Name: Key

# Matrix Quiz 1 Applications Review

Spring '19

Find the following given $A = \begin{bmatrix} 4 & x & -2 \\ y & 1 & z \\ 6 & w & v \end{bmatrix}$, and $B = \begin{bmatrix} a & -4 & z \\ 5 & y & 3 \\ x & 0 & 8 \end{bmatrix}$. If a matrix operation is undefined, explain why. Show work!

1. Find A + B

$$\begin{bmatrix} 4+a & -4+x & -2+z \\ 5+y & y+1 & z+3 \\ x+6 & w & v+8 \end{bmatrix}$$

2. 3B − A

$$3\begin{bmatrix} a & -4 & z \\ 5 & y & 3 \\ x & 0 & 8 \end{bmatrix} - \begin{bmatrix} 4 & x & -2 \\ y & 1 & z \\ 6 & w & v \end{bmatrix} = \begin{bmatrix} 3a-4 & -12-x & 3z+2 \\ 15-y & 3y-1 & 9-z \\ 3x-6 & -w & 24-v \end{bmatrix}$$

3. AB

$$\begin{bmatrix} 4 & x & -2 \\ y & 1 & z \\ 6 & w & v \end{bmatrix} \cdot \begin{bmatrix} a & -4 \\ 5 & y \\ x & 0 \end{bmatrix} = \begin{bmatrix} 4a+5x-2x & -16+xy+0(-2) & 4z+3x-16 \\ ay+5+xz & -4y+y+z(0) & yz+3(1)+8z \\ 6a+5w+vx & (-4)+wy+v(0) & 6z+3w+8v \end{bmatrix} = \begin{bmatrix} 4a+3x & -16+xy & 3x+4z-16 \\ ay+5+xz & -3y & yz+3+8z \\ 6a+5w+vx & -24+wy & 6z+3w+8v \end{bmatrix}$$

4. $\begin{bmatrix} 4c & 2-d & 5 \\ -3 & -1 & 2 \\ 0 & -10 & 15 \end{bmatrix} + \begin{bmatrix} 2c+5 & 4d & g \\ -3 & h & f-g \\ 0 & -4c & 15 \end{bmatrix}$

$$\begin{bmatrix} 4c+2c+5 & 2-d+4d & 5+g \\ -3+-3 & -1+h & 2+f-g \\ 0+0 & -10-4c & 15+15 \end{bmatrix} = \begin{bmatrix} 6c+5 & 3d+2 & g+5 \\ -6 & h-1 & 2+f-g \\ 0 & -4c-10 & 30 \end{bmatrix}$$

5. $\begin{bmatrix} x^2 & 4 \\ -2 & y^2 \end{bmatrix} - \begin{bmatrix} 9 & 5 \\ -2 & 5y \end{bmatrix} = \begin{bmatrix} x^2-9 & -1 \\ 0 & y^2-5y \end{bmatrix}$

$$\begin{bmatrix} x^2-9 & 4-5 \\ -2-2 & y^2-5y \end{bmatrix}$$

6. $\begin{bmatrix} x^2 & 4 \\ -2 & y^2 \end{bmatrix} \cdot \begin{bmatrix} 9 & 5 \\ -2 & 5y \end{bmatrix}$ (2x2)(2x2) matching middles → gives 2x2

$$\begin{bmatrix} x^2 & 4 \\ -2 & y^2 \end{bmatrix} \cdot \begin{bmatrix} 9 & 5 \\ -2 & 5y \end{bmatrix} = \begin{bmatrix} 9x^2-8 & 5x^2+20y \\ -18-2y^2 & -10+5y^3 \end{bmatrix}$$

7. $\begin{bmatrix} x^2 & 4 \\ -2 & y^2 \end{bmatrix} \cdot \begin{bmatrix} 4c & 2-d & 5 \\ -3 & -1 & 2 \\ 0 & -10 & 15 \end{bmatrix}$

(2x2)·(3x3) → undefined because "inside dimensions" don't match

8. If $A_{2x4} \cdot B_{4x1}$, what are the dimensions of the product?

(2x4)·(4x1) matches → 2 x 1

9. Is the multiplication of matrices commutative? Show examples to prove, disprove, or both.

No. Order matters because the "inside dimensions" must match. Ex: If $A_{2x3} \cdot B_{3x4}$ → defined, answer 2x4 VS $B_{3x4} \cdot A_{2x3}$ is undefined, no match.

# Matrix Application Problems

**Show your work for each problem using matrix operations to solve the best way.**

11. A travel agent offers three different travel packages to Williamsburg, Virginia. Package A consists of 4 nights at a hotel, 3 passes to local attractions, and 5 meals. Package B consists of 3 nights at a hotel, 4 passes, and 7 meals. Package C consists of 5 nights at a hotel, 4 passes, and no meals. The agent can book a hotel room for $90 per night, get passes for $28 and provide meal vouchers at a local restaurant for $15 per meal. She wants to run an ad featuring the least expensive package. Which plan should she advertise? Explain your reasoning. Label the rows and columns of your matrices in your work.

$$\begin{matrix} \\ A \\ B \\ C \end{matrix}\begin{bmatrix} 4 & 3 & 5 \\ 3 & 4 & 7 \\ 5 & 4 & 0 \end{bmatrix} \cdot \begin{matrix} \text{Hotels} \\ \text{Passes} \\ \text{meal} \end{matrix}\begin{bmatrix} 90 \\ 28 \\ 15 \end{bmatrix} = \begin{matrix} \\ A \\ B \\ C \end{matrix}\begin{bmatrix} \text{Cost} \\ 519 \\ 487 \\ 562 \end{bmatrix}$$

Package B costs the least so the travel agent should advertize that one.

12. An ultimate frisbee team has to order jerseys, shorts, and hats. They have a budget of $1350 to spend on $50 jerseys, $20 shorts, and $15 hats. They want to buy 40 items in preparation for the oncoming season and must order as many jerseys as shorts and hats combined. How many of each item should they order? Write a system of equations to help you solve this problem. $j$ = # jerseys, $x$ = # shorts, $h$ = # hats

$$50j + 20x + 15h = 1350$$
$$j + x + h = 40 \Rightarrow j = 1x + h$$

$$50j + 20x + 15h = 1350$$
$$1j + 1x + 1h = 40$$
$$1j - 1x - 1h = 0$$

$$\begin{bmatrix} 50 & 20 & 15 \\ 1 & 1 & 1 \\ 1 & -1 & -1 \end{bmatrix}^{-1} \cdot \begin{bmatrix} 1350 \\ 40 \\ 0 \end{bmatrix} = \begin{bmatrix} 20 \\ 10 \\ 10 \end{bmatrix}$$

$A^{-1} \cdot B$

20 jerseys, 10 shorts, 10 hats

13. The math club holds a Fall and Spring candy sale for a fundraiser. Each individual bar of candy sold at the following prices: $1.00 (Trail Treats) $1.00 (Carob Chews) $0.50 (Fruit Clusters) $1.50 (Nut Bars). Last year, their sales totals each semester were:

| | | | | | |
|---|---|---|---|---|---|
| Fall: | 40 | Trail Treats | Spring: | 75 | Trail Treats |
| | 100 | Carob Chews | | 108 | Carob Chews |
| | 0 | Fruit Clusters | | 80 | Fruit Clusters |
| | 40 | Nut Bars | | 65 | Nut Bars |

(a) What is the total revenue from last Fall? Label the rows + columns of your matrices. Last Spring?

$$\begin{matrix} \text{Sale} \\ \text{price} \end{matrix}\begin{bmatrix} TT & CC & FC & NB \\ 1 & 1 & 0.5 & 1.5 \end{bmatrix} \cdot \begin{matrix} \\ TT \\ CC \\ FC \\ NB \end{matrix}\begin{bmatrix} \text{Fall} & \text{Spring} \\ 40 & 75 \\ 100 & 108 \\ 0 & 80 \\ 40 & 65 \end{bmatrix} = \begin{matrix} \\ \text{Revenue} \end{matrix}\begin{matrix} \text{Fall} & \text{Spring} \\ \end{matrix}\begin{bmatrix} 200 & 320.50 \end{bmatrix}$$

(b) If the club made a 40% profit last Fall and a 50% profit last Spring, what was the yearly profit for the group?

$$\begin{matrix} \\ \text{Revenue} \end{matrix}\begin{matrix} \text{Fall} & \text{Spring} \\ \end{matrix}\begin{bmatrix} 200 & 320.50 \end{bmatrix} \cdot \begin{matrix} \\ \text{Fall} \\ \text{Spring} \end{matrix}\begin{bmatrix} \text{Profit} \\ 0.40 \\ 0.50 \end{bmatrix} = \boxed{\$240.25}$$

14. Barrett's bookstore sells pencils for $0.10 each and erasers for $0.15 each. Last Tuesday, the store sold 17 more pencils than erasers for a total of $23.45. How many of each item was sold? $x$ = # pencils, $y$ = # erasers

$$\$ \rightarrow 0.10x + 0.15y = 23.45$$
$$\text{total items} \rightarrow x - y = 17 \text{ (17 more pencils than erasers)}$$

$$\begin{bmatrix} .10 & .15 \\ 1 & -1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 23.45 \\ 17 \end{bmatrix} \Rightarrow A \cdot X = B$$

$$\text{Do } A^{-1} \cdot B \Rightarrow$$

104 pencils & 87 erasers were sold

15. You are designing a plumbing system for a new office building. Three pipes, A, B, and C enter the building from the main water line. The total flow in all three pipes is 100 gal/min. If pipes B and C together carry 40 gal/min, and pipe A carries twice as much water as pipe B, how much water must flow in each pipe?

$$a + b + c = 100$$
$$b + c = 40$$
$$a = 2b$$

$$a + b + c = 100$$
$$a + b + c = 40$$
$$a - 2b + 0c = 0$$

$a$ = # flow with pipe A, $b$ = flow with B, $c$ = # flow with pipe C

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 1 & -2 & 0 \end{bmatrix} \cdot \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 100 \\ 40 \\ 0 \end{bmatrix} \Rightarrow A \cdot X = B$$

$$\text{Do } A^{-1} \cdot B \Rightarrow$$

Gallons per minute for pipe A was 60, pipe B was 30 & pipe C was 10