Honors ICM
Unit 3 Homework Packet

Name Key

Day 1: p.12–14 #5-10, 12 SECTION 1.2

5. For this example determine the percentage of voters that ranked each choice first and last.

a. Enter the results in a table like the following: Round to the nearest hundredth. *Be careful.. do not just count the "lines"

CHOICE	% FIRST	% LAST
A	8/26 ≈ 30.77%	69.23% 18/26
B	5/26 ≈ 19.23%	0%
C	23.08%	0%
D	26.92%	30.77% ≈ 8/26

A	B	C	D
B	C	B	B
C	D	D	C
D	A	A	A
8	5	6	7

= 26 voters

Figure 1.1

Answers vary

b. On the basis of these percentages only, which choice do you think would be the most fair to voters? The least fair? Explain your answers. Most fair: B because there is 19.23% 1st + 80.76% 2nd
Least fair: A because 30.77% 1st and 69.23% last

c. Which choice do you think most deserves to be ranked first for the group? Explain your reasoning.
B because it was 1st 19.23% of the time and 2nd the other 80.76% of the time

d. Give at least one argument against your choice.
① A should win because most 1st place votes - with 8 votes
② B has least amount of 1st place votes - with just 5 1st place votes

6. The 1998 race for governor of Minnesota has three strong candidates. The following are unofficial results from the general election.

Jesse Ventura	768,356
Norm Coleman	713,410
Hubert Humphrey III	581,497
Others	12,017

total = 2,075,280

a. What percentage of the vote did the winner receive? Is the winner a majority winner?

$768356/2075280 \approx 37.024\%$ No because he won <50%.

b. What is the smallest percentage the plurality winner can receive in a race with exactly three candidates? Explain.

$$\frac{100\%}{3} = 33.\overline{3}\% \rightarrow > 33.3\%$$

because must get the most votes... this extra 0.1% breaks a tie

7. The Borda method determines a complete group ranking, but the other methods examined in this lesson produce only a first. Each of these methods may be extended, however, to produce a complete group ranking.

a. Describe how the plurality method could be extended to determine a second, third, and so forth. Apply this to the example in Figure 1.1 and list the second, third, and fourth that your extension produces.

Plurality: 1st) A with 8 1st place votes ⇒ Find one with most 1st place
2nd) D with 7 1st place votes ⇒ Find one with next most 1st places
3rd) C with 6 1st place votes ⇒ Find one with third most 1st places, etc.
4th) B with 5 1st place votes

11. When runoff elections are used in the U.S., voters do not rank the candidates and therefore must return to the polls to vote in the runoff. In some countries, such as Ireland, a method commonly called "instant runoff" is used. In an instant runoff, the voters rank the candidates and do not return to the polls. **Examine** the vote totals in the two runoffs below. **Do the totals tell you anything about the merits of the instant runoff? Explain.**

Answers may vary

The instant runoff is good because in the Ireland case, the winner was by far better in each case. Also, in the Texas House race, where the vote was close, the runoff caused a reversal in the winner, though only 59% returned to the polls.

President of Ireland: 1997 Results

	General Election	Runoff
votes total	1,269,836	1,203,775
Mary Banotti	372,002	497,516
Mary McAleese	574,424	706,259
Derek Nally	59,529	
Adi Roche	88,423	
Dana Scallon	175,458	

U.S. House Texas District 9: 1996 Results

	General Election	Runoff
Total votes	189838	112070
Nick Lampson	83,781	59,217
Steve Stockman	88,171	52,853
Geraldine Sam	17,886	

12. A procedure for solving a problem is called an algorithm. This section has presented various algorithms for determining a group ranking from individual preferences. Algorithms are often written in numbered steps in order to make them easy to apply. The following is an EXAMPLE algorithmic of the runoff method.

1. For each choice, determine the number of preference schedules on which the choice was ranked.
2. Eliminate all choices except the two that were ranked first most often.
3. For each preference schedule, transfer the vote total to the remaining choice that ranks highest on that schedule.
4. Determine the vote total for the preference schedules on which each of the remaining choices is ranked first.
5. The winner is the choice ranked first most.

a. Write an algorithmic description of the sequential runoff method. (Describe how to use the runoff method)

1. Determine # of preference schedules
2. Eliminate the one with the least 1st place votes
3. For each preference schedule, transver vote total to ~~the~~ highest now on that pref. schedule
4. Determine # 1st place votes now
5. Repeat steps 1-4 until 2 choices remain

b. Write an algorithmic description of the Borda method. (Describe how to use the Borda method)

1. Determine # of preference schedules
2. If n = # items ranked, assign n points to 1st place, n-1 pts to 2nd place, etc
3. For each item ranked, multiply points by # voted for each preference schedule and sum that for all the preference schedules
4. The item with the most points wins!

8. Determine the plurality, Borda, runoff, and sequential runoff winners for the following set of preferences.

	16	20	12	7
4	A	B	C	C
3	D	D	B	D
2	C	A	D	A
1	B	C	A	B

Borda method

A: $16(4) + 20(2) + 12(1) + 7(2) = 130$

B: $16(1) + 20(4) + 12(3) + 7(1) = 139$

C: $16(2) + 20(1) + 12(4) + 7(4) = 128$

D: $16(3) + 20(3) + 12(2) + 7(3) = 153$

Plurality Winner: B (got most 1st place votes, 20)

Runoff Winner: C → see pink X's

Keep B + C with most 1st place votes (2nd, 1st) be careful

Then B: 20 / C: 35

Borda Winner: D

Sequential Runoff Winner: C (16+7+12 = 35)

1st) Eliminate D: no 1st place votes

2nd) Eliminate A: only 16 1st place

9. Each year the Heisman Trophy recognizes one of the country's outstanding college football players. The year 1997 marked the first time a defensive player received the award. The results of the voting follow. Each voter selects a player to rank first, another to rank second, and another to rank third.

	1st	2nd	3rd	Points
Charles Woodson, Michigan	3(433) +	2(209) +	1(98) =	1,815
Peyton Manning, Tennessee	3(281) +	2(263) +	1(174) =	1,543
Ryan Leaf, Washington State	70	205	174	861
Randy Moss, Marshall	17	56	90	253
Ricky Williams, Texas	4	31	61	135
Curtis Enis, Penn State	3	18	20	65
Tim Dwight, Iowa	5	3	11	32
Cade McNown, UCLA	0	7	12	26
Tim Couch, Kentucky	0	5	12	22
Amos Zereoue, West Virginia	3	1	10	21

a. How many points are awarded for a first-place vote? For a second place? For a third place vote?

3 2 1

b. Would the ranking produced by this system have differed if the plurality method had been used? Explain.

No. Charles Woodson still received the most 1st place votes, as the plurality method requires, so the results would have stayed the same as with the Borda method.

Day 2: p. 28 – 29 #9-13 SECTION 1.4

9. What are the advantages and disadvantages of approval voting? Disadv: weighs all choices equally, voting for 2nd choice could cause 1st choice to lose

Adv: more freedom, less negative campaigning, better voter turnout

10. What is the effect on a group ranking if casting approval votes for all choices? Or casting approval votes for none of the choices?

• No effect on group ranking if vote for all choices

• No effect on group ranking if vote for 0 choices

11. The voters whose preferences are represented below all feel strongly about their first choices but are not sure about their second and third choices. They all dislike their fourth and fifth choices. Use the Condorcet Method to determine the winner. Show all work.

A	D	E
B	B	C
C	E	B
D	C	D
E	A	A
22	20	18

	A	B	C	D	E
A	X	B	C	D	E
B	X	X	B	B	B
C	X	X	X	C	E
D	X	X	X	X	D
E	X	X	X	X	X

Winner: B because won versus ALL other candidates

B: 4 wins
C: 2 wins
D: 2 wins
E: 2 wins

Ex: A 22 vs B: 38 → B wins
A 22 vs C 38 → C wins

12. Consider the preference list shown with lines added to indicate an "approval line". Voters will cast approval votes for candidates above the approval line and will not cast approval votes for candidates below that line. What candidate would win by the approval method? Explain.

	Number of Voters (170 total)				
	33	33	34	36	34
1st	A •	B ✱	E —	D)	B ✱
2nd	D)	E —	C	A •	(C
3rd	C	(C	D	B ✱	A •
4th	B	A	A	E	D)
5th	E	D	B	C	E

A: 103 votes = 33 + 36 + 34
B: 103 votes = 33 + 36 + 34
C: 67 votes = 33 + 34
D: 103 votes = 33 + 36 + 34
E: 67 votes = 33 + 34

Not a true winner
There's a tie between A, B, + D with = amount of votes.

13. Kenneth Arrow is an American mathematician and gained worldwide recognition for his mathematical applications to election theory. The many paradoxes in election methods led Mr. Arrow to formulate a list of conditions he thought were necessary for a group ranking to be fair: **1-Non Dictatorship, 2-Individual Sovereignty, 3-Unanamity, 4-Freedom from Irrelevant Choices, 5-Uniqueness of Group Ranking.** Use Arrow's Five Conditions for fair ranking methods to answer the following questions.

a. Suppose that there are only two choices in a list of preferences and that the plurality method is used to decide the group ranking. Which of Arrow's conditions could be violated? None.

b. There often are situations in which insincere voting results. Do any of Arrow's conditions state that insincere voting should not be part of a fair group-ranking procedure?

No. None of his conditions speak to insincere voting issues.

Day 3: Classwork 1.4-1.5

1. Ten committee members vote by approval voting on four candidates for a new chairperson of the committee. The following table indicates the results; an X denotes approval of the candidate.

Candidate	#1	#2	#3	#4	#5	#6	#7	#8	#9	#10	Total
A	X	X	X	X		X	X				6
B	X	X	X	X		X			X	X	7
C	X	X			X		X	X	X		6
D						X	X				2

a. Which candidate wins and which finishes last?

wins: B last: D

b. If committee members #5 and #8 are adamantly opposed to candidates B and D and they have prior knowledge of the others' votes, how might they have voted differently when using the approval voting method in part (a)?

They might have also voted for A .. to give more votes to the candidates that they don't adamantly oppose.

2. Determine whether any voter is a dictator and whether any is a dummy. Explain your choice.

20 votes are needed to pass an issue {A,B,C;24}, {A,B,D;21}, {A,B,C,D;27}

A: 10 votes B: 8 votes C: 6 votes D: 3 votes

Power: 3 3 1 1

B and A are dictators because no issues can be passed without B's or A's votes.

3. Consider the weighted voting situation in which voters A, B, C, and D have 15, 12, 6, and 3 votes respectively, and 24 votes are needed to pass an issue.

a. Is the coalition formed by B, C, and D a winning coalition?

No. The coalition of B, C, and D only has 21 votes, not the 24 needed.

b. Which players are essential in the coalition {A, B, C; 33}?

A and B because removing either of their votes leaves the coalition with not enough votes to win.

c. Which players are essential in the coalition {A, B, C, D; 36}?

A is essential because removing 15 votes leaves the coalition with only 21 votes.

d. List all the winning coalitions.

{A,B;27}, {A,B,C;33}, {A,B,D;30}, {A,C,D;24}, {A,B,C,D;36}

e. Find the power index for each voter.

A: 5 B: 3 C: 1 D: 1

f. Will each voter think that he or she has a fair share of the power? If not, who received more and who received less? Explain.

No. The voters don't have a fair share of the power because the distribution of votes and power indices is not proportional. A and D have 3 points/votes for each 1 power index, but for B its 4:1 and for C its 6:1.

3 ☐Fix

Day 2: p. 35 – 36 # 1- 4 SECTION 1.5

1. Consider a situation in which A, B, and C have 3, 2, and 1 votes, respectively, and in which 4 votes are required to pass an issue.

KEY→ Possible Coalitions: Winning Coalitions:

a. List all possible coalitions and all winning coalitions

{none; 0}, {A; 3}, {B; 2}, {C; 1}, {A, B; 5}, {A, C; 4}, {B, C; 3}, {A, B, C; 6}

b. Determine the power index for each voter.

A: 3 B: 1 C: 1

c. If the number of votes required to pass an issue is increased from 4 to 5, determine the power index of each voter.

{A, B; 5}, {A, B, C; 6}

A: 2 B: 1 C: 0

2. In a situation with three voters, 51 votes are required to win. A has 49 votes, B has 48, and C has 3.

{A, B; 97}, {A, C; 52}, {B, C; 51}, {A, B, C; 100}

a. Determine the power index of each voter

A: 2 B: 2 C: 2

b. A dictator is a member of a voting body who has all the power. A dummy is a member who has no power. Are there any dictators or dummies in this situation?

No. Each voter has an equal power index.

3. Four partners decide to start a business. P_1 buys 8 shares, P_2 buys 7 shares, P_3 buys 3 shares and P_4 buys 2 shares. One share = one vote. The quota is set at two-thirds of the total number of votes. 20 shares

→ Quota = 14 shares/votes

a. Describe as a weighted voting system.

b. The partnership above decides the quota is too high and changes the quota to 10 votes. Describe the winning coalitions. $\{P_1, P_2; 15\}$, $\{P_1, P_3; 11\}$, $\{P_1, P_4; 10\}$, $\{P_2, P_3; 10\}$, $\{P_1, P_2, P_3; 18\}$, $\{P_1, P_2, P_4; 17\}$, $\{P_2, P_3, P_4; 12\}$, $\{P_1, P_2, P_3, P_4; 20\}$

c. The partnership above decides to make the quota equal 21 votes. Describe the winning coalitions.

None. There are only 20 total votes, so there could not be winning coalitions with quota at 21 votes.

4. Weighted voting is commonly used to decide issues at meetings of corporate stockholders. Each member is given one vote for each share of stock held. {A, B; 51}, {A, C; 51}, {A, D; 50}, {B, C; 50}, {A, B, C, D; 100}, {A, B, C; 76}, {A, B, D; 75}, {B, C, D; 74}

a. A company has four stockholders: A, B, C, and D. They own 26%, 25%, 25%, and 24% of the stock, respectively, and more that 50% of the vote is needed to pass an issue. Determine the power index of each stockholder. Use your results from Exercise 4 as an aid.

A: 4 B: 3 C: 3 D: 1

b. Another company has four stockholders. They own 47%, 41%, 7%, and 5% of the stock. Find the power index of each stockholder. {A, B; 88}, {A, C; 54}, {A, D; 52}, {B, C, D; 53}, {A, B, C, D; 100}, {A, B, C; 95}, {A, B, D; 93}, {A, C, D; 59}

A: 6 C: 2
B: 2 D: 2

c. Compare the percentage of stock owned by the smallest stockholder in parts a and b. Do the same for the power index of the smallest stock holder in each case. %s: 47 41 7 5

power indices: 6 2 2 2

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KEY

Day 3: Quiz Review 1.1-1.5

1. A basketball ranking poll is trying to rank the top teams in the nation. The leading contenders are: Villanova (V), Illinois (I), UConn (C), or Duke (D). The preference ballots are organized in the following preference schedule.

NUMBER OF VOTERS	13	11	4	11	7
1ST CHOICE	V	I	D	C	C
2ND CHOICE	I	V	I	I	D
3RD CHOICE	D	D	V	V	I
4TH CHOICE	C	C	C	D	V

a). How many votes were cast? 46

b). How many votes would be needed for there to be a majority winner? 24 (23 is exactly half... majority winner requires over half)

Is there a majority winner? no If so, who is it? –

c). Find the winner by the plurality method. UConn

d). Find the winner by the 4-3-2-1 Borda Count Method. Illinois

POINT TOTAL FOR V: 122
POINT TOTAL FOR I: 142
POINT TOTAL FOR C: 100
POINT TOTAL FOR D: 96

V: 4(13) + 3(11) + 2(4) + 2(11) + 1(7)
I: 3(13) + 4(11) + 3(4) + 3(11) + 2(7)
C: 1(13) + 1(11) + 1(4) + 4(11) + 4(7)
D: 2(13) + 2(11) + 4(4) + 1(11) + 2(7)

e). Find the winner by the runoff method. Villanova

f). Find the winner by the sequential runoff method. Illinois

g). Find the winner by the Condorcet method. Illinois

for Condorcet winner must win head-to-head versus everyone else

	V	I	C	D
V	X	I	V	V
I	X	X	I	I
C	X	X	X	D
D	X	X	X	X

2. Suppose that this election is conducted by the approval method instead and that each voter approves of the choices marked on the chart below.

VOTERS	A	B	C	D	E	G	H	I	J	K	10 total voters
Villanova	X		X			X		X			4
Illinois	X	X		X		X	X			X	6
U Conn	X		X		X			X			4
Duke	X		X	X	X		X	X	X	X	8

a) Determine the approval winner. Duke

b) If any team with 50% or more of the votes can be selected and they will have a run off, which teams would be in this run-off?

Duke + Illinois
(8 votes) of 10 (6 votes) of 10

3. How many possible preference schedules are there if there are 7 choices to choose from in an election? 7! = $\boxed{5{,}040}$

4. Name and describe each of Arrow's Conditions.

① Non-dictatorship: The preference of a single individual should not become the group ranking without considering the preferences of others
② Individual Sovereignty: Each individual should be able to rank choices how they want - and to indicate ties
③ Unanimity: If everyone prefers one choice, the group ranking should reflect that preference
④ Freedom from Irrelevant Alternatives: The winning choice should still win if one choice - the irrelevant alternative - is removed
⑤ Uniqueness of Group Ranking: The method of producing the group ranking should give the same result each time (it should be repeatable)

5. Consider the weighted voting situation in which voters A, B, C, & D have 11, 8, 6, & 2 votes, respectively, and 14 votes are needed to pass an issue.

a) Is the coalition formed by A and C a winning coalition? yes

{A, C ; 17}
11+6

This A and C coalition has 17 votes, which is above the amount (14) to pass.

b) Which players are essential in the coalition {A, B, C, D; 27}? none

taking 11, 8, 6, or 2 votes away from the coalition still leaves enough votes (≥14) needed to pass

c) List all the winning coalitions (USE CORRECT NOTATION – see question above).

{A, B; 19}, {A, C; 17}, {B, C; 14}
{A, B, C; 25}, {A, B, D; 21}, {A, C, D; 19}, {B, C, D; 16}
{A, B, C, D; 27}

d) Find the Power index for each voter. A: 4 B: 4 C: 4 D: 0

A: 1111 C: 1111
B: 1111 D: 0

e) Are there any voters who are dummies or dictators? Explain.

- There are no dictators because no voters are essential in every winning coalition.
- There is a dummy - D - because D is not essential in any winning coalition

f) Do the power indices reflect the distribution of the votes? Explain.

No. The power indices do not reflect the distribution of votes because A, B, and C have the same power index but not the same weight in votes

Day 5: p. 54-55 # 1 – 9 odd SECTION 2.1-2.2

1. a. The application of any fair division algorithm requires certain assumptions, or axioms. For example, the success of the estate division algorithm requires that each heir be capable of placing a value on each object in the estate. If any heir considers an object priceless or is otherwise capable of placing a dollar value on an object, the algorithm fails. Give at least one other axiom that you think is necessary for the success of the algorithm.

answers vary

Ex:
- heir(s) are able to pay cash into the estate, if needed
- no ties for highest bid on the same item

3. Amy, Bart, and Carol are heirs to an estate that consists of a valuable painting, a motorcycle, a World Series ticket, and $5,000 in cash. They submit the bids shown:

	Painting	Motorcycle	Ticket
AMY	$2, 000	$4,000	$500
BART	$5,000	$2,000	$100
CAROL	$3,000	$3,000	$300

a. Us the estate division algorithm to divide the estate among the heirs. For each heir, state the fair share, the items received, the amount of cash, and the final settlement. Show all of your work and summarize your results in a matrix.

Fair Share

	Fair Share	Items Received	Cash received from estate	TOTAL Final SETTLEMENT
Amy: (2000 + 4000 + 500 + 5000)/3 =	3833.33	Motorcycle, Ticket (4000, 500)	288.89	4788.89
Bart: (5000 + 2000 + 100 + 5000)/3 =	4033.33	Painting (5000)	− 11.11 (pays)	4988.89
Carol: (3000 + 3000 + 300 + 5000)/3 =	3766.67	None	4722.22	4722.22

Final Settlement

all above their Fair Share

You may wish to use a similar table:

		work for cash to/from estate	work for cash to heirs
Cash in the estate	$5,000		
Received from Amy	+666.67	← 4500 − 3833.33	−666.67 + 955.56
Received from Bart	+966.67	← 5000 − 4033.33	−966.67 + 955.56
Paid to Carol	− 3766.67	← total fair share because Carol got no items	3766.67 + 955.56
Cash remaining	= 2866.67		

÷3 = $955.56 to each heir

b. It is common for one or more heirs to pay into an estate. This lesson's algorithm fails if an heir who must pay into the estate cannot do so. Suggest a way the algorithm could be modified to account for situations in which one or more heirs cannot raise the cash necessary to complete the division.

- the item could go to a second highest bidder if the highest bidder can't pay into estate

5. If two heirs submit an identical highest bid for an item, how would you resolve the tie?

- if one heir won another item, the tied item could go to the other heir

7. Could the estate division algorithm of this lesson encourage insincerity by any of the heirs? Explain.

- Heirs might want to bid higher on items to make fair share higher to owe less to estate

9. Two friends have decided to share an apartment in order to obtain a nicer apartment than either could afford individually. They choose a two-bedroom that rents for $900 monthly, including utilities. One bedroom is larger than the other. Propose a procedure for deciding which of the friends gets the nicer bedroom.

- the friends could bid on the value of the larger room then use that to determine each roommates' rent

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	Painting	Motorcycle	World Series Ticket	$5000 Cash	Fair Share
Amy:	(2000	+ 4000	+ 500	+5000)/3	= 3833.33
Bart:	(5000	+ 2000	+ 100	+5000)/3	= 4033.33
Carol:	(3000	+ 3000	+ 300	+5000)/3	= 3766.67

(sum of personal bids + cash / 3 heirs = Fair Share

	Amy	Bart	Carol
Fair Share	$3833.33	$4,033.33	$3,766.67
Items Received	motorcycle, Ticket	Painting	No items
− Fair Share Value of Items Received	−4000 −500	−5,000	−0
= Initial Cash Received (or owe to estate if negative)	−666.67	−966.67	3,766.67
Share of Extra $	+955.56	+955.56	+955.56
FINAL SETTLEMENT	motorcycle, WS Ticket, and gets $288.89 from estate	Painting and owes $11.11 to estate	No items and gets $4722.22 from estate

Remaining Cash in Estate = $5000 originally in estate + 666.67 from Amy + 966.67 from Bart − 3,766.67 to complete Carol's Fair Share

= $2,866.67

Share of Remaining Cash = $2,866.67 / 3 heirs = $955.56 to each heir (Extra Cash)

Section 2.3-2.4: Methods of Apportionment

Example: A country has 6 states with **populations 27774, 25178, 19947, 14614, 9225, and 3292.** Its House of Representatives has 36 seats. Find the apportionment using the methods of Hamilton, Jefferson, Webster, and Hill.

** All methods obtain a standard divisor / ideal ratio

$$s = \frac{\textit{total population}}{\textit{number of seats}} \qquad \frac{100{,}030}{36}$$

\# of seats = 36 ; total population = 100,030 (27774 + 25178 + 19947 + 14614 + 9225 + 3292) s = 2778.6 = standard divisor or ideal ratio

(store in calc) STO then X (by 2 key) (by Alpha key)

- To obtain quotas, divide the population of each state by the idea ratio (s).

State	Population	Quota
A	27,774	$\frac{27774}{S \text{ or } X} = \frac{27774}{2778.6} = 9.99564$
B	25,178	$\frac{25178}{S} = \frac{25178}{X} = 9.06136$
C	19,947	$\frac{19947}{X} = 7.17876$
D	14,614	$\frac{14614}{X} = 5.25946$
E	9,225	$\frac{9225}{X} = 3.32000$
F	3,292	$\frac{3292}{X} = 1.18476$

- **THE HAMILTON METHOD** – each state receives either its lower quota or its upper quota.

State	Quota	Tentative Apportionment	Final Apportionment
A	9.99564	9	→ 10
B	9.06136	9	9
C	7.17876	7	7
D	5.25946	5	5
E	3.32000	3	→ 4
F	1.18476	1	1

Sum is 34 (2 seats short)

Sum is 36

- Round each quota down to get a tentative apportionment. Since the resulting house size is too small (by 2), consider the two quotas with the largest decimal values. Increase their apportionments by 1.
- The Hamilton method always satisfies the quota condition (each states apportionment is equal to either its lower quota or its upper quota).

lower quota: truncated value

upper quota: "bumped" up value

- Ex: 9.06 → lower Quota 9, upper quota 10
- Ex: 5.25 → LQ 5, UQ 6

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- **THE JEFFERSON METHOD** (tends to favor large states)

Remember s = 2778.611

State	Tentative Apportionment (Hamilton ↑ = Truncated Quota)	Jefferson Adjusted Ratio $\frac{\text{State Size}}{(\text{tentative app} + 1)}$	Next Tentative Apportionment	Final Apportionment
A	9	$\frac{27774}{(9+1)} = 2777.4$	10 $\frac{27774}{(10+1)} =$	2524.91 ← still biggest ratio so give them the last leftover seat 11
B	9	$\frac{25178}{(9+1)} = 2517.8$	→ 9	→ 9
C	7	$\frac{19947}{(7+1)} = 2493.38$	→ 7	→ 7
D	5	$\frac{14614}{(5+1)} = 2435.67$	→ 5	→ 5
E	3	$\frac{9225}{(3+1)} = 2306.25$	→ 3	→ 3
F	1	$\frac{3292}{(1+1)} = 1646.0$	→ 1	→ 1

SAME this time

sum of 34 | sum of 35 seats | sum of 36 seats

- The tentative apportionment is the same as the Hamilton method (found by dividing each states population by s and rounding down). Since the resulting house size is too small, calculate the adjusted ratio for each state.

$$\text{Jefferson Adjusted Ratio} = \frac{\textit{state size}}{\textit{tentative apportionment} + 1}$$

- Give the state with the adjusted ratio closest to s (that is the state with the largest adjusted ratio) an additional seat.

- Recompute the state's adjusted ratio based on its new tentative apportionment. If more seats are to be given out, give the state with the largest adjusted ratio another seat. Continue in this manner until all seats are allocated

- The Jefferson method may not satisfy the Quota Condition

↳ In this example, state A gets 11 seats which does not equal its upper quota or lower quota so it violates the quota condition.

Notes Day 8 (continued) Packet p12

- **THE WEBSTER METHOD** (favors neither large nor small states)

State	Quota	Tentative Apportionment	Webster Adjusted Ratio	Final Apportionment
A	9.996	10	$\frac{27774}{10.5}$ = 2645.14	→ 10
B	9.061	9	$\frac{25178}{9.5}$ = 2650.32	→ 9
C	7.179	7 *	$\frac{19947}{7.5}$ = 2659.60	→ 8 (got extra seat b/c * largest adj. ratio)
D	5.259	5	$\frac{14614}{5.5}$ = 2657.09	→ 5
E	3.320	3	$\frac{9225}{3.5}$ = 2635.71	→ 3
F	1.185	1	$\frac{3292}{1.5}$ = 2194.67	→ 1

sum 35 sum 36 seats

- To obtain the tentative apportionment, round each quota (round up if the decimal is .5 or higher and round down if it is smaller than .5)

- When too few seats are given (as is the case here), compute the adjusted ratio as follows:

$$\text{Webster Adjusted Ratio} = \frac{\textit{state size}}{\textit{tentative apportionment} + 0.5}$$

(for too few seats)

- *Choose the state with the largest adjusted ratio (that is, the adjusted ratio that is closest to "s"). Increase that state's apportionment by 1.

- Recompute the state's adjusted ratio based on its new tentative apportionment. If more seats are to be added, compare the adjusted ratio to s as before. Continue in this manner until all seats have been allocated.

- When too many seats are given, compute the adjusted ratio as follows:

$$\text{Webster Adjusted Ratio} = \frac{\textit{state size}}{\textit{tentative apportionment} - 0.5}$$

(for too many seats)

- Choose the state with the smallest adjusted ratio (that is, the adjusted ratio that is closest to "s"). Decrease that state's apportionment by 1.

- Recompute the state's adjusted ratio based on its new tentative apportionment. If more seats are to be taken away, compare the adjusted ratio to s as before. Continue in this manner until the house size is reached.

- The Webster method does not satisfy the quota condition.

Notes Day 8 (continued)

- **Hill-Huntington Method** (tends to favor small states)

s = 2778.6111 $\sqrt{LQ \cdot UQ}$

State	Quota	Geometric Mean	Tentative Apportionment	Adjusted Ratio	Final Apportionment
A 27,774	9.9956 >	$\sqrt{9 \cdot 10} \approx 9.49$	↑ 10	$\frac{27774}{\sqrt{10 \cdot 11}} = 2648.15$	10
B 25,178	9.061 <	$\sqrt{9 \cdot 10} \approx 9.49$	↓ 9	$\frac{25178}{\sqrt{9 \cdot 10}} = 2653.99$	9
C 19,947	7.179 <	$\sqrt{7 \cdot 8} \approx 7.48$	↓ 7	$\frac{19947}{\sqrt{7 \cdot 8}} = 2665.53$	7
D 14,614	5.259 <	$\sqrt{5 \cdot 6} \approx 5.48$	↓ 5	$\frac{14614}{\sqrt{5 \cdot 6}} = 2668.14$ * →	6
E 9,225	3.320 <	$\sqrt{3 \cdot 4} \approx 3.46$	↓ 3	$\frac{9225}{\sqrt{3 \cdot 4}} = 2663.03$	3
F 3,292	1.185 <	$\sqrt{1 \cdot 2} \approx 1.41$	↓ 1	$\frac{3292}{\sqrt{1 \cdot 2}} = 2327.80$	1
			sum 35 seats		36 seats

- For each quota, compute the geometric mean as follows:

$$\text{Geometric Mean} = \sqrt{\textit{lower quota} \cdot \textit{upper quota}}$$

- To get the tentative apportionment, compare the quota to the geometric mean

Round the geometric mean:

UP if the quota is bigger $Q > GM \rightarrow$ Round GM UP ↑

DOWN if the quota is smaller $Q < GM \rightarrow$ Round GM Down ↓

- When too few seats are given (as is the case here), compute the adjusted ratio as follows:

$$\text{Hill – Huntington Adjusted Ratio} = \frac{\textit{state size}}{\sqrt{\textit{tentative apportionment}(\textit{tentative appportionment}+1)}}$$

- *Choose the state with the largest adjusted ratio and increase that states apportionment by 1 seat. Recompute the adjusted ratio and continue until the desired size is reached.

- When too many seats are given, compute the adjusted ratio as follows:

$$\text{Hill – Huntington Adjusted Ratio} = \frac{\textit{state size}}{\sqrt{\textit{tentative apportionment}(\textit{tentative appportionment}-1)}}$$

- Choose the state with the smallest adjusted ratio (closest to s) and decrease that states apportionment by 1 seat. Recompute the adjusted ratio and continue until the desired size is reached.

Hamilton, Jefferson, Webster, and Hill

Ex. 1) City College is made up of 5 different departments: communications, accounting, marketing, psychology, and technology. A total of 110 teaching positions are to be apportioned based on the school's enrollment as shown below.

Department	Communications	Accounting	Marketing	Psychology	Technology	Total
Enrollment	2425	745	497	230	1053	4950

a. Find the total enrollment. 4950 (sum of # enrolled)

b. Find the ideal ratio, s. What does the ideal ratio represent in this problem?
$s = \frac{4950}{110} = 45$ = size of each class or # of students per teacher

c. Find the number of faculty members apportioned to each department using the methods of Hamilton, Jefferson, Webster, and Hill-Huntington and record in the table below. **Split the table columns in two to represent initial apportionment and final apportionment!**

Truncate, give extras (to highest decimal)

Truncate quota

state size (tent. app +1) the extras to highest adj ratio

Round initial quotas then do adj ratio if too many or too few

Initial & Final Apportionments

Department	Initial Quota	Hamilton Initial	Hamilton Final	Jefferson Initial	Jefferson FINAL	Webster Initial	Webster FINAL	Hill-Huntington Initial	Hill-Huntington FINAL
Communications 2425/45=	53.8888	53 →	54	53 →	55	54 →	54	54 →	54
Accounting 745/45=	16.5556	16 →	17	16 →	16	17 →	17	17 →	17
Marketing 497/45=	11.0444	11 →	11	11 →	11	11 →	11	11 →	11
Psychology 230/45=	5.1111	5 →	5	5 →	5	5 →	5	5 →	5
Technology 1053/45=	23.40	23 →	23	23 →	23	23 →	23	23 →	23
sum		108	110	108	110	110	110	110	110

For each method that requires an adjusted ratio, state the adjusted ratio that you use.

State size (tent. app. +1) 44.09 = 2425/55 THEN 2425/(54) 745/17 497/12 230/6 1053/24

Jefferson's Adjusted Ratios: C: 44.91 A: 43.82 M: 41.42 P: 38.33 T: 43.875

Webster's Adjusted Ratios: C: n/a A: n/a M: n/a P: n/a T: n/a

tentative apport. sum was 110 already :)

Hill-Huntington Geometric Means: C: 53.5 = $\sqrt{53 \cdot 54}$ A: 16.5 $\sqrt{16 \cdot 17}$ M: 11.5 = $\sqrt{11 \cdot 12}$ P: 5.48 = $\sqrt{6 \cdot 6}$ T: 23.5 = $\sqrt{23 \cdot 24}$

Hill-Huntington Adjusted Ratios C: n/a A: n/a M: n/a P: n/a. T: n/a } sum 110 already :)

C: Q 53.89 > GM 53.5 so ↑GM A: Q 16.56 > GM 16.5 so ↑GM ...

Which department prefers each apportionment method and why?

Communications prefers Jefferson's method as it gives them the most seats (55). Accounting prefers Hamilton, Webster or Hill-Huntington which give them the most (17). Marketing, Psychology, and technology have no preference since they get the same seats no matter what method.

Do any methods violate the quota condition?

Jefferson violates the quota condition here because communications ends up with 55 teachers which is above their upper quota (54).

Ex. 2) A country has six states with populations 27,770; 25,193; 19,418; 14,612; 9,217; 3,790. It's House of Representatives has 40 seats.

a. Find the apportionment using the methods of Hamilton, Jefferson, Hill-Huntington, and Webster and record in the table below. For Jefferson, Hill-Huntington, and Webster, state the adjusted ratio that you use. Total Pop = 100000 S = Ideal Ratio = 2500 = 100000/40

Initial & Final Apportionments

Population	Initial Quota	Hamilton Init. → FINAL	Jefferson Initial → FINAL	Webster Initial → FINAL	Hill-Huntington Initial → FINAL
A: 27,770/2500	11.108	11 → 11	11 →(3rd) 12	11 → 11	Q<∴ GM↓ 11 → 11
B: 25,193/2500	10.077	10 → 10	10 → 10	10 → 10	Q<∴ GM↓ 10 → 10
C: 19,418/2500	7.767	7 → 8	7 →(2nd) 8	8 → 8	Q>∴ GM↑ 8 → 8
D: 14,612/2500	5.845	5 → 6	5 →(1st) 6	6 → 6	Q>∴ GM↑ 6 → 5
E: 9,217/2500	3.687	3 → 4	3 → 3	4 → 4	Q>∴ GM↑ 4 → 4
F: 3,790/2500	1.516	1 → 1	1 → 1	2 → 1	Q>∴ GM↑ 2 → 2
		37 / 40	37 / 40	41 / 40	41 / 40

Adj. Ratio closest to S (smallest Adj. Ratio) so has least need for a seat

2nd calc. D: 2087.4
3rd calc. C: 2157.6

b. Jeff. Adjusted Ratios: A: 2314.2 (3rd) B: 2290.3 C: 2427.25 (2nd) D: 2435.33 (1st) E: 2304.3 F: 1895

c. Webs. Adjusted Ratios: A: 2644.8 B: 2651.9 C: 2589.07 D: 2656.7 E: 2633.4 F: 2526.67 ← closest to S 50 ↓ one seat

d. H-H Geometric Means: A: 11.49 B: 10.49 C: 7.48 D: 5.48 E: 3.46 F: 1.414

e. H-H Adjusted Ratios: A: 2647.8 B: 2655.6 C: 2594.8 D: 2435.3 E: 2660.7 F: 2679.9

do Webster Adj. Ratio = $\frac{\text{State size}}{\text{tent. app.} - 0.5}$ because too many seats

H-Hill Adj Ratio = $\frac{\text{State size}}{\sqrt{(\text{tent. app.})(\text{tent. app.} - 1)}}$ because need to take away a seat

Ex. 3) Central High School has sophomore, junior, and senior classes of 464, 240, and 196 students respectively. The 20 seats on the school's student council are divided among the classes according to population. Total student pop. = 900 S = ideal ratio = 900/20 = 45

a. Find the apportionment using the methods of Hamilton, Jefferson, Hill-Huntington, and Webster and record in the table below.

Class Size	Initial Quota	Hamilton Init.	Hamilton FINAL	Jefferson Init. → FINAL	Webster Init. →	Hill-Huntington Tentative →
Sophomores 464/45 =	10.311	10	10	10 → 11	10 → 11	Q<∴ GM↓ 10 → 11
Juniors 240/45 =	5.333	5	5	5 → 5	5 → 5	Q<∴ GM↓ 5 → 5
Seniors 196/45 =	4.356	4	5	4 → 4	4 → 4	Q<∴ GM↓ 4 → 4
		19		19 / 20	19 / 20	19 / 20

b. Jeff. Adjusted Ratios: Sophomores: 42.18 (464/11) Juniors: 40 (240/6) Seniors: 39.2 (196/5)

c. Webs. Adjusted Ratios: Sophomores: 44.19 (464/(10.5)) Juniors: 43.636 (240/5.5) Seniors: 43.556 (196/4.5)

d. H-H Geometric Means: Sophomores: 10.49 ($\sqrt{10 \cdot 11}$) Juniors: 5.48 ($\sqrt{5 \cdot 6}$) Seniors: 4.47 ($\sqrt{4 \cdot 5}$)

e. H-H Adjusted Ratios: Sophomores: 44.24 ($464/\sqrt{10 \cdot 11}$) Juniors: 43.82 ($240/\sqrt{5 \cdot 6}$) Seniors: 43.83 ($196/\sqrt{4 \cdot 5}$)

f. Did any methods violate quota condition? If so, which ones?

No. All methods gave a value of seats equal to the lower quota or upper quota.