Unit 2 ICM Matrices and Game Theory **Name:** _____

Quiz 2 Review

Remember to show your work to receive credit!

space – 0 A = 1 B = 2 C = 3 D = 4 E = 5 F = 6 G = 7 H = 8 I = 9 J = 10 K = 11 L = 12 M = 13
N = 14 O = 15 P = 16 Q = 17 R = 18 S = 19 T = 20 U = 21 V = 22 W = 23 X = 24 Y = 25 Z = 26

1. Encode the message, The Lion King, using matrix A $\begin{bmatrix} 3 & -2 & 4 \\ 0 & 1 & -1 \\ 2 & -3 & 5 \end{bmatrix}$. Show work.

T H E L I O N K I N G

20-8-5-0-12-9-15-14-0-11-9-14-7

$\begin{bmatrix} 20 & 8 & 5 \\ 0 & 12 & 9 \\ 15 & 14 & 0 \\ 11 & 9 & 14 \\ 7 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} 3 & -2 & 4 \\ 0 & 1 & -1 \\ 2 & -3 & 5 \end{bmatrix} = \begin{bmatrix} 70 & -47 & 97 \\ 18 & -15 & 33 \\ 45 & -16 & 46 \\ 61 & -55 & 105 \\ 21 & -14 & 28 \end{bmatrix}$

2. Decode the message using encoding matrix A $\begin{bmatrix} 2 & -1 \\ -3 & 0 \end{bmatrix}$. Show work.

-15 -6 16 -14 -24 -9 14 -7 13 -14 -19 -13

$\begin{bmatrix} -15 & -6 \\ 16 & -14 \\ -24 & -9 \\ 14 & -7 \\ 13 & -14 \\ -19 & -13 \end{bmatrix} \cdot \begin{bmatrix} 2 & -1 \\ -3 & 0 \end{bmatrix}^{-1} = \begin{bmatrix} 6 & 9 \\ 14 & 4 \\ 9 & 14 \\ 7 & 0 \\ 14 & 5 \\ 13 & 15 \end{bmatrix}$

6 9 14 4 9 14 7 0
F I N D I N G

N E M O
14 5 13 15

3. The local shop sells 3 types of pies. Beef pies cost $3 each, chicken pies cost $4 each and vegetable pies cost $2 each.

Create a matrix operation to show the total amount of money the local shop made each day of the week on pies. Show the resulting matrix and label the rows and columns.

	Mon	Tue	Wed	Thu
Beef	13	9	7	15
Chicken	8	7	4	6
Vegetable	6	4	0	3

Income $\begin{bmatrix} 3 & 4 & 2 \end{bmatrix}$ (Beef, Chic., Veg) $\cdot$ $\begin{matrix} \\ \text{Beef} \\ \text{Chic} \\ \text{Veg} \end{matrix}\begin{matrix} \text{M} & \text{Tu} & \text{W} & \text{Th} \end{matrix}\begin{bmatrix} 13 & 9 & 7 & 15 \\ 8 & 7 & 4 & 6 \\ 6 & 4 & 0 & 3 \end{bmatrix} = \begin{matrix} \\ \text{Income} \end{matrix}\begin{matrix} \text{M} & \text{Tu} & \text{W} & \text{Th} \end{matrix}\begin{bmatrix} 83 & 63 & 37 & 75 \end{bmatrix}$

$(1 \times 3) \cdot (3 \times 4)$
income B,C,V B,C,V M,T,W,Th

1 x 4 In M,T,W,Th

4. A landlord owns 3 condominiums, a 1-bedroom condo, a 2-bedroom condo, and a 3-bedroom condo. The total rent she receives is $1240. She needs to make repairs on the condos, and it costs 10% of the 1-bedroom condo's rent for its repairs, 20% of the 2-bedroom for its repairs, and 30% of the 3-bedroom condo's rent for its repairs. The total repair bill was $276. The 3-bedroom condo's rent is twice the 1-bedroom condo's rent. How much is the rent for each of the condos?

See next page for work and tips

Rent for the 1-bedroom is $280,
for the 2-bedroom is $400, and
for the 3-bedroom condo is $560

Quiz 2 Review (continued)

4)

Word Problem Practice

A landlord owns 3 condominiums, a 1-bedroom condo, a 2-bedroom condo, and a 3-bedroom condo. The total rent she receives is $1240. She needs to make repairs on the condos, and it costs 10% of the 1-bedroom condo's rent for its repairs, 20% of the 2-bedroom for its repairs, and 30% of the 3-bedroom condo's rent for its repairs. The total repair bill was $276. The 3-bedroom condo's rent is twice the 1-bedroom condo's rent. How much is the rent for each of the condos?

Total Rent → $x+y+z=1240$

Repair Cost → $.10x+.20y+.30z = 276$

x = rent for 1 bed condo

y = rent for 2 bed condo

z = rent for 3 bed condo

$z = 2x$ ← rent for 3 bedroom is twice 1 bedroom rent

Ex to test: one bed $1000, 3 bed $2000 ✓ with sentence

$2000 = 2(1000)$, $z = 2y$ ✓ works with equation too!!

Equations: $x + y + z = 1240$ ← Total Rent

$.10x + .20y + .30z = 276$ ← Repair Cost

$2x + 0y - z = 0$ ← 3 Br Rent is twice the 1 Br Rent

Matrix Equations:

$$\begin{bmatrix} 1 & 1 & 1 \\ .10 & .20 & .30 \\ 2 & 0 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1240 \\ 276 \\ 0 \end{bmatrix}$$

Enter into calculator

Coefficient matrix = A

Constant matrix = B

To solve, do $A^{-1} \cdot B$ on main screen of calc

(Remember, you cannot solve matrices using division, so to find the variables and get A to the other side you do $A^{-1} \cdot B$ because $A \cdot X = B \rightarrow A^{-1} \cdot A \cdot X = A^{-1} \cdot B \rightarrow X = A^{-1} \cdot B$)

Rent for the 1 bedroom is $280, for 2 bedroom is $400, and for 3 bedroom is $560.

Quiz 2 Review Continued

Unit 2 ICM Matrices and Game Theory **Name:** _____

5. Suppose an animal population has the characteristics described in the table.

Age Groups (years)	0-3	3-6	6-9	9-12	12-15
Survival rate	0.6	0.7	0.8	0.6	0
Birth rate	0	.6	1.3	0.7	0.2

Be careful!

a. Construct the Leslie matrix for this animal.

Birth rates, survival rates in super diagonal

$$L = \begin{bmatrix} 0 & 0.6 & 0 & 0 & 0 \\ 0.6 & 0 & 0.7 & 0 & 0 \\ 1.3 & 0 & 0 & 0.8 & 0 \\ 0.7 & 0 & 0 & 0 & 0.6 \\ 0.2 & 0 & 0 & 0 & 0 \end{bmatrix}$$

*Pay attention to the cycle length ... Here 1 cycle = 3 years

b. For the initial female populations given in the table below, find the population distribution after one cycle **by hand. Show ALL your calculations.**

Age Group	0-3	3-6	6-9	9-12	12-15
Number	10	18	25	16	3
Amount after one cycle	55.1	6	12.6	20	9.6

$= P_0$ ↑ Initial Population Distribution

newborns $= 10(0) + 18(0.6) + 25(1.3) + 16(0.7) + 3(0.2) = 55.1$

amount moving up: $10(0.6) = 6$ ↑ $18(.7) = 12.6$ ↑ $25(0.8) = 20$ $16(.6) = 9.6$

c. Find the female age distribution **and** the total female population after 6 years.

3 years = 2 cycles

Fill in the table below after 6 years:

Age Group	0-3	3-6	6-9	9-12	12-15
Number	35.9	33.06	4.2	10.08	12

$= P_0 \cdot L^2$

$P_0 \cdot L^n$ gets population distribution

$P_0 \cdot L^n \cdot C$ gets total population

Total Population: $P_0 \cdot L^2 \cdot C = P_2 \cdot C^* =$ **95.24 animals in 6 years**

* use $C = \begin{bmatrix} 1 \\ \vdots \\ 1 \end{bmatrix}$ for total

d. How long until a total of 200 female animals?

n	$P_0 \cdot L^n \cdot C$	n	$P_0 \cdot L^n \cdot C$
12	160.57	15	190.39
13	169.96	16	201.48
14			

have enough

16 cycles · 3 yrs/cycle

48 years

e. What is the long term growth rate? Express your final answer as a percent to one decimal place.

n	$P_0 \cdot L^n \cdot C$	GR
20	252.7377	
21	267.4656	5.83%
22	283.0522	5.83%
⋮	⋮	⋮
30	445.3226	5.83%
31	471.2763	5.83%
32	498.7425	5.83%

5.8% growth between cycles long-term

$GR = \frac{\text{new} - \text{previous}}{\text{previous}}$

$GR_{21} = \frac{267.4656 - 252.7377}{252.7377} = .0583$