the yearbook on time ?

Task	Time (days)	Prerequisite Task
Start	0	---
A. Buy film	1	none
B. Load cameras	1	A
C. Take photos of clubs	3	B
D. Take sports photos	2	C
E. Take photos of teachers	1	B
F. Develop film	2	D, E
G. Design layout	5	D, E
H. Print and mail pages	3	F, G



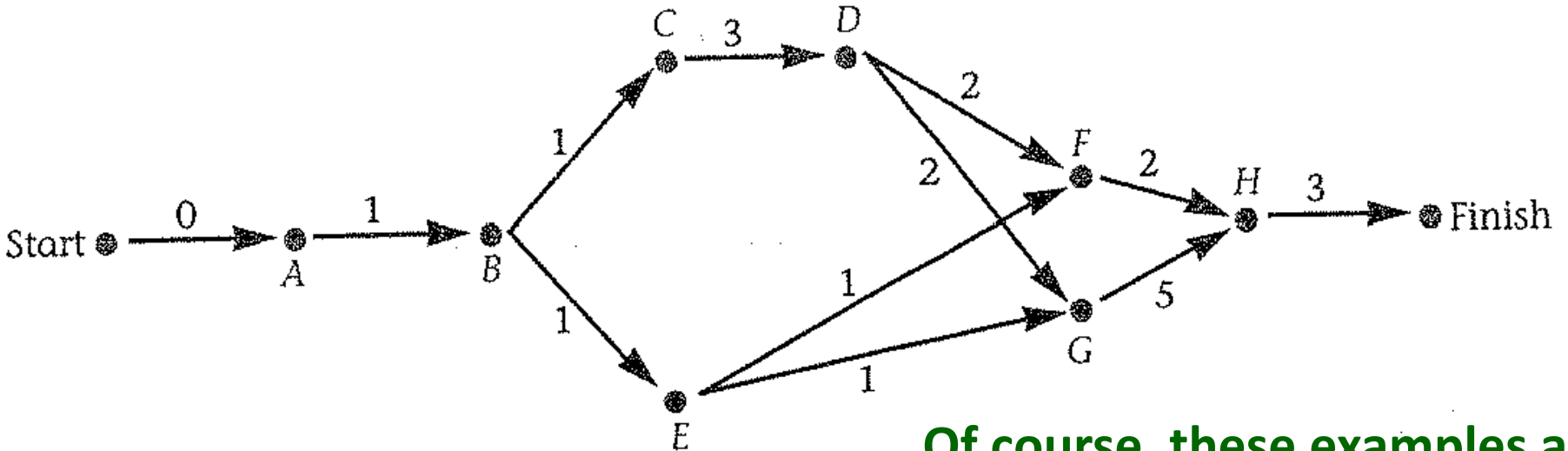
**18**

You may have noticed, some of the yearbook jobs can be done simultaneously, while several of them cannot be started until others have been completed.

Assume these prerequisites. Now, how soon can the project be completed ?



Drawing a graph of this information makes it easier to see the relationships among the tasks. Consider the **TASK GRAPH** below.



Of course, these examples are more simple than constructing a submarine, but the concept is the same.

Examine the construction of the graph based on the information in the table.

- Prerequisites
- Task Times

Task	Time (days)	Prerequisite Task
Start	0	---
A. Buy film	1	none
B. Load cameras	1	A
C. Take photos of clubs	3	B
D. Take sports photos	2	C
E. Take photos of teachers	1	B
F. Develop film	2	D, E
G. Design layout	5	D, E
H. Print and mail pages	3	F, G

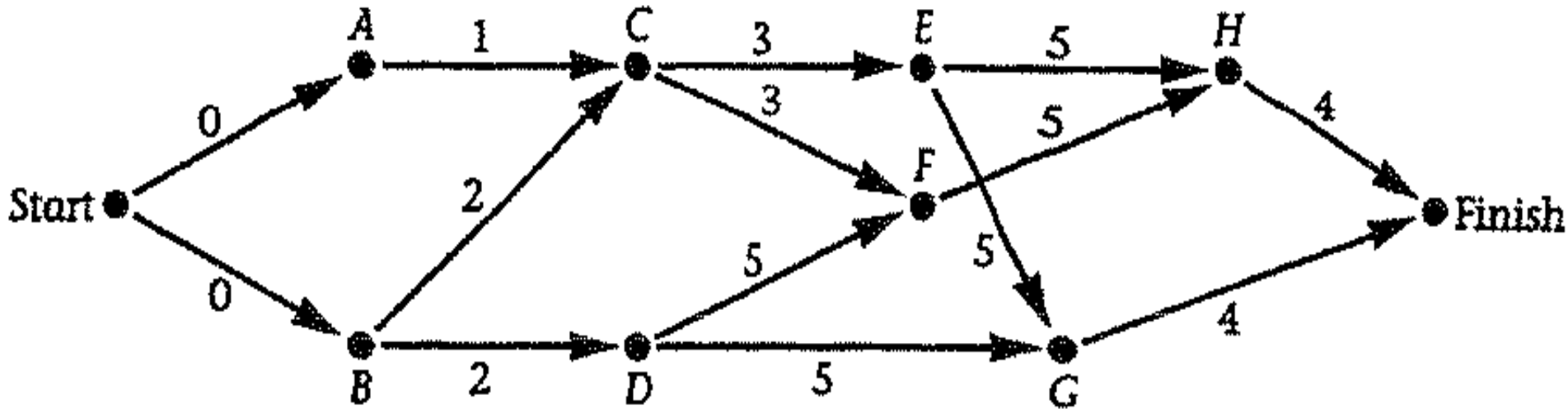
# For now, let's practice drawing the graphs!

Use the task table to draw a task graph with labeled vertices and edges.

Task	Time	Prereqs
Start	0	---
A	1	none
B	2	none
C	3	A, B
D	5	B
E	5	C
F	5	C, D
G	4	D, E
H	4	E, F
Finish		

# ANSWER: For now, let's practice drawing the graphs!

Use the task table to draw a task graph with labeled vertices and edges.



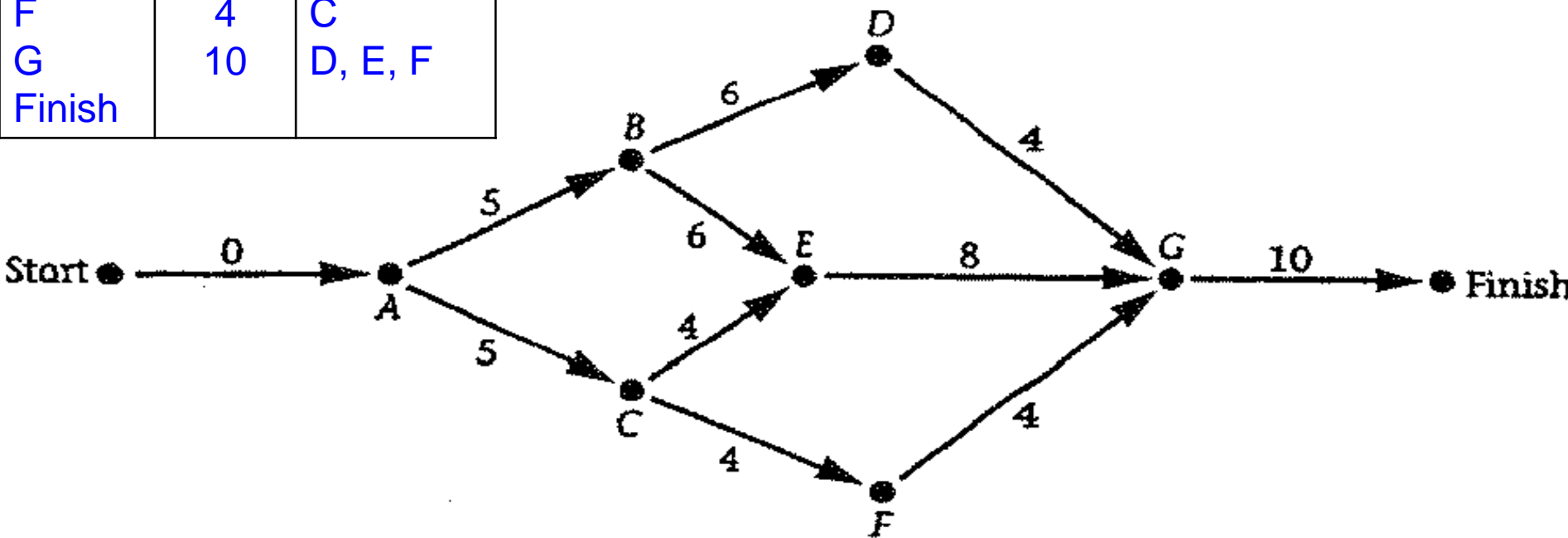
Task	Time	Prereqs
Start	0	---
A	1	none
B	2	none
C	3	A, B
D	5	B
E	5	C
F	5	C, D
G	4	D, E
H	4	E, F
Finish		

Task	Time	Prereqs
Start	0	---
A	5	none
B	6	A
C	4	A
D	4	B
E	8	B, C
F	4	C
G	10	D, E, F
Finish		

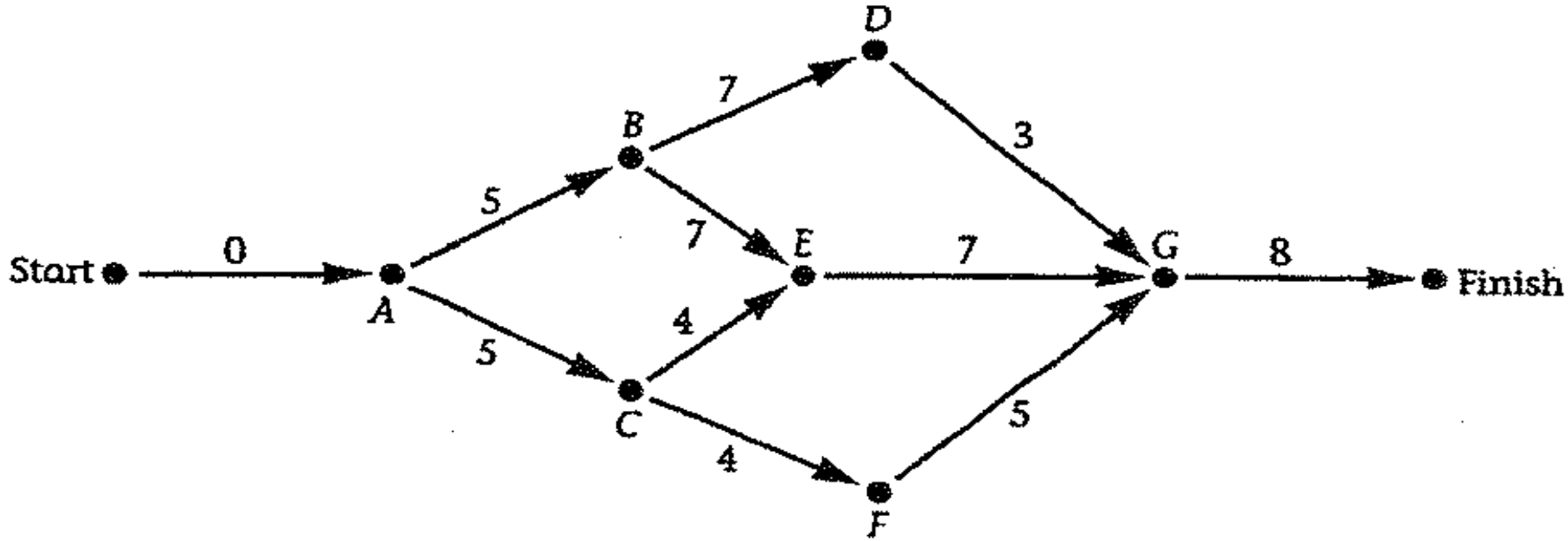
Use the task table to draw a task graph with labeled vertices and edges.

Task	Time	Prereqs
Start	0	---
A	5	none
B	6	A
C	4	A
D	4	B
E	8	B, C
F	4	C
G	10	D, E, F
Finish		

**ANSWERS:** Use the task table to draw a task graph with labeled vertices and edges.

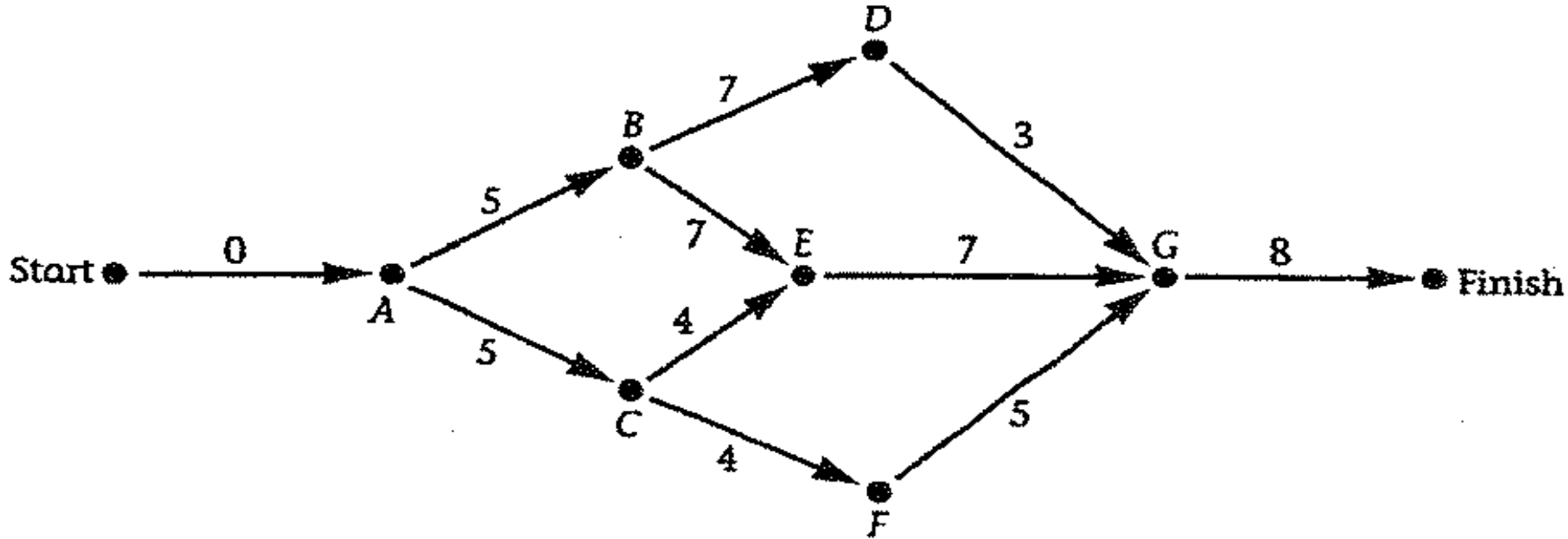


Create task table based on the graph.



Task	Time	Prereqs
Start	0	---
A		
B		
C		
D		
E		
F		
G		
Finish		

**ANSWERS:** Create task table based on the graph.



Task	Time	Prereqs
Start	0	---
A		
B		
C		
D		
E		
F		
G		
Finish		

Task	Time	Prereqs
Start	0	---
A	5	none
B	7	A
C	4	A
D	3	B
E	7	B, C
F	5	C
G	8	D, E, F
Finish		

# Critical Paths

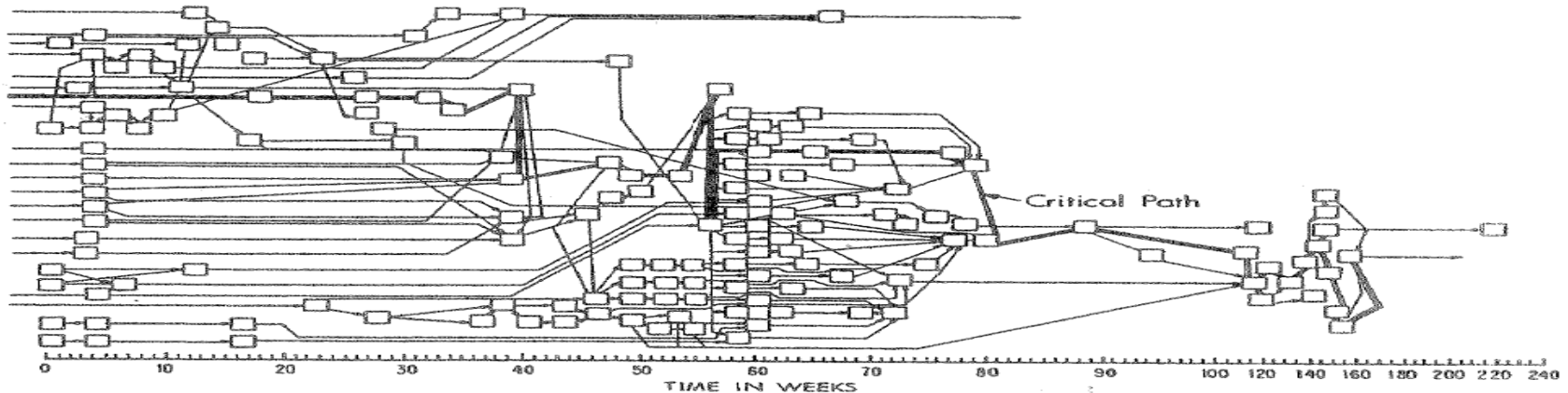
## Section 4.2





# Task Graphs – Critical Paths

➤ It is relatively simple to find the shortest time needed to complete a project if the project consists of only a few activities. As the tasks increase in number, the problem becomes more difficult to solve by inspection alone.



System Flow Plan – Propulsion Component.

➤ In the 1950s the government was faced with the need to complete very complex systems such as the US Navy Polaris Submarine project. In order to do this efficiently, a method was developed called

**PERT** (*Program Evaluation & Review Technique*)

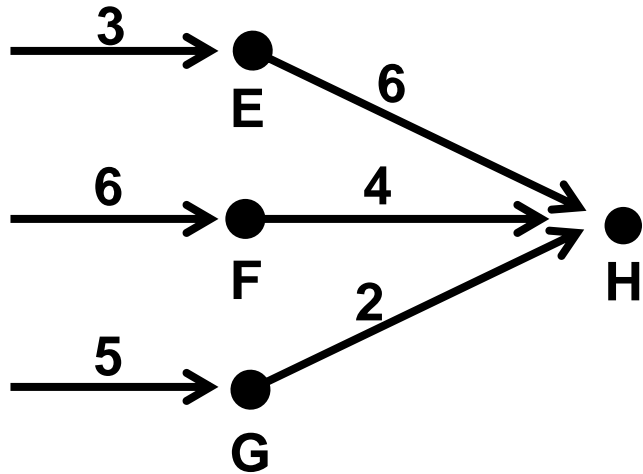
in which those tasks which were critical to the earliest completion of the project were targeted.

The path of targeted tasks from the start to the finish of the project became known as the **CRITICAL PATH**.

# Task Graphs – Critical Paths

- We need a systematic way to identify the critical path of a project.
- An Earliest Start Time (EST) must be found for each task/activity. (the earliest that an activity can begin if all the activities preceding it begin as early as possible)

Consider this portion of a task graph:  
(times are in minutes)



E to H is a total of 9 mins.

F to H is a total of 10 mins.

G to H is a total of 7 mins.

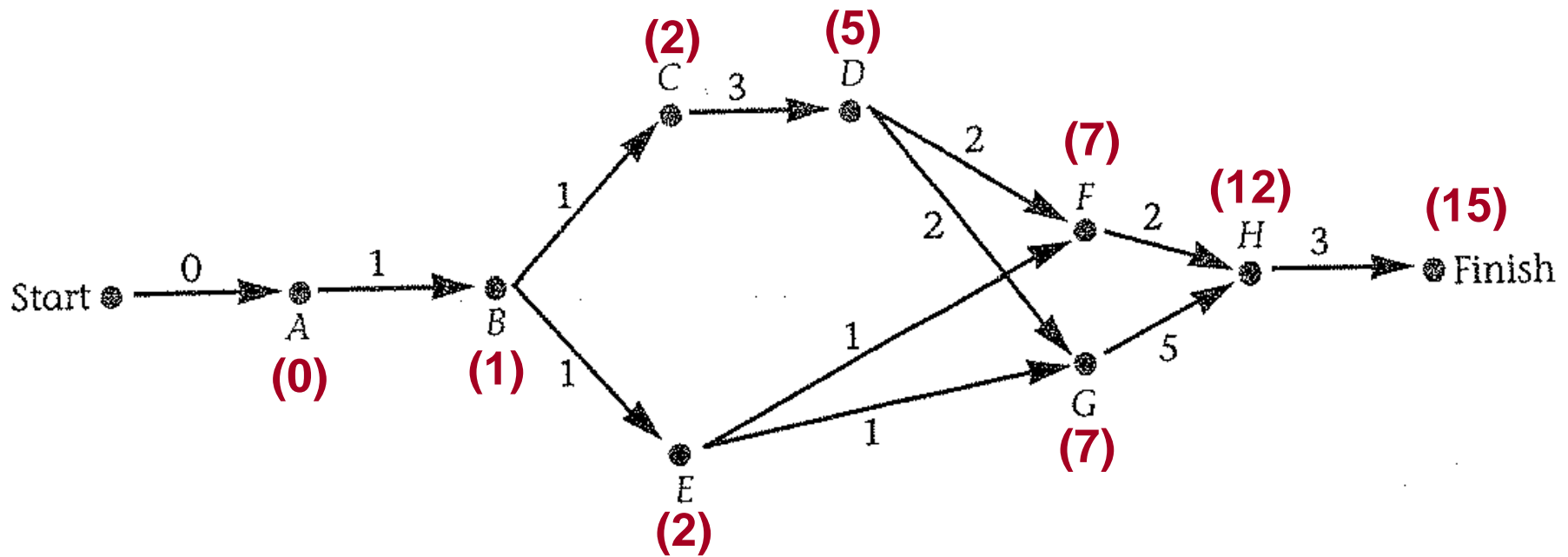
For H to start, ALL of its prerequisites have to finish.

So, the EST for H is 10 mins.

# Task Graphs – Critical Paths

## Earliest Start Time (EST)

Recall the Yearbook example from section 4.1.



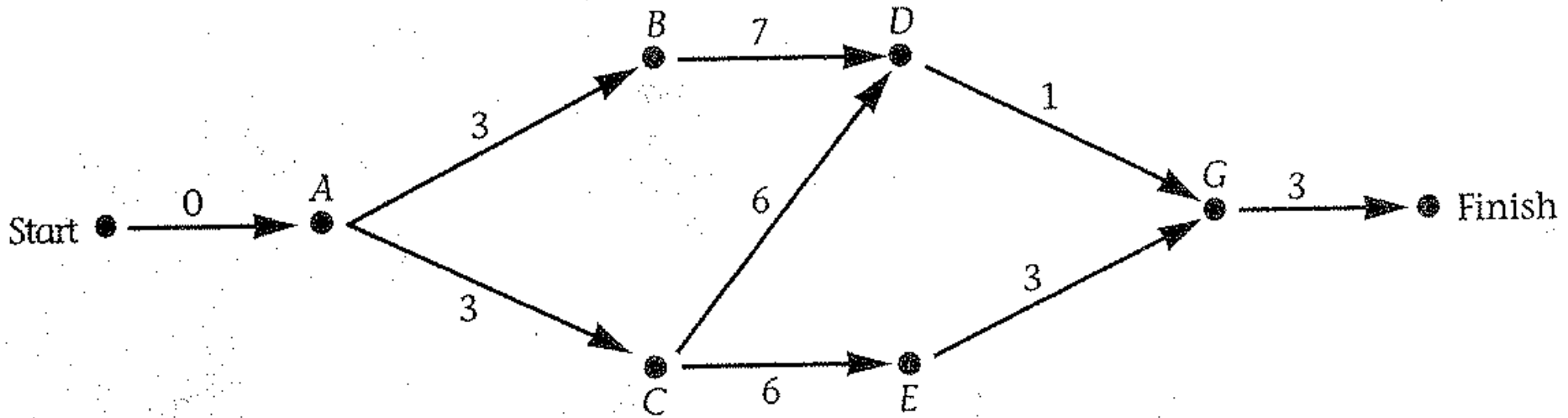
To calculate the EST for each task, begin at the start and label each vertex with the smallest possible time that is needed before the task can begin.

The EARLIEST TIME in which the Yearbook project can be completed is **15** days. It seems paradoxical, but the least amount of time it takes to complete all the tasks in the project is the same as the **LONGEST PATH** through the graph.

That longest path is the **CRITICAL PATH**.

# Task Graphs – Critical Paths

To figure the CRITICAL PATH, trace through the path backwards.



Identify the EST ( Earliest Start Time ) for each task.

Identify the Critical Path.

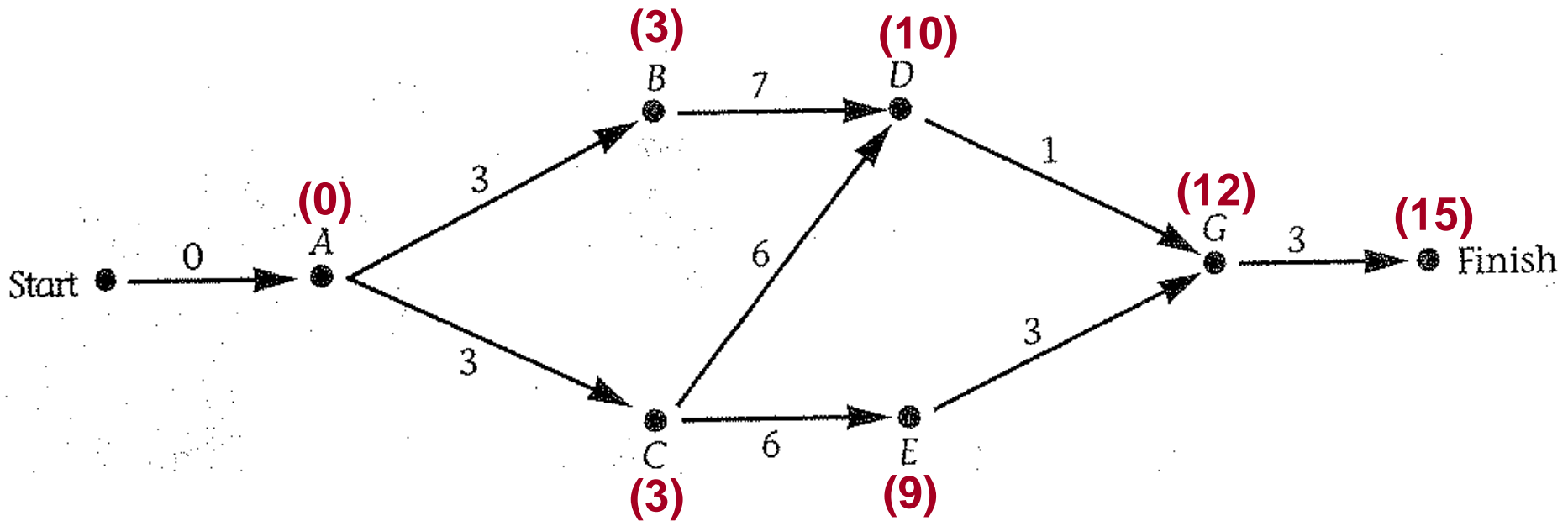
Trace through the path backwards.

Which path did each EST come from? \_\_\_\_\_

If Task B took 5 minutes instead of 7, would that shorten the completion time? \_\_\_\_\_

If Task E took 2 minutes instead of 3, would that shorten the completion time? \_\_\_\_\_

# ANSWERS: Task Graphs – Critical Paths



Identify the EST ( Earliest Start Time ) for each task.

Identify the Critical Path.

Trace through the path backwards.

Which path did each EST come from? **Start-ACEG-Finish**

If Task B took 5 minutes instead of 7, would that shorten the completion time? **No**

If Task E took 2 minutes instead of 3, would that shorten the completion time?

**Yes, it would cut the project time to 14 minutes.**

# You Try

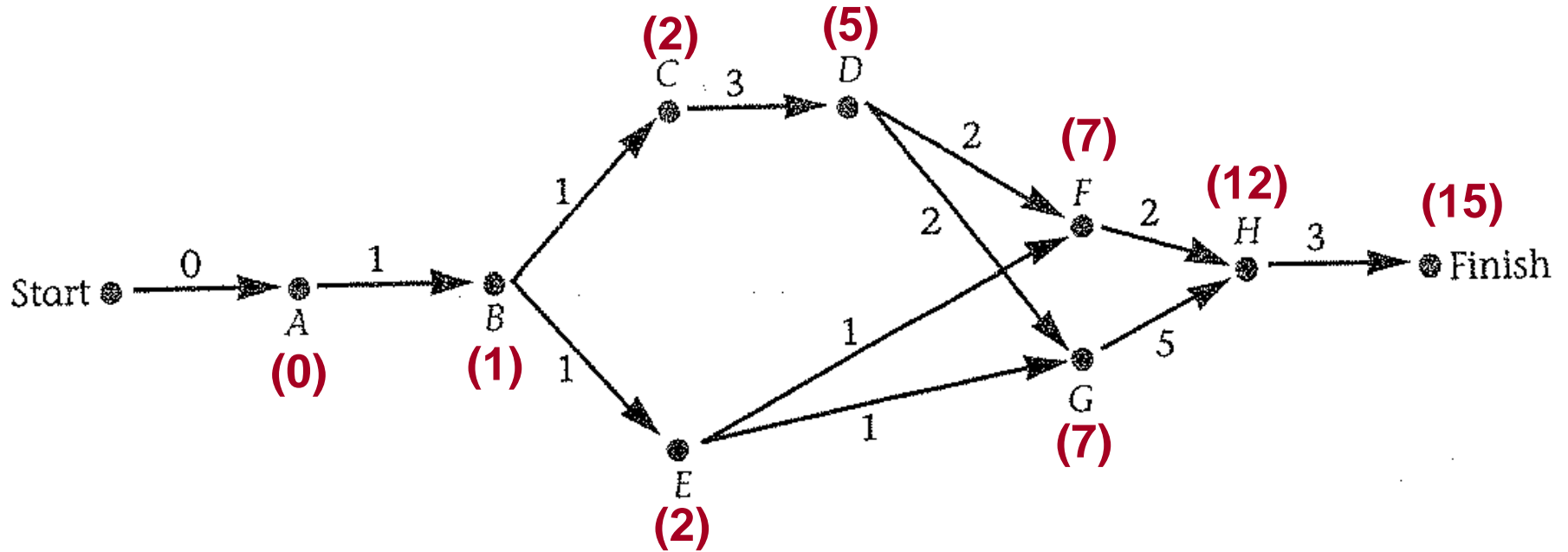
- Packet p. 1 #7b (HINT: Find ESTs then the Critical Path to find the shortest time to finish the project).

# Latest Start Time (LST)

If an activity is not on the critical path, it is possible for it to start later than its earliest start time and not delay the project. The latest a task can begin without delaying the project's minimum completion time is known as the **LATEST START TIME (LST)** for the task.

# Task Graphs – Critical Paths

To figure the **CRITICAL PATH** of the Yearbook example, trace through the path backwards.



The Critical Path for the Yearbook project is

Find the Latest Start Time for task F.

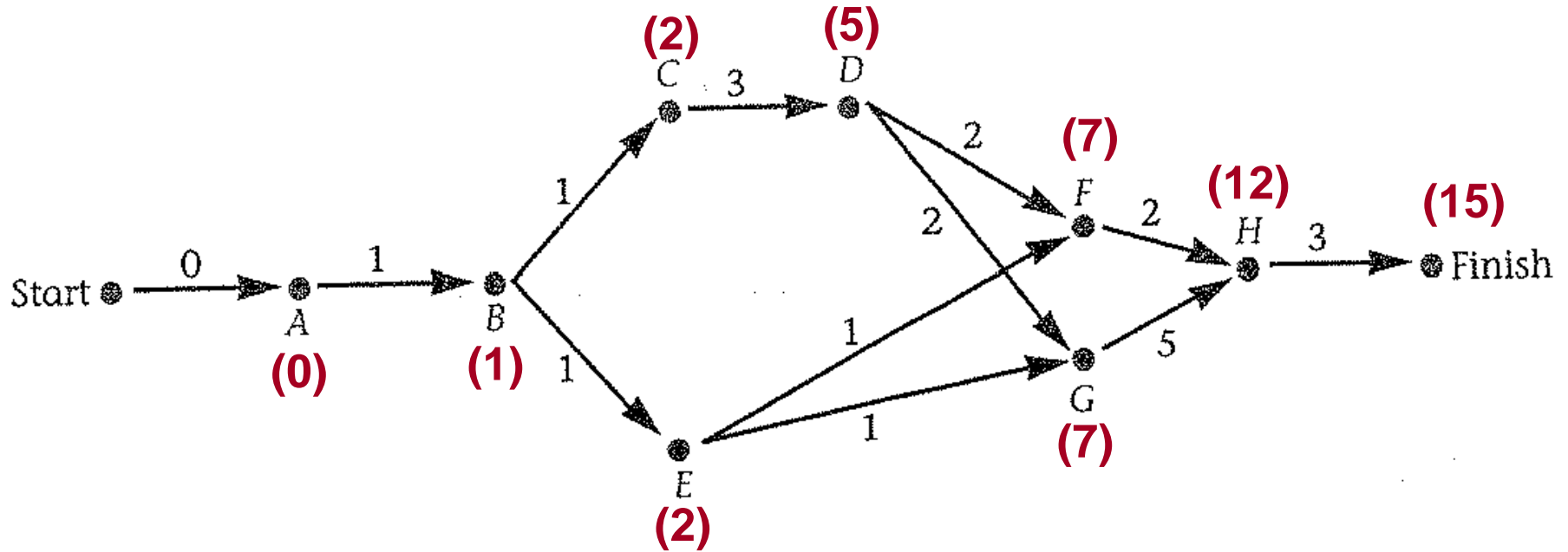
Find the Latest Start Time for Task D.

← same as EST because  
it is on the Critical Path



# ANSWERS: Task Graphs – Critical Paths

To figure the **CRITICAL PATH** of the Yearbook example, trace through the path backwards.



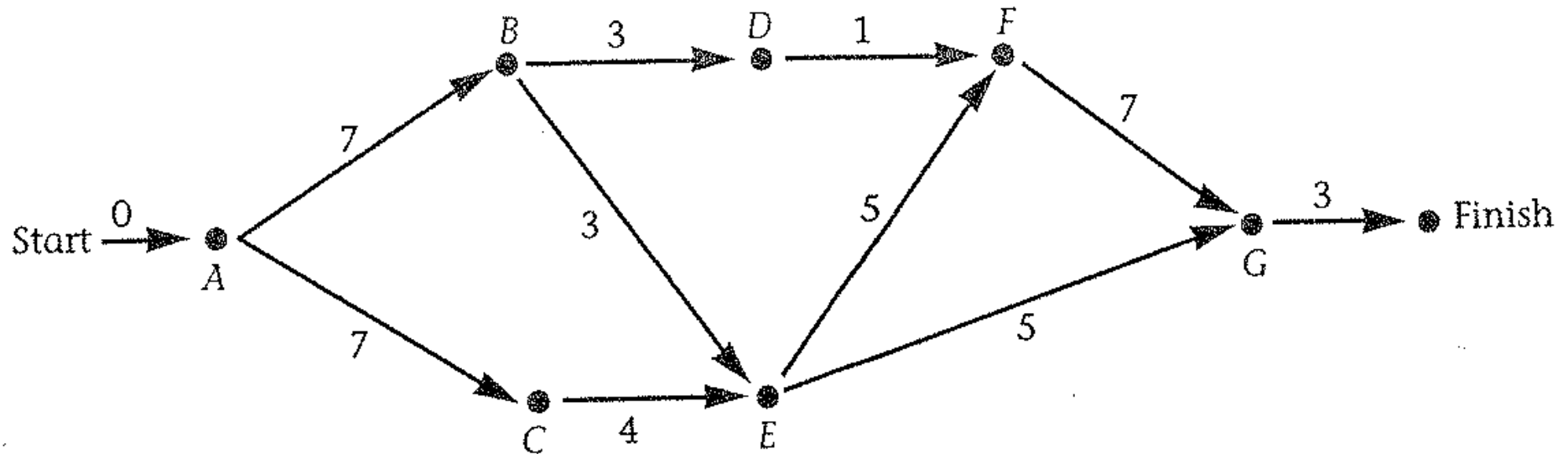
The Critical Path for the Yearbook project is **Start-ABCDGH-Finish** .

Find the Latest Start Time for task F. **10**

Find the Latest Start Time for Task D. **5** ← same as EST because it is on the Critical Path

# Try this:

Complete the following.



1.

Task	EST
A	0
B	7
C	
D	
E	
F	
G	

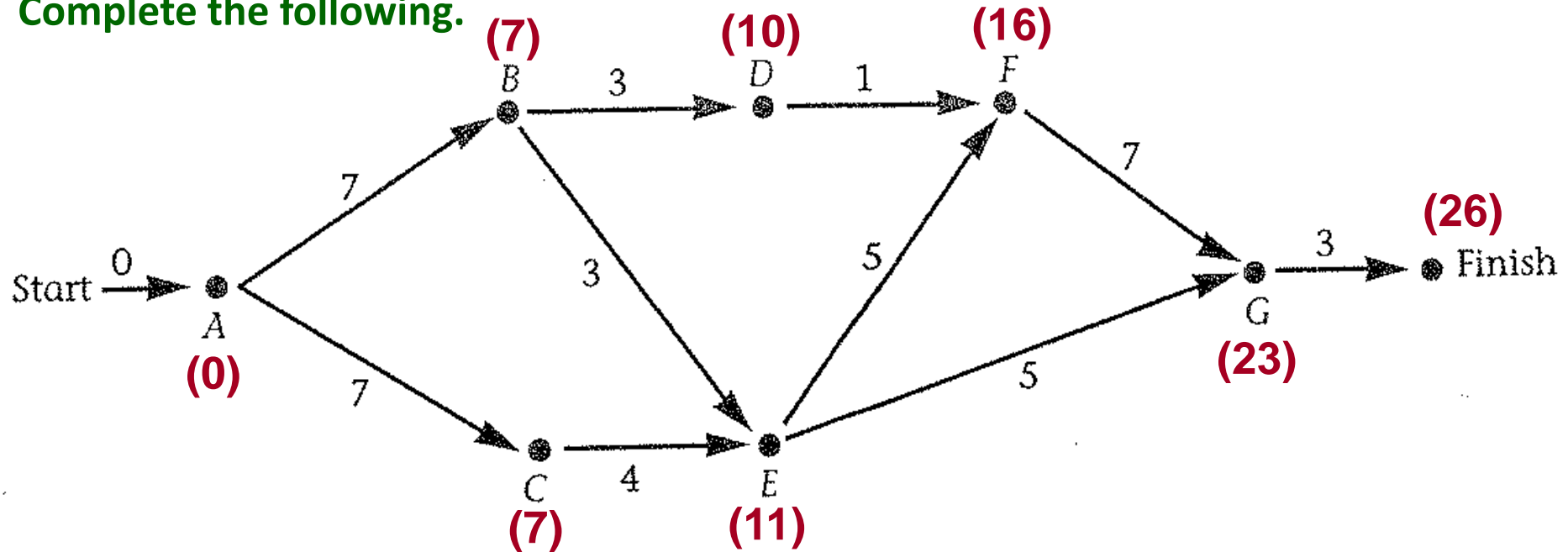
Minimum Project Time =

Critical Path(s) =

Latest Start Time for D =

# Try this: ANSWERS

Complete the following.



1.

Task	EST
A	0
B	7
C	7
D	10
E	11
F	16
G	23

**Minimum Project Time = 26**

**Critical Path(s) = Start-ACEFG-Finish**

**Latest Start Time for D = 15**

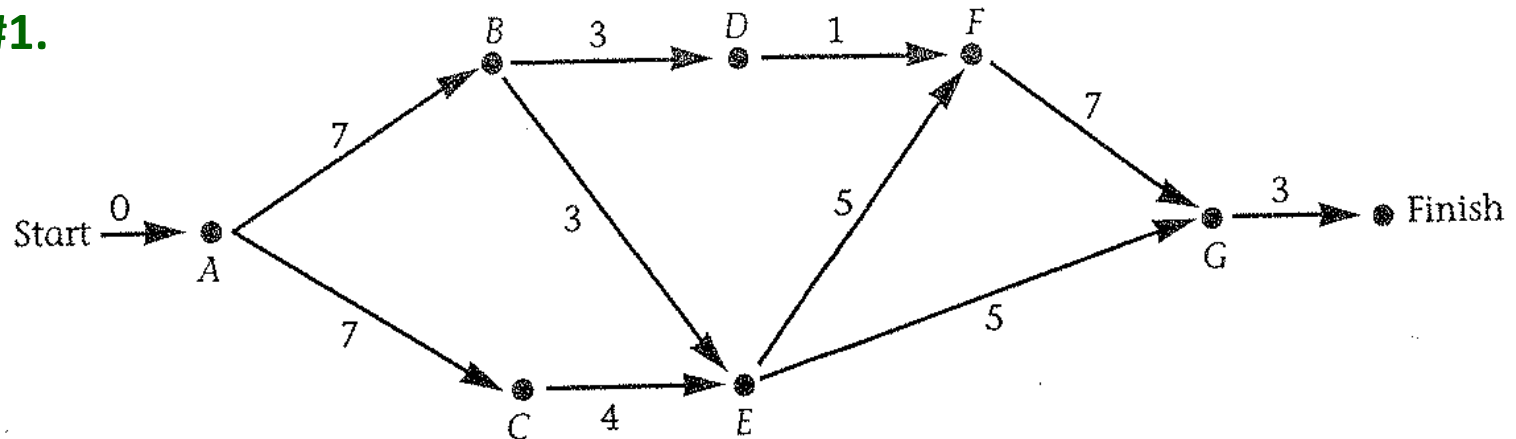
## Section 4.2 – Exercise #9,10

# LATEST START TIME Algorithm ☺

The general algorithm for finding the LST for each task in a graph is :

1. Begin with Finish and use the minimum project time.
2. Subtract from the minimum project time the time that it takes to complete the task(s) preceding it. This is the LST for the preceding tasks. Label the vertex with its LST.
3. If the time of a preceding task is dependent on more than one edge, choose the earliest time.
4. Continue until all vertices are labeled.

Try the algorithm on the graph  
from exercise #1.



**Key**

\_\_\_ = EST

\_\_\_ = LST

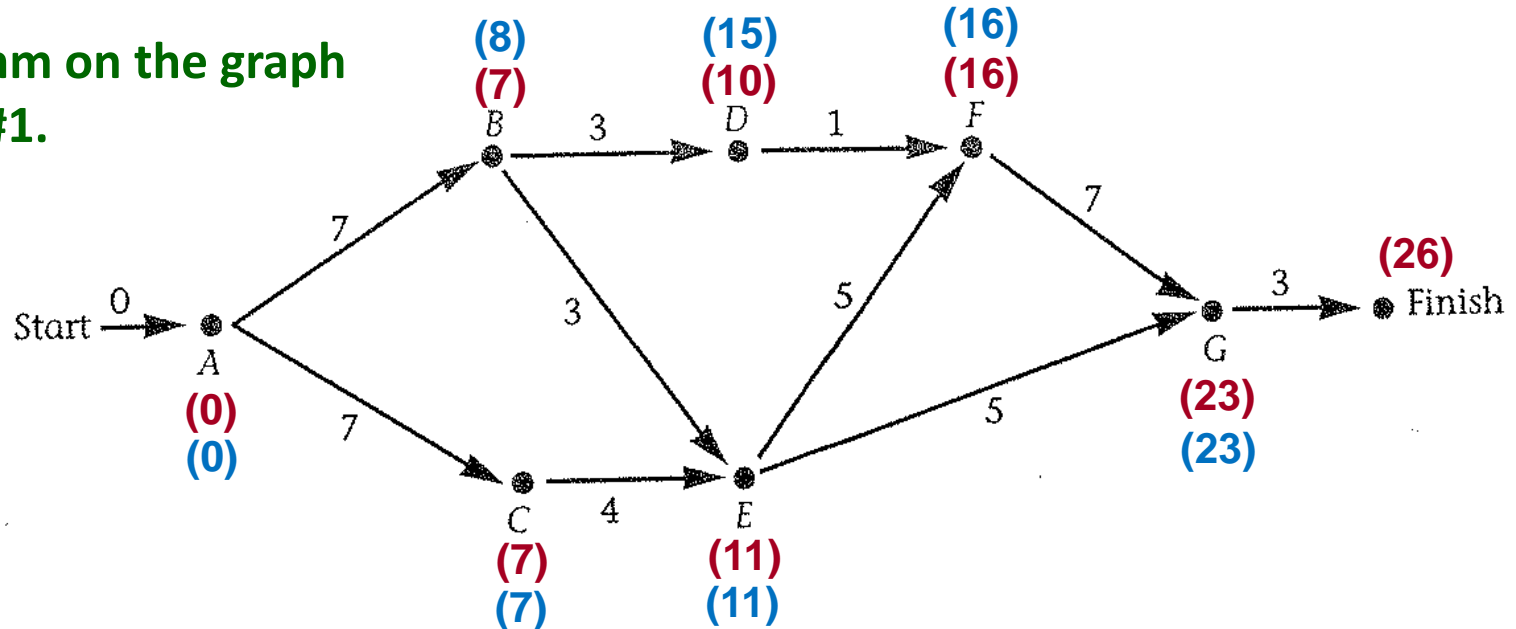
## Section 4.2 – Exercise #9,10

# LATEST START TIME Algorithm ANSWERS ☺

The general algorithm for finding the LST for each task in a graph is :

1. Begin with Finish and use the minimum project time.
2. Subtract from the minimum project time the time that it takes to complete the task(s) preceding it. This is the LST for the preceding tasks. Label the vertex with its LST.
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Try the algorithm on the graph from exercise #1.



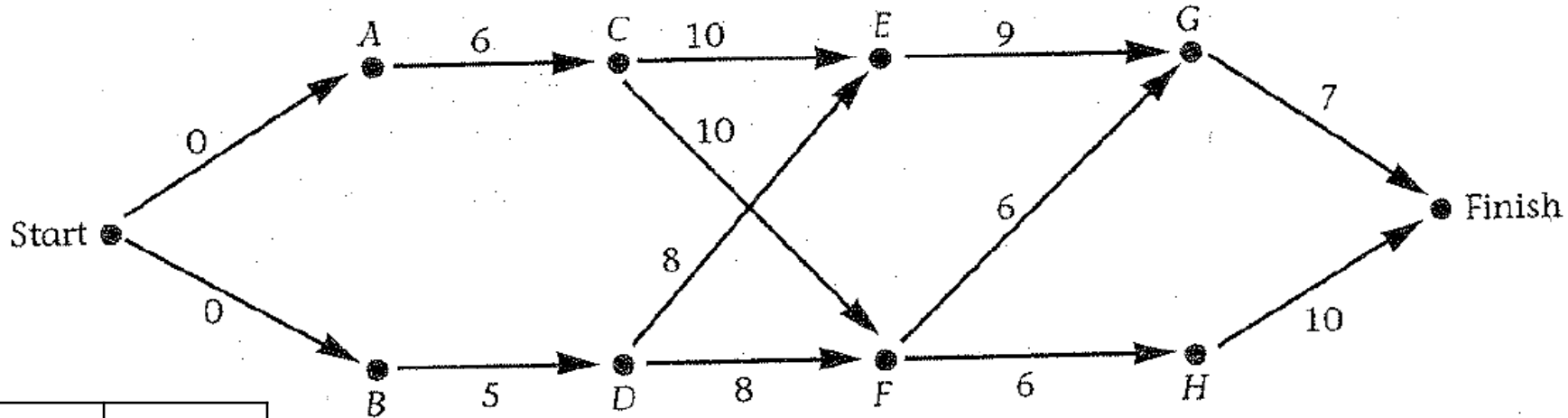
**Key**

\* = EST

\* = LST

# And this...

List the tasks and earliest start times in a table, as in exercise #1.  
Determine the minimum project time and all the critical paths.



2.

Task	EST
A	
B	
C	
D	
E	
F	
G	
H	

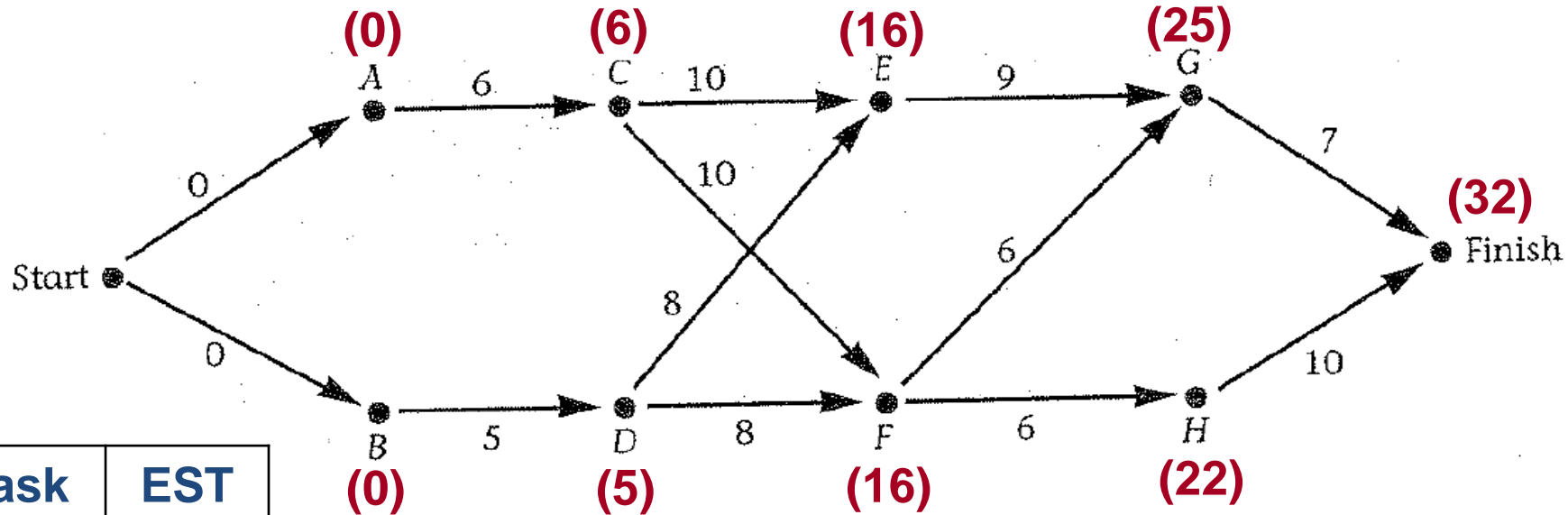
Minimum Project Time = \_\_\_\_\_

Critical Path(s) = \_\_\_\_\_  
 \_\_\_\_\_

# And this... ANSWERS

List the tasks and earliest start times in a table, as in exercise #1.

Determine the minimum project time and all the critical paths.



2.

Task	EST
A	0
B	0
C	6
D	5
E	16
F	16
G	25
H	22

**Minimum Project Time = 32**

**Critical Path(s) = Start-ACEG-Finish  
Start-ACFH-Finish**

# Homework

- Packet p. 1-2 (Day 1)
- Finish College Tour Activity