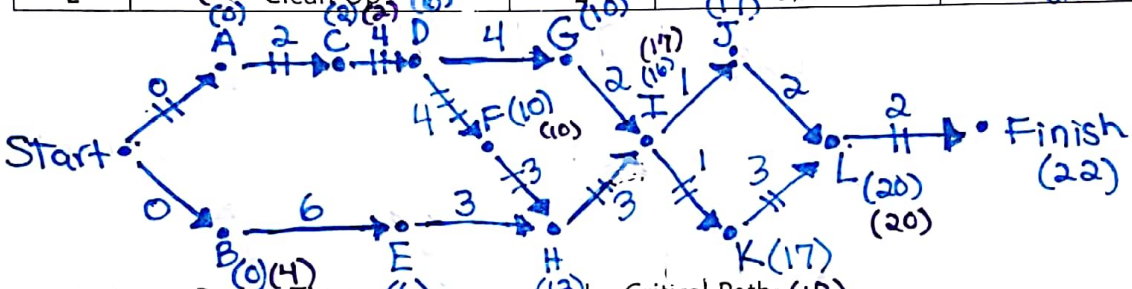
ICM ~ UNIT 7 TEST REVIEW

Name Key

1) Ms. Collins wants to remodel her house. The following task table lists the task, time needed in days, and immediate prerequisites for each task. Draw a directed graph to represent the situation, complete the table with the earliest start time and latest start time for each task, calculate the minimal project time required to complete the project, and state the critical path.

Remember: Use arrows because Task Graphs are directed graphs

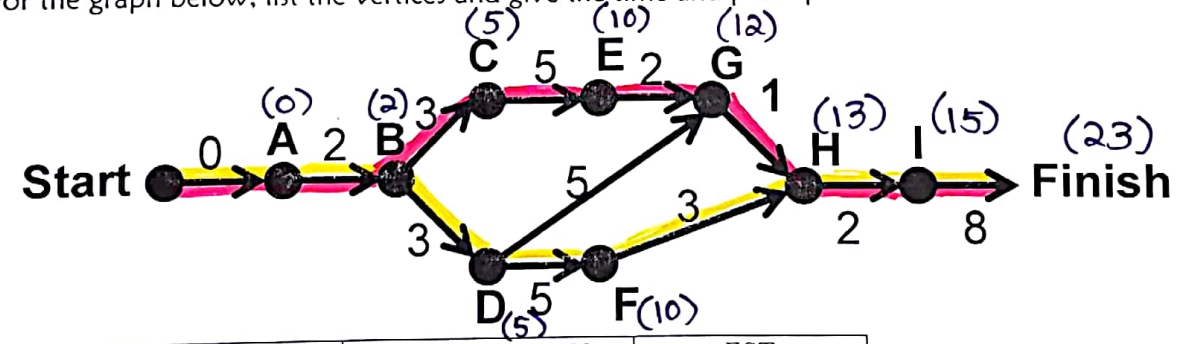
	Task	Time	Prerequisites	EST	LST
A	Initial Clean-Up	2	None	0	0
B	Find a Contractor	6	None	0	4
C	Get an Inspection	4	A	2	2
D	Framing	4	C	6	6
E	Plumbing	3	B	6	10
F	Electrical Work	3	D	10	10
G	Install Windows	2	D	10	14
H	Finish Carpentry	3	E,F	13	13
I	Flooring	1	H,G	16	16
J	Install appliances	2	I	17	18
K	Reassemble house	3	I	17	17
L	Clean Up	2	J,K	20	20



a. Minimum Project Time: 22 days

b. Critical Path: Start-ACDFHIKL-Finish

2) For the graph below, list the vertices and give the time and prerequisite for each task.



TASK	TIME	PREREQUISITES	EST
A	2	none	0
B	3	A	2
C	5	B	5
D	5	B	5
E	2	C	10
F	3	D	10
G	1	D,E	12
H	2	F,G	13
I	8	H	15

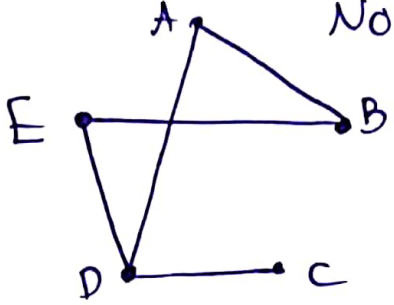
Minimum Project time: 23

Critical path:
Start-ABDFHI-Finish
Start-ABCEGHI-Finish

Not "digraph" so NO arrows for direction

3) Construct a graph for the following adjacency matrix. Is the graph complete? Connected?

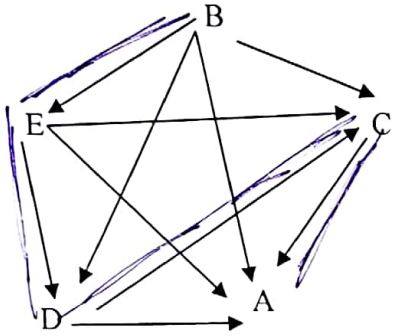
A	0	1	0	1	0
B	1	0	0	0	1
C	0	0	0	1	0
D	1	0	1	0	1
E	0	1	0	1	0



No.

yes connected because you can eventually get from any vertex to any other

4) Given the following tournament, find a Hamiltonian Path. Then use the path to rank the players from 1st place – 5th place

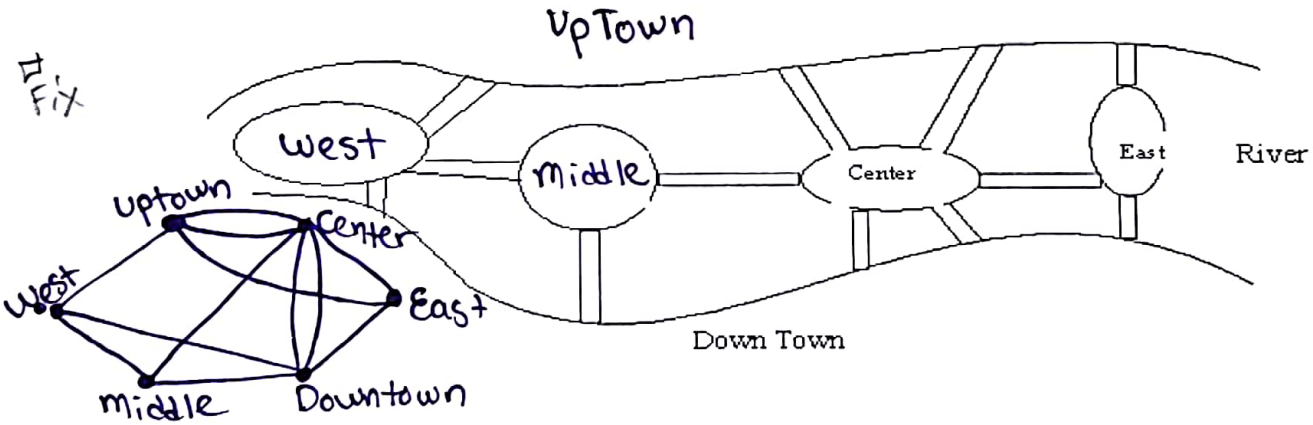


a) Hamiltonian Path: BEDCA

b) Who beats player C? B, E, D

c) In the game D vs. E, who wins? E

5) A river runs through a city. There are 4 islands and 12 bridges as shown below. Draw a graph to help you answer the following questions. (hint: each island or town represents a vertex, each bridge represents an edge.)



a. How many vertices and how many edges would a graph that models this situation have?

Vertices 6 (4 Islands, Uptown, Downtown) Edges 12 (12 bridges)

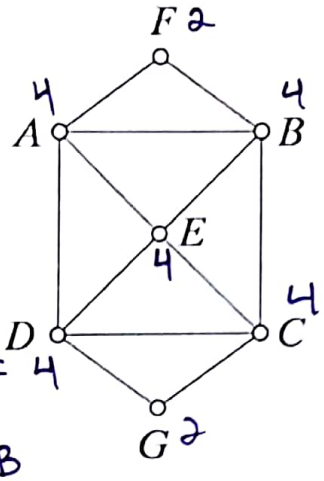
b. In a graph that models this situation, what is the degree of the vertex that represents:

East Island 3 Middle Island 3 Downtown 5

c. Is it possible to take a walk through this town, starting at **East Island**, crossing each bridge exactly once. If so, sketch such a path.

No. This walk would represent an Euler Path but one is not possible as there are more than 2 odd degree vertices.

6) Consider the graph to the right:



a. Is there an Euler circuit? If so, find one. If not, explain.

Yes. Example: FADGCDECBEABF

b. Is there an Euler path? If so, find one. If not, explain.

No. All vertices are even degree.

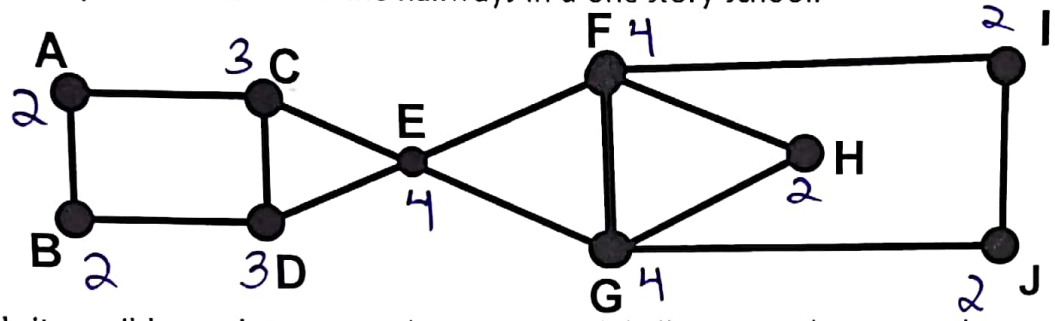
c. Is there a Hamiltonian circuit? If so, find one; if not, explain why not.

Yes. Examples: A D G C E B F A or FAEDGCBF

d. Is there a Hamiltonian path? If so, find one; if not, explain why not.

Yes. Examples: A D G C E B F or FAEDGCB

7) The map below shows all the hallways in a one story school.



a) Is it possible to plan a route that covers each hallway exactly once and returns to where it started? If so give the route. If not, tell WHY?

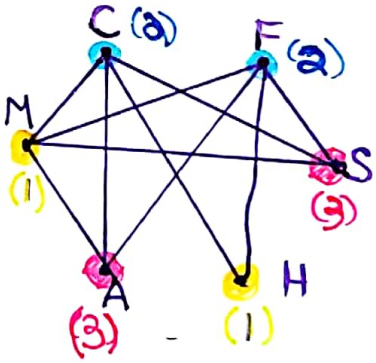
It is Not possible because C and D are degree 3 and Euler circuits need all even degree.

b) Is it possible to plan a route that covers each hallway exactly once, but does NOT return to where it started? If so give the route. If not, tell WHY? CABDEGJIFHGFECD.

Yes. DEFIJGFHGECABDC is an example or ↗

In scheduling the final exams for summer school at Green Hope High School six different exams must be scheduled. The following table shows the exams that are needed for seven different students. How many different time slots are necessary to schedule finals in a way that no student has two finals at the same time?

	Students						
Exam	1	2	3	4	5	6	7
(C) Civics	X	-	X	-	X	-	X
(F) French	-	X	-	X	-	X	-
(S) Science	X	X	-	-	-	-	X
(H) History	-	-	X	-	-	X	-
(A) Art	-	-	-	X	X	-	-
(M) Math	X	X	-	X	X	-	X



8) Determine the minimum number of time slots needed to schedule the six exams.

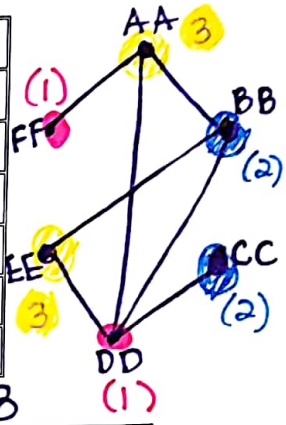
3 time slots (see example graph above with 3 colors)

9) The Federal Communications Commission (FCC) monitors radio stations to make sure their signals do not interfere with each other. They prevent interference by assigning appropriate frequencies to each station. ⇒ conflict if ≤ 150 miles

Two stations cannot use the same frequency when they are within 150 miles of each other. The table below shows the distances in miles between six particular radio stations.

Draw and label a graph to determine how many different frequencies are needed for the six radio stations.

	KQAA	KQBB	KQCC	KQDD	KQEE	KQFF
KQAA	–	25	202	77	375	106
KQBB	25	–	175	51	148	222
KQCC	202	175	–	111	365	411
KQDD	77	51	111	–	78	297
KQEE	375	148	365	78	–	227
KQFF	106	222	411	297	227	–



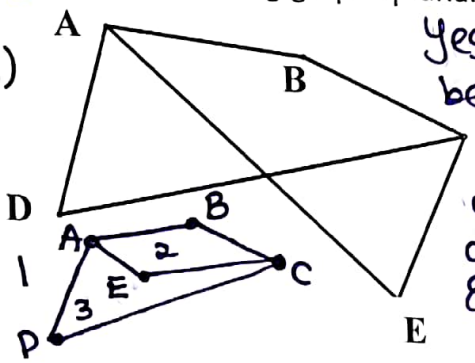
Number of Frequencies 3

CHAPTER 4 DEFINITIONS:

Sec. 4.1	1. Vertices
	2. Graphs
	3. Edges
Sec. 4.2	4. Critical Path
	5. Earliest Start Time (EST)
Sec. 4.3	6. Connected Graph
	7. Complete Graph
	8. Adjacent
	9. Degree
	10. Loop
	11. Multigraph
Sec. 4.4	12. Euler Circuit (pronounced OILER)
	13. Euler Path
	14. Digraph
	15. Indegree/ Outdegree
	16. Hamiltonian Path
	17. Hamiltonian Circuit
	18. Tournament
Sec. 4.5	19. Chromatic Number
	20. Circuit (Cycle)

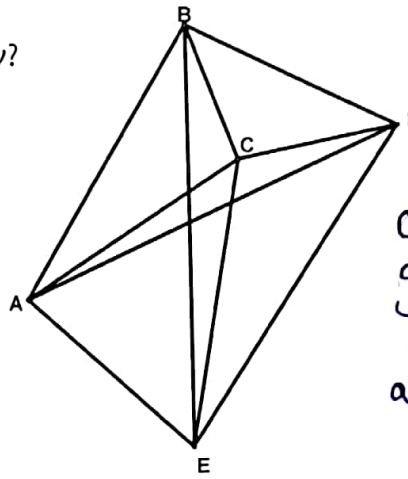
10) Are the following graphs planar? How do you know?

a)



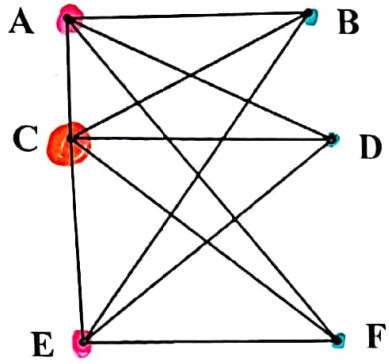
Yes. Planar because you can draw it without crossing edges and it fits Euler's theorem
$r + v = e + 2$
$3 + 5 = 6 + 2$ ✓

b)

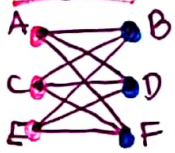


No. It's a K_5 graph. complete graphs with ≥ 5 vertices are non planar.

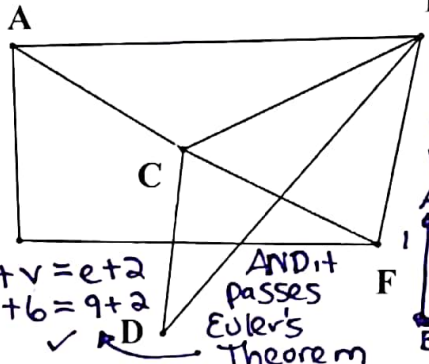
c)



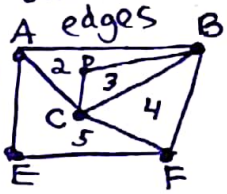
No. Not planar because it contains a $K_{3,3}$ subgraph



d)

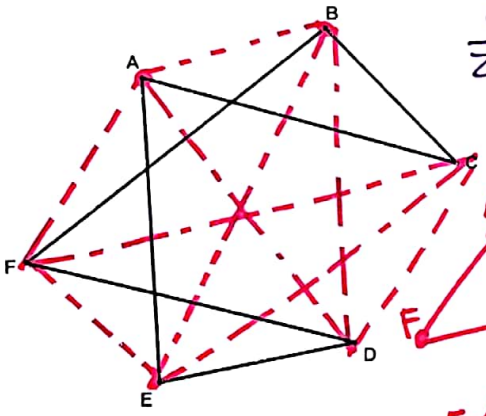


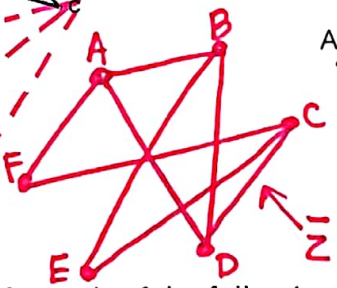
Yes, planar because it can be drawn without crossing edges



$r + v = e + 2$
$5 + 6 = 9 + 2$ ✓ AND it passes Euler's Theorem

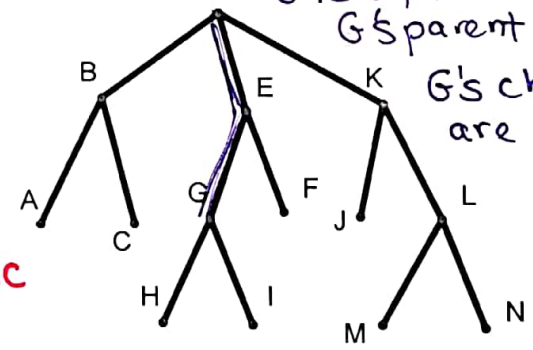
11) Below is graph Z. Draw $\overline{Z}$. What is the chromatic number of $\overline{Z}$?





2 because $\overline{Z} = K_{3,3}$

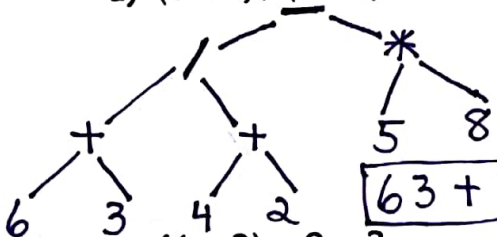
12) Name the level, parent and children of vertex G.



G is on level 2
G's parent is E
G's children are H and I

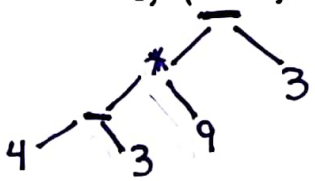
13) Draw an expression tree for each of the following and write the post-order traversal listing for the expression.

a) (6 + 3) / (4 + 2) – 5 * 8



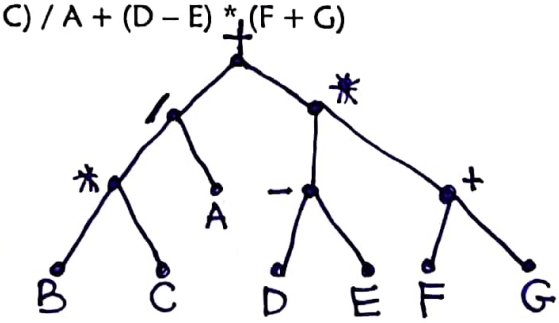
63 + 42 + / 58 * –

b) (4 – 3) x 9 – 3



4 3 – 9 * 3 –

c) (B * C) / A + (D – E) * (F + G)



BC * A / DE – FG + * +

14) Solve each RPN expression:

a) 2 5 * 4 + 3 2 * 1 + /

10 4 + 3 2 * 1 + /

14 3 2 * 1 + /

14 6 1 + /

14 7 /

2

b) 3 4 + 3 6 / +

7 3 6 / +

7 0.5 +

7.5

c) 7 1 + 3 x 4 2 - +

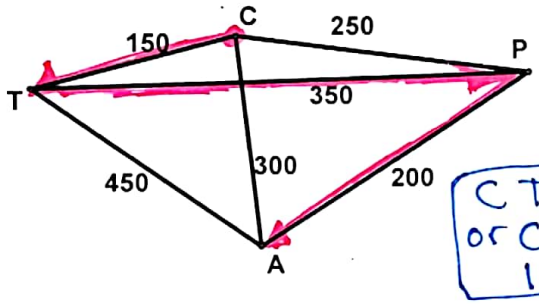
8 3 x 4 2 - +

24 4 2 - +

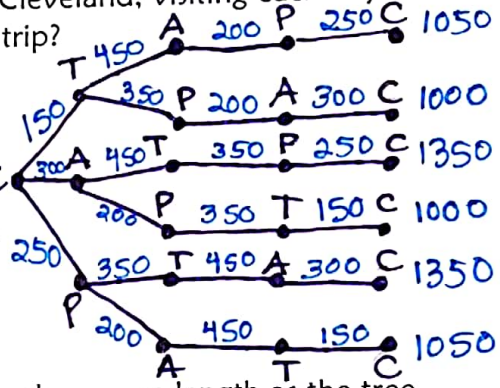
24 2 +

26

15) Use our tree technique to find the shortest round trip, starting in Cleveland, visiting each city in the graph and returning to Cleveland. What is the length of the trip?



C - Cleveland
T - Toledo
A - Athens
P - Pittsburgh



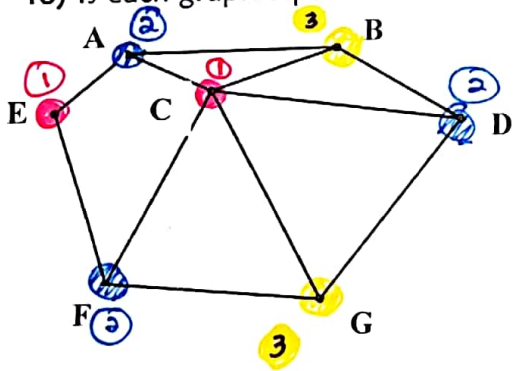
CTPAC
or CAPTC
1000

What route does the nearest neighbor technique produce? Does it produce same length as the tree technique? see pink above

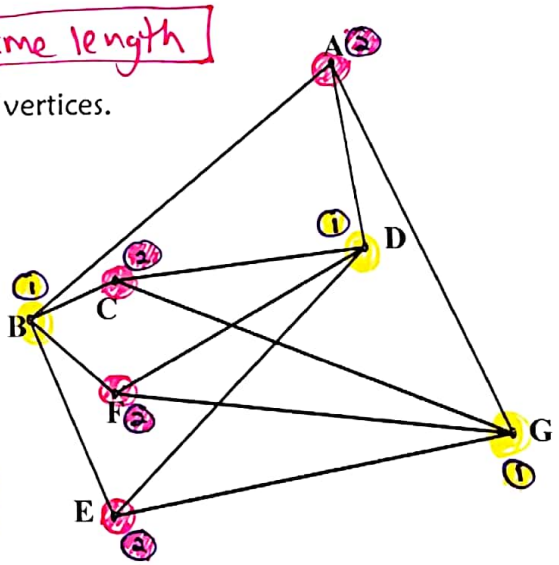
150 + 350 + 200 + 300

CTPAC | 1000 | Yes Same length

16) Is each graph bipartite? If it is, list the two distinct sets of vertices.

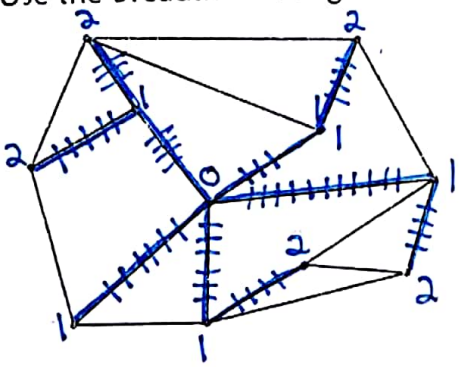


No. Need 3 colors.

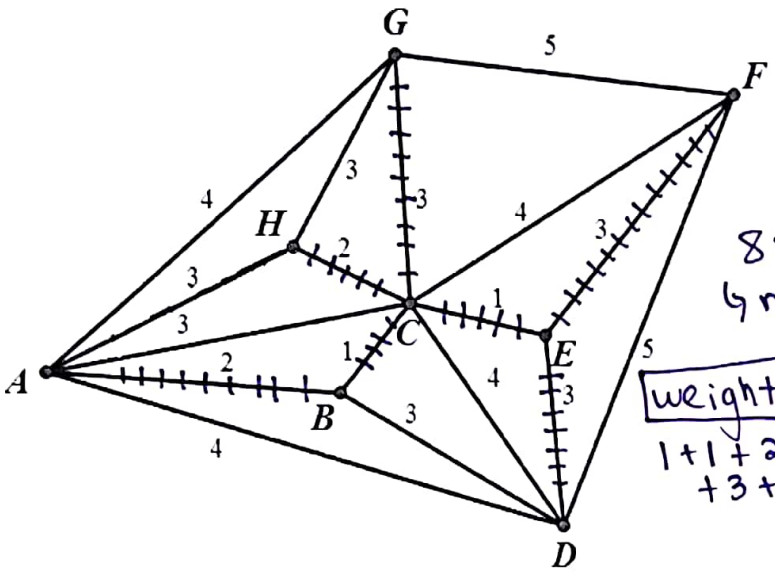


Yes
Bipartite graph.
{B, D, G}
{A, C, E, F}

17) Use the Breadth-First Algorithm to trace a spanning tree for this graph.



18) Use Kruskal's algorithm to find the **minimum spanning tree** of the graph using a table. What is the weight of the minimum spanning tree? Be sure to show your ordered list of edges.



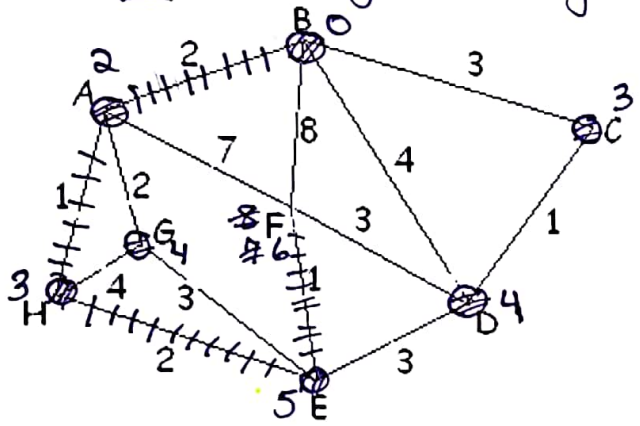
8 vertices
↳ need 7 edges

weight = 15
1+1+2+2
+3+3+3

Table:

BC	1✓	CF	4
CE	1✓	CD	4
CH	2✓	AD	4
AB	2✓	AG	4
GC	3✓	GF	5
~~GH~~	~~3~~	DF	5
~~AH~~	~~3~~		
EF	3✓		
ED	3✓		
BD	3		
AC	3		

19) The graph below shows distances between various towns. Find the length of the shortest path from B to F. ↳ Dijkstra's Algorithm



BAHEF

6

20) The vertices of the following graph represent buildings on a small college campus. Administrators on the campus want to connect the buildings with fiber-optic cable and are interested in finding the least expensive way of doing so. The costs of connecting the buildings (in thousands of dollars) are shown as the weighted edges of the graph.

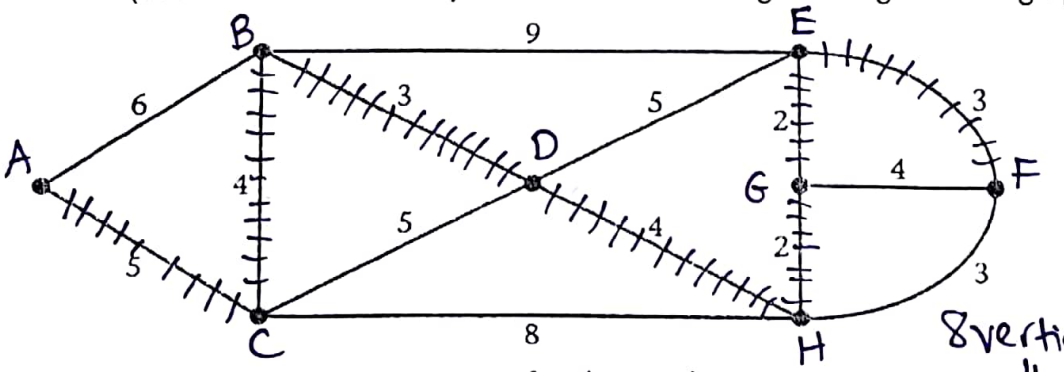


Table:

EG	2✓	AC	5✓
GH	2✓	DE	5
EF	3✓	~~CD~~	~~5~~
~~HF~~	~~3~~	~~AB~~	~~6~~
BD	3✓	~~CH~~	~~8~~
DH	4✓	~~BE~~	~~9~~
BC	4✓		
~~GE~~	~~4~~		

8 vertices
↓
need
7 edges

Find the minimum spanning tree for the graph using a table.
Darken the edges on the graph above.
What is the minimum total cost of connecting the buildings?

$23,000

2+2+3+3+4+4+5