Test Review Stations ANSWERS

Unit 2 Matrices & Game Theory

Test Review: Station A ANSWERS

A local designer clothes manufacturer tries to keep up with her sales in Target, Old Navy, and Kohl's. The inventories of her most popular items in all three **stores** are recorded in the table.

	T-shirts	Jackets	Cardigans
Target	15	20	23
Old Navy	18	23	21
Kohl's	17	26	19

Label your rows and columns in your work and your answer.

1. The designer makes the T-shirts for \$10, the jackets for \$13, and the cardigans for \$17. Use matrices to calculate the manufacturer's cost of making these popular items for each store. Call the resulting cost matrix C.

$$\begin{array}{c} \\ \text{Target} \\ \text{Old Navy} \\ \text{Kohl's} \end{array} \begin{array}{c} \begin{array}{ccc} T & J & C \end{array} \\ \begin{bmatrix} 15 & 20 & 23 \\ 18 & 23 & 21 \\ 17 & 26 & 19 \end{bmatrix} \end{array} \bullet \begin{array}{c} \\ \text{T} \\ \text{J} \\ \text{C} \end{array} \begin{array}{c} \text{Cost} \\ \begin{bmatrix} 10 \\ 13 \\ 17 \end{bmatrix} \end{array} = \begin{array}{c} \\ \text{Target} \\ \text{Old Navy} \\ \text{Kohl's} \end{array} \begin{array}{c} \text{Cost} \\ \begin{bmatrix} 801 \\ 836 \\ 831 \end{bmatrix} \end{array} = C$$

Test Review: Station A ANSWERS (continued)

A local designer clothes manufacturer tries to keep up with her sales in Target, Old Navy, and Kohl's. The inventories of her most popular items in all three **stores** are recorded in the table.

	T-shirts	**Jackets**	**Cardigans**
Target	15	20	23
Old Navy	18	23	21
Kohl's	17	26	19

Label your rows and columns in your work and your answer.

2. The designer sells the T-shirts to the stores for $13, the jackets for $18, and the cardigans for $16. Use matrices to calculate the income, I, that the designer makes from selling these popular items to the stores.

$$\begin{array}{c} \\ \text{Target} \\ \text{Old Navy} \\ \text{Kohl's} \end{array} \begin{array}{c} \begin{array}{ccc} T & J & C \end{array} \\ \begin{bmatrix} 15 & 20 & 23 \\ 18 & 23 & 21 \\ 17 & 26 & 19 \end{bmatrix} \end{array} \bullet \begin{array}{c} \\ \text{T} \\ \text{J} \\ \text{C} \end{array} \begin{array}{c} \text{Income} \\ \begin{bmatrix} 13 \\ 18 \\ 16 \end{bmatrix} \end{array} = \begin{array}{c} \\ \text{Target} \\ \text{Old Navy} \\ \text{Kohl's} \end{array} \begin{array}{c} \text{Income} \\ \begin{bmatrix} 923 \\ 984 \\ 993 \end{bmatrix} \end{array} = I$$

Test Review: Station A ANSWERS (continued)

A local designer clothes manufacturer tries to keep up with her sales in Target, Old Navy, and Kohl's. The inventories of her most popular items in all three **stores** are recorded in the table.

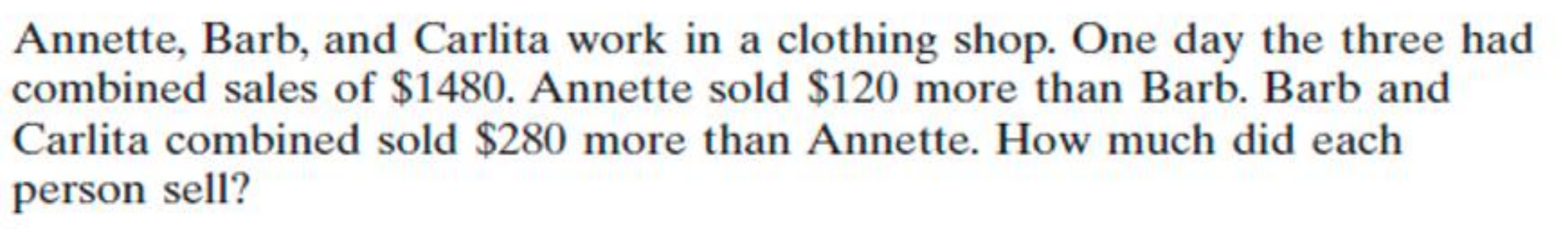
	T-shirts	Jackets	Cardigans
Target	15	20	23
Old Navy	18	23	21
Kohl's	17	26	19

Label your rows and columns in your work and your answer.

3. Use matrices to calculate the profit that the designer makes on her sales to each store.

$$\text{Profit} = P = I - C = \begin{array}{c} \\ \text{Target} \\ \text{Old Navy} \\ \text{Kohl's} \end{array} \begin{array}{c} \text{Income} \\ \begin{bmatrix} 923 \\ 984 \\ 993 \end{bmatrix} \end{array} - \begin{array}{c} \text{Cost} \\ \begin{bmatrix} 801 \\ 836 \\ 831 \end{bmatrix} \end{array} = \begin{array}{cc} & \text{Profit} \\ \text{Target} & \\ \text{Old Navy} & \begin{bmatrix} 122 \\ 148 \\ 162 \end{bmatrix} \\ \text{Kohl's} & \end{array}$$

Test Review: Station B ANSWERS


Annette, Barb, and Carlita work in a clothing shop. One day the three had combined sales of $1480. Annette sold $120 more than Barb. Barb and Carlita combined sold $280 more than Annette. How much did each person sell?

1. Create a system of equations for the problem.

Let a = Annette's sales, b = Barb's sales, and c = Carlita's sales

$$\begin{aligned} a + b + c &= 1480 \\ a &= 120 + b \\ b + c &= a + 280 \end{aligned} \quad \rightarrow \quad \begin{aligned} a + b + c &= 1480 \\ a - b + 0c &= 120 \\ -a + b + c &= 280 \end{aligned}$$

Test Review: Station B ANSWERS (continued)

Let a = Annette's sales, b = Barb's sales, and c = Carlita's sales

$$\begin{aligned} a + b + c &= 1480 \\ a &= 120 + b \\ b + c &= a + 280 \end{aligned} \quad \rightarrow \quad \begin{aligned} a + b + c &= 1480 \\ a - b + 0c &= 120 \\ -a + b + c &= 280 \end{aligned}$$

2. Write a matrix equation to represent the scenario.

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & -1 & 0 \\ -1 & 1 & 1 \end{bmatrix} \bullet \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 1480 \\ 120 \\ 280 \end{bmatrix}$$

A ● X = B

A●X = B

→ type in A and B in calc.

Test Review: Station B ANSWERS (continued)

Let a = Annette's sales, b = Barb's sales, and c = Carlita's sales

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & -1 & 0 \\ -1 & 1 & 1 \end{bmatrix} \bullet \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 1480 \\ 120 \\ 280 \end{bmatrix}$$

A●X = B

→ type in A and B in calc.

3. Use matrices to solve the problem.

Do **A^{-1} ● B** in calc to solve for X (to find a, b, and c)

$$\mathbf{A^{-1} \bullet B} = \begin{bmatrix} 600 \\ 480 \\ 400 \end{bmatrix}$$

That day, Annette sold $600, Barb sold $480, and Carlita sold $400 of merchandise.

Test Review Station C ANSWERS

Suppose that Sol and Tina change their game. If Sol displays heads and Tina shows tails, Sol wins 3 cents. If Sol shows tails and Tina displays heads, then Sol wins 2 cents. If they both show heads, then Tina wins 1 cent. If they both show tails, then Tina wins 4 cents.

1. What is the payoff matrix for this game with Sol as the row player?

$$\begin{array}{cc} & \begin{array}{cc} H & T \end{array} \\ \begin{array}{c} H \\ T \end{array} & \begin{pmatrix} -1 & 3 \\ 2 & -4 \end{pmatrix} \end{array}$$

Test Review Station C ANSWERS (continued)

2. Is the game strictly determined? Explain.

$$\begin{array}{cccc} & H & T & \text{Maximin} \\ H & -1 & 3 & \boxed{-1} \\ T & 2 & -4 & -4 \\ \text{Minimax} & \boxed{2} & 3 & \end{array}$$

No. The game is NOT strictly determined because maximin does not equal minimax (there is not a saddle point).

3. Use the row matrix $\begin{bmatrix} p & 1-p \end{bmatrix}$ to find Sol's best strategy for this game.

Sol should play heads 6 out of 10 times and tails 4 out of 10 times.

4. Use the column matrix $\begin{bmatrix} q \\ 1-q \end{bmatrix}$ to find Tina's best strategy for this game.

Tina should play heads 7 out of 10 times and tails 3 out of 10 times.

Test Review Station C ANSWERS (continued)

Suppose that Sol and Tina change their game so that the payoffs to Sol are

$$\begin{array}{c|cc} & H & T \\ \hline H & -1 & 3 \\ T & 2 & -4 \end{array}$$

5. What is Sol's expected **payoff matrix equation** for this game?

$$\begin{bmatrix} .6 & .4 \end{bmatrix} \begin{bmatrix} -1 & 3 \\ 2 & -4 \end{bmatrix} \begin{bmatrix} .7 \\ .3 \end{bmatrix} = Sol's\ Expected\ Payoff$$

6. Solve problem 5 and interpret the results.

$$\begin{bmatrix} .6 & .4 \end{bmatrix} \begin{bmatrix} -1 & 3 \\ 2 & -4 \end{bmatrix} \begin{bmatrix} .7 \\ .3 \end{bmatrix} = 0.2$$

Sol should expect to win 2 cents every 10 times.

(or 0.2 cents each game).

Test Review: Station D Answers

1. Each of the following matrices represents a payoff matrix for a game. Determine if the game is strictly determined or not. If it is, find the best strategies for the row and column players, and the saddle point of the game.

d.

$$\begin{array}{cc} \begin{bmatrix} 0 & 1 & 2 \\ 3 & -2 & 0 \end{bmatrix} & \begin{array}{c} \boxed{0} \\ -2 \end{array} \\ \begin{array}{ccc} 3 & \boxed{1} & 2 \end{array} & \end{array}$$

e.

$$\begin{array}{cc} \begin{bmatrix} 0 & -6 & 1 \\ -4 & 8 & 2 \\ 6 & 5 & 4 \end{bmatrix} & \begin{array}{c} -6 \\ -4 \\ \boxed{4} \end{array} \\ \begin{array}{ccc} 6 & 8 & \boxed{4} \end{array} & \end{array}$$

f.

$$\begin{array}{cc} \begin{bmatrix} 0 & 3 & 1 \\ -3 & 0 & 2 \\ -1 & -4 & 0 \end{bmatrix} & \begin{array}{c} \boxed{0} \\ -3 \\ -4 \end{array} \\ \begin{array}{ccc} \boxed{0} & 3 & 2 \end{array} & \end{array}$$

- Not a Strictly Determined Game because maximin does not equal minimax.

- Strictly Determined Game because Saddle Point at 4.
- Best strategies are row 3 and column 3.

- Strictly Determined Game because Saddle Point at 0.
- Best strategies are row 1 and column 1.

Test Review: Station D Answers (cont.)

2. Use the concept of dominance to solve each of the following games. Give the best row and column strategies and the saddle point of each game.

a.

$$\begin{array}{c|ccc|c} & E & F & G & \\ \hline A & 3 & 1 & 7 & 1 \\ B & 0 & 1 & 3 & 0 \\ C & 4 & 3 & 4 & \textcircled{3} \\ D & 1 & 3 & 6 & 1 \\ \hline & 4 & \textcircled{3} & 7 & \end{array}$$

b.

$$\begin{array}{c|ccc|c} & E & F & G & \\ \hline A & 4 & -1 & -2 & -2 \\ B & 0 & 1 & 1 & 0 \\ C & 0 & -2 & 5 & -2 \\ D & 3 & 2 & 4 & \textcircled{2} \\ \hline & 4 & \textcircled{2} & 5 & \end{array}$$

- Saddle Point at 3.
- Best strategy for the Row Player is option C, and for the Column Player is option F.

- Saddle point at 2.
- Best strategy for the Row Player is option D, and for the Column Player is option F.

Test Review: Station E ANSWERS

There are 30 students in the Math Club and every week they bring snacks to their meeting. This week 13 brought chips, 9 brought drinks and 8 brought a dessert. 16% of those who brought chips to the first meeting brought chips again and 38% brought drinks. Of those that brought drinks, 30% brought drinks again and the rest brought dessert to the next meeting. And of those that brought a dessert to the first meeting, 24% brought a dessert again and 40% brought chips.

1. What is the initial distribution matrix for the math club? $D_0 = \begin{bmatrix} 13 & 9 & 8 \end{bmatrix}$

2. What is the transition matrix for the math club?

$$T = \begin{array}{c} \\ Chips \\ Drinks \\ Dessert \end{array} \begin{array}{c} \begin{array}{ccc} Chips & Drinks & Dessert \end{array} \\ \begin{bmatrix} 0.16 & 0.38 & 0.46 \\ 0 & 0.30 & 0.70 \\ 0.40 & 0.36 & 0.24 \end{bmatrix} \end{array}$$

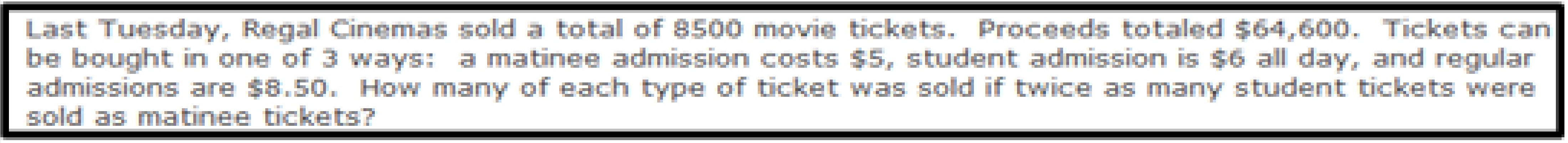
3. Approximately how many students will bring desserts to the 4th meeting??

   13.34 so ~13 students (Find $D_4 = D_0 \bullet T^4$, then dessert column.)

4. In the long run, how many of these students will bring each item to a meeting?

   Approximately 6.35 will bring chips, 10.31 will bring drinks, and 13.34 will bring desserts (Find $D_{20} = D_0 \bullet T^{20}$, which = $D_{30} = D_0 \bullet T^{30}$)

Test Review: Station F More System of Equations



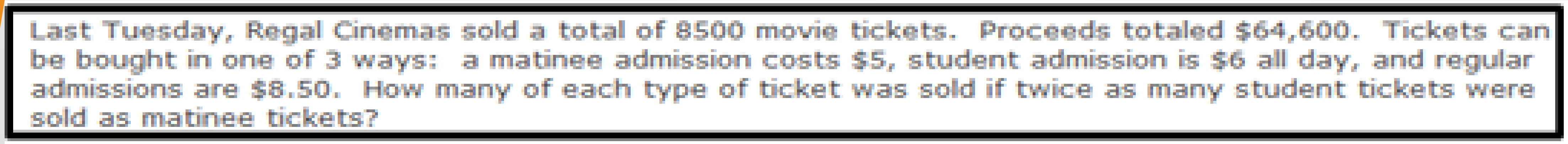
Last Tuesday, Regal Cinemas sold a total of 8500 movie tickets. Proceeds totaled $64,600. Tickets can be bought in one of 3 ways: a matinee admission costs $5, student admission is $6 all day, and regular admissions are $8.50. How many of each type of ticket was sold if twice as many student tickets were sold as matinee tickets?

1. Create a system of equations for the problem.

   Let m = # of matinee tickets, s = # of student tickets,
   and r = # of regular tickets

$$\begin{aligned} m + s + r &= 8500 \\ 5m + 6s + 8.5r &= 64600 \\ s &= 2m \end{aligned} \quad \rightarrow \quad \begin{aligned} m + s + r &= 8500 \\ 5m + 6s + 8.5r &= 64600 \\ -2m + s + 0r &= 0 \end{aligned}$$

Test Review: Station F More System of Equations



Last Tuesday, Regal Cinemas sold a total of 8500 movie tickets. Proceeds totaled $64,600. Tickets can be bought in one of 3 ways: a matinee admission costs $5, student admission is $6 all day, and regular admissions are $8.50. How many of each type of ticket was sold if twice as many student tickets were sold as matinee tickets?

Let m = # of matinee tickets,
s = # of student tickets,
and r = # of regular tickets

$$\begin{aligned} m + s + r &= 8500 \\ 5m + 6s + 8.5r &= 64600 \\ -2m + s + 0r &= 0 \end{aligned}$$

2. Write a matrix equation to represent the scenario.

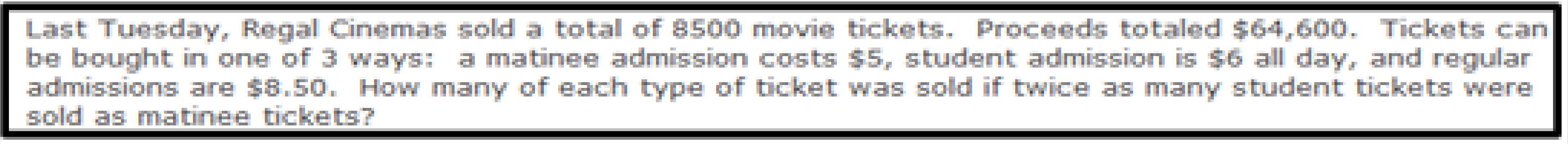
$$\underbrace{\begin{bmatrix} 1 & 1 & 1 \\ 5 & 6 & 8.5 \\ -2 & 1 & 0 \end{bmatrix}}_{\textbf{A}} \bullet \underbrace{\begin{bmatrix} m \\ s \\ r \end{bmatrix}}_{\textbf{X}} = \underbrace{\begin{bmatrix} 8500 \\ 64600 \\ 0 \end{bmatrix}}_{\textbf{B}}$$

A ● X = B

A•X = B

→ type in A and B in calc.

Test Review: Station F More System of Equations


Last Tuesday, Regal Cinemas sold a total of 8500 movie tickets. Proceeds totaled $64,600. Tickets can be bought in one of 3 ways: a matinee admission costs $5, student admission is $6 all day, and regular admissions are $8.50. How many of each type of ticket was sold if twice as many student tickets were sold as matinee tickets?

Let m = # of matinee tickets, s = # of student tickets, and r = # of regular tickets

$$\begin{bmatrix} 1 & 1 & 1 \\ 5 & 6 & 8.5 \\ -2 & 1 & 0 \end{bmatrix} \bullet \begin{bmatrix} m \\ s \\ r \end{bmatrix} = \begin{bmatrix} 8500 \\ 64600 \\ 0 \end{bmatrix}$$

A•X = B

→ type in A and B in calc.

3. Use matrices to solve the problem.

Do **A^{-1} ● B** in calc to solve for X (to find a, b, and c)

$$\mathbf{A^{-1} \bullet B} = \begin{bmatrix} 900 \\ 1800 \\ 5800 \end{bmatrix}$$

That day, Regal Cinemas sold 900 matinee, 1800 student, and 5800 regular tickets.