KEY
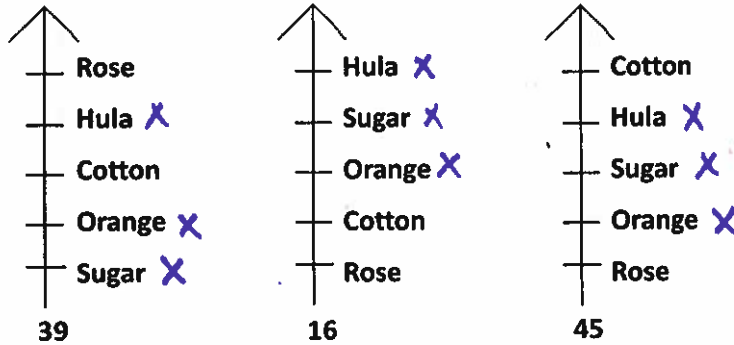
Station 1: Voting Methods Review

A university marching band has been invited to march in 5 different parades. The rankings of the band members are shown below. Determine the winner – with amount – for each method below.
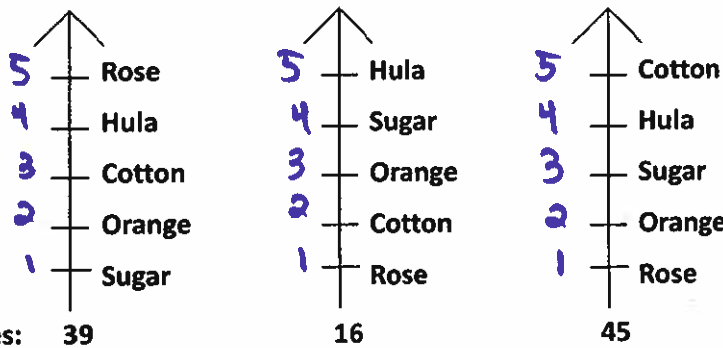
(39 votes)	(16 votes)	(45 votes)
Rose	Hula ✗	Cotton
Hula ✗	Sugar ✗	Hula ✗
Cotton	Orange ✗	Sugar ✗
Orange ✗	Cotton	Orange ✗
Sugar ✗	Rose	Rose

Votes: 39   16   45

Plurality: Cotton with 45 1st place

Majority: None (all < 50% 1st place)

Borda: Hula with 416 points (see pg 2)

Runoff: Cotton with 61 1st place

Seq. Runoff: Cotton with 61

Condorcet: Hula (see below)

Approval: Hula (see below ↓) (the top 3 votes were approved)

Sequential Runoff
1st) eliminate Sugar + Orange with 0 1st place votes
2nd) eliminate Hula with only 16 1st place votes now
3rd) eliminate Rose with only 39 1st place votes now

Condorcet: Hula wins

	R	H	C	O	S
R	✗	H	C	O	S
H	✗	✗	H	H	H
C	✗	✗	✗	C	C
O	✗	✗	✗	✗	S
S	✗	✗	✗	✗	✗

Ex: R vs H ⟹ Hula wins
    39   61

Ex: R vs C ⟹ Cotton wins
    39   61

Borda: Hula with most points

Station 1: Voting Methods Review

A university marching band has been invited to march in 5 different parades. The rankings of the band members are shown below. Determine the winner – with amount – for each method below.

points	(39)	points	(16)	points	(45)
5	Rose	5	Hula	5	Cotton
4	Hula	4	Sugar	4	Hula
3	Cotton	3	Orange	3	Sugar
2	Orange	2	Cotton	2	Orange
1	Sugar	1	Rose	1	Rose

Votes: 39   16   45

Plurality:

Majority:

Borda:

Runoff:

Seq. Runoff:

Condorcet: Hula beat everyother one.

Approval: (the top 3 votes were approved)

Approval: Hula wins

Rose: 39
Hula: 100   39+16+45
Cotton: 84   39+45
Sugar 61   16+45
Orange: 16

Rose: $39(5) + 16(1) + 45(1) = 256$
Hula: $39(4) + 16(5) + 45(4) = 416$
Cotton: $39(3) + 16(2) + 45(5) = 374$
Orange: $39(2) + 16(3) + 45(2) = 216$
Sugar: $39(1) + 16(4) + 45(3) = 238$

4/23/2019

Station 2: Coalition Review

- Voter A gets 6 votes.
- Voter B gets 5 votes.
- Voter C gets 4 votes.
- Voter D gets 2 votes.
- A winning coalition must have 10 votes.

a) How many votes would the coalition formed by voters A and C have? 6 + 4

$\{A, C ; 10\}$ [10]

b) Write down all of the winning coalitions. Use proper notation.

$\{A, B ; 11\}$, $\{A, C ; 10\}$, $\{A, B, C ; 15\}$, $\{A, B, D ; 13\}$
$\{A, C, D ; 12\}$, $\{B, C, D ; 11\}$, $\{A, B, C, D ; 17\}$

c) Which voters are essential in the coalition $\{A, B, C ; 15\}$? Explain.

• A because if its 6 votes were removed, the coalition would have only 9 votes, not enough to pass.

d) What is the Power Index for each voter?

A: 5   C: 3
B: 3   D: 1

e) Are there any dictators or dummies? Explain why for each.

No dictators b/c no one voter is essential to every winning coalition.

No dummies because no voter is essential to zero winning coalitions.

Remember:
Power index is the number of times a voter is essential to a winning coalition.

Station 2: Coalition Review

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- A winning coalition must have 10 votes.

a) How many votes would the coalition formed by voters A and C have?

b) Write down all of the winning coalitions. Use proper notation.

c) Which voters are essential in the coalition $\{A, B, C ; 15\}$? Explain.

d) What is the Power Index for each voter?

A: ___   C: ___
B: ___   D: ___

e) Are there any dictators or dummies? Explain why for each.

1

4. Four heirs, Skyler, Flynn, Holly, and Marie divide fairly an estate consisting of four items— a car wash, a house, a car, and a watch— using the **Method of Sealed Bids**. The players' bids are given in the table below. *and $60,000*

(a) **(The Bids)** Determine each player's opinion of the total value of the estate and the value that each player places on a fair share. Enter these values in rows (1) and (2) of the table.

(b) **(The Allocation)** Determine the allocation of the items in the estate. Fill in the item(s) allocated to each player in row (3) of the table.

(c) **(The Payments)** After the items are allocated, some players will owe the estate money and others will be owed money by the estate. Determine the amount of money each player owes or is owed and enter this in row (4) *OR* (5) of the table.

(d) **(Dividing the Surplus)** After the payments are all made, there might be a surplus left in the estate. Determine each player's share of the surplus and enter it in row (6) of the table.

(e) **(Final Settlement)** Find the net settlement (items and money) for each player and enter it in row (7) of the table.

			Skyler	Flynn	Holly	Marie
Bids		Car Wash	$800,000	$500,000	$750,000	$100,000
		House	$200,000	$120,000	$250,000	$180,000
		Car	$10,000	$9,000	$6,000	$5,000
		Watch	$1,000	$1,500	$1,200	$1100
Sum of Bids + Cash =	(1)	Total Value of Estate	+60,000 1,071,000	+60,000 690,500	+60,000 1,067,200	+60,000 346,100
	(2)	Value of a <u>Fair</u> Share	267,750	172,625	266,800	86,525	÷4
Allocation	(3)	Item(s) Allocated	Carwash + Car =800000 =10000	watch -1500	house -250,000	none -0
Payments	(4) Player Owes Estate *if fairshare - item = (-)*		542,250	n/a	n/a	n/a
	(5) Estate Owes Player *if fairshare - item = (+)*		n/a	171,125	16,800	86,525
Surplus *(remaining cash)*	(6)	Share of Surplus	81,950	81,950	81,950	81,950
Final Settlement	(7) Final Settlement Items: *owes/owed by estate + surplus* Money (+/-): *share*		Car wash, car, and pays estate $460,300	watch and $253,075 received	house and gets $98,750	no items and gets $168,475

Surplus (remaining cash) = 60000 + 542,250 - 171,125 - 16,800 - 86,525 = 327800
                            initial $ to estate   Skyler owes estate   pay Flynn   pay Holly   pay Marie

Discrete Math Uneven
Estate Division Worksheet

Name__KEY__

1. John, Jerry, and Jill are heirs to their mother's estate that includes their family house, an automobile, a small mountain cabin, and $125,000 in cash (from investments and a life insurance policy). Use the algorithm we learned in class to fairly divide this inheritance among the three siblings.

Assume John, Jerry, and Jill have submitted the following bids on the family house, the car, and the mountain cabin.

	Family House	Car	Cabin	Cash
John	90,000	4,000	20,000	125000
Jerry	95,000	5,000	16,000	125000
Jill	92,000	4,000	18,000	125000

Due to some other items being distributed prior to their mother's death, John receives 40% of the inheritance, Jerry receives 25%, and Jill receives 35% of the inheritance.

	John	Jerry	Jill
Total of bids & cash	239,000	241,000	239,000
Fair share	95,600	60,250	83,650
Items received	Cabin	house + car	none
Value of item(s) received	20,000	100,000	0
Initial cash received	75,600	−39,750 (paid estate)	83,650
Share of remaining cash	2200 (5500*0.40)	1375 (5500*0.25)	1925 (5500*0.35)
Final settlement	Cabin and gets $77,800	house, car, and pays $38,375	no items but gets $85,575

Fair share − Value of item

Remaining cash = 125000 initially in estate − 75600 to pay John + 39750 from Jerry − 83650 to pay Jill = 5500 remaining cash

2. It is common for one or more heirs to pay into an estate when this algorithm is used. However, this algorithm fails if an heir who must pay into the estate doesn't have the money to do so. If this happened in your family, what would you suggest to finish the estate division process?

Ex: If an heir doesn't have money to pay into an estate, the item they bid highest on could go to the second highest bidder instead.

3. Could this estate division algorithm of this lesson encourage insincerity by any of the heirs? Explain why or why not. Ex: Yes if collusion was going on especially!

Ex: An heir might bid highly on items to "jack up" his or her fair share value, but then he/she might get an item he/she didn't want instead of getting more money.