Summary & Practice: Rational Functions

A rational function is an equation that can be written as a ratio (a fraction)

Types of Discontinuities of Rational Functions

1) Holes

also called removable discontinuities

Step 1) Factor top and bottom

2) Slash out (cancel out) any common factors

3) Find root of slashed factor (in other words, set slashed factor = 0 and solve) → gets x-value of hole

To find the y-value for the hole, substitute the x-value into the remaining equation (after factoring and crossing out shared factors)

2) Vertical Asymptotes

Are written as $x = \#$

a.k.a. "non-removable disc." or "infinite disc."

Step 1) Factor top and bottom

2) Slash out (cancel out) any common factors

2) Find root of denominator (in other words, set remaining factor = 0 and solve)

3) Horizontal Asymptotes

Are written as $y = \#$

There are 3 scenarios) 1st, Find degree of top and bottom

a) small degree < large degree → y = 0 (Bottom degree bigger)

b) same degree → y = ratio of leading coefficients

c) large degree > small degree → none (NO Horizontal Asymptotes) (Top degree bigger)

Degree is largest exponent of expanded polynomials

0 } NO H.A.

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Rational Functions Practice **Remember to show work with Algebra for credit!** ☺

For each problem:

a) Find holes, vertical asymptotes, and horizontal asymptotes.

b) Find domain, x-intercept, and y-intercept.

1) $f(x) = \frac{x^2 - 4}{x - 2} = \frac{(x+2)(x-2)}{x-2}$

$x - 2 = 0$

$x = 2$

$y = 2 + 2$

Hole at (2, 4)

VA: none → no denom. remaining

HA: none → top degree 2, bottom degree 1

D: $(-\infty, 2) \cup (2, \infty)$

x-int: (-2, 0) and (2, 0)

y-int: (0, 2)

2) $f(x) = \frac{1x^2 - 3x - 10}{3x^2 - 11x - 20} = \frac{(x-5)(x+2)}{(3x+4)(x-5)}$

$y = \frac{x+2}{3x+4}$ remains

Hole at $(5, 7/19)$ $y = \frac{5+2}{3 \cdot 5 + 4}$

VA: $x = -4/3$ ← leftover denom.

HA: $y = 1/3$ same degree → ratio of leading coeff.

D: $(-\infty, -4/3) \cup (-4/3, 5) \cup (5, \infty)$

x-int: (-2, 0) and (5, 0)

y-int: $(0, 1/2)$ $y = 2/4$

3) $f(x) = \frac{x - 3}{x^2 - 9} = \frac{x-3}{(x+3)(x-3)}$

leftover equation $y = \frac{1}{x+3}$

Hole at $(3, 1/6)$ $y = \frac{1}{3+3}$

VA: $x = -3$ leftover denom. = 0

HA: $y = 0$ small degree < large degree

D: $(-\infty, -3) \cup (-3, 3) \cup (3, \infty)$

x-int: none (3, 0) hole there

y-int: $(0, 1/3)$

4. Find the vertical asymptotes, if any, of the graph of the rational function.

$f(x)=\dfrac{3}{x^2-3x-4} = \dfrac{3}{(x-4)(x+1)}$

$x=4, -1$ V.A.

- x = 0
- no vertical asymptotes
- x = 4 and x = −1
- x = 4 and y = −1

5. Find the all the asymptotes, if any, of the graph of the rational function.

$f(x)=\dfrac{x^3-1}{x^2-9} = \dfrac{(x-1)(x^2+x+1)}{(x+3)(x-3)}$ → large degree / small degree ⇝ No H.A. because top degree is bigger

$x=-3, x=3$ V.A.

- A. y = 0, x = 3, x = 0
- B. x = 3, x = −3
- C. y = x, y = 0
- D. y = x, x = 3, x = −3

6. Find the all the asymptotes, if any, of the graph of the rational function.

$f(x)=\dfrac{x^3-27}{x^2-9} = \dfrac{(x-3)(x^2+3x+9)}{(x-3)(x+3)}$

Hole at (3, 27)

V.A.: $x=-3$

large degree / small degree ⇝ No H.A. because top degree is bigger

- A. y = 0, x = 3, x = 0
- B. x = 3, x = −3
- C. x = 3
- D. x = −3

7. Find the location of all of the removable discontinuities, if any, of the graph of the rational function.

$f(x)=\dfrac{x^3-27}{x^2-9}$

Hole at $x=3$

$y=\dfrac{(3)^2+3(3)+9}{(3+3)}$

$y=\dfrac{9+9+9}{3+3}$

$(3, \frac{27}{6})$

- A. x = 3
- B. x = −3
- C. x = −27
- D. none

8. Find the horizontal asymptotes, if any, of the rational function.

$f(x)=\dfrac{2x^2}{x^2+4}$

same degree → ratio of leading coefficient $y=\dfrac{2}{1}$ H.A.

- A. x = 2
- B. y = 0
- C. y = 2
- D. no horizontal asymptotes