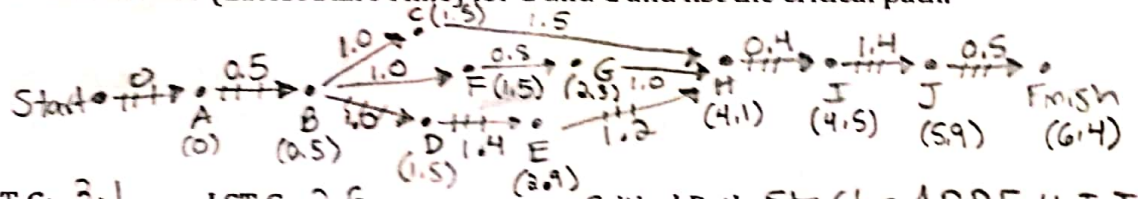
4.1-4.5 Practice

1. Use the table below #1-2, showing the steps to produce a padlock, in order to construct a graph. Label each vertex with the earliest start time. Determine the minimum project time and critical path.

Activity	Description	Immediate Predecessor	Duration (Hours)
A	Receive raw materials	-	0.5
B	Bolt cutting	A	1.0
C	Transfer Machine (series of drilling and cutting operations)	B	1.5
D	Transfer Machine (barrels)	B	1.4
E	Barrel pinning	D	1.2
F	Shackle groove cutting	B	0.8
G	Shackle Bending	F	1.0
H	Insert shackle into body	C,E,G	0.4
I	Insert barrel into body and test key set	H	1.4
J	Packaging of padlock	I	0.5

Source: http://criticalpathmethod.weebly.com/solved-problem.html

2. Now find the LST (Latest Start Time) for G and C and list the critical path.

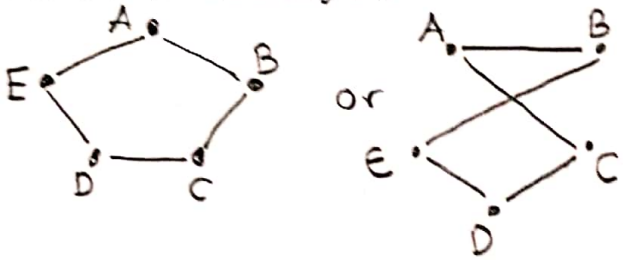


LST G: 3.1 LST C: 2.6 Critical Path: Start - A B D E H I J - Finish

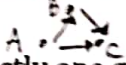
6.4-.5-1.4-.4-1 6.4-.5-1.4-0.4-1.5

3. Central High School is a member of a five-team hockey league. Each team in the league plays exactly two games, which must be against different teams. Show that there is only one possible graph for this schedule.

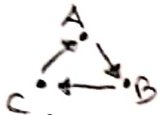
each vertex has degree 2
showing 2 games for team



4. Draw a tournament with three vertices in which:
   a) One player wins all the games he or she plays.



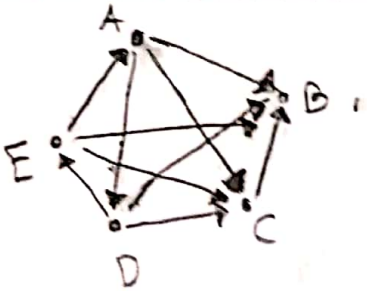
   b) Each player wins exactly one game.



   c) Two players lose all of the games they play.

not possible

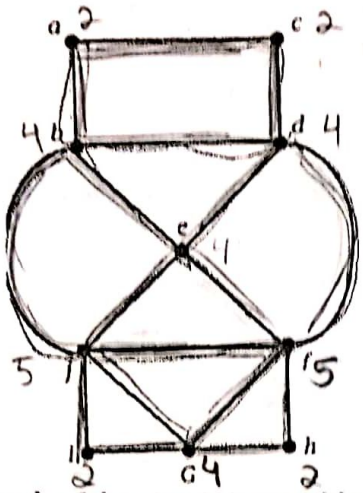
5. Draw a tournament with five vertices in which there is a three-way tie for first place.



A beats B, C, D
D beats B, C, E
E beats A, B, C

6. The street network of a city can be modeled with a graph in which the vertices represent the street corners, and the edges represent the streets. Suppose you are the city street inspector and it is desirable to minimize time and cost by not inspecting the same street more than once.

G edges once
⇓
use Euler's methods



a) In this graph of the city, is it possible to begin at the garage (G) and inspect each street only once? Will you be back at the garage at the end of inspection? No. You need to start and end at odd degree vertices to make Euler path.

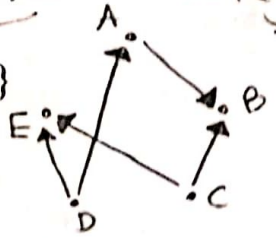
No Euler circuit because not all vertices are even degree

b) Find a route that inspects all streets, repeats the least number of edges possible, and returns to the garage. Ghfdcabdefjebjfgjig or Gijbacdebdfgjfhgfjg only one repeat!!

7. Construct the following digraphs.

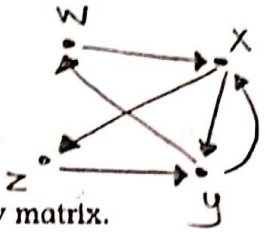
a) $V = \{A, B, C, D, E\}$
$E = \{AB, CB, CE, DE, DA\}$

means A beats B



b) $V = \{W, X, Y, Z\}$
$E = \{WX, XZ, ZY, YW, XY, YX\}$

means W beats X

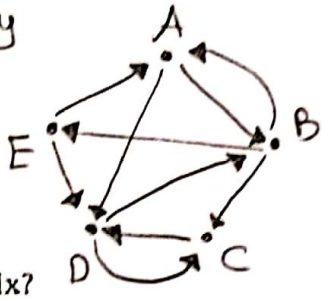


8. a. Construct a digraph for the following adjacency matrix.

Remember digraphs have arrows to show direction

	A	B	C	D	E
A	0	1	0	1	0
B	1	0	1	0	1
C	0	0	0	1	0
D	0	1	1	0	0
E	1	0	0	1	0

. For digraphs, 1 shows a win versus opponent

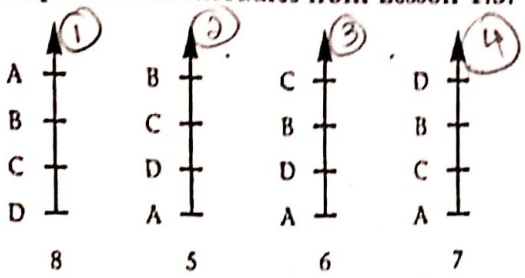


b. Is there symmetry along the main diagonal of the adjacency matrix? Explain why or why not. No because a candidate may get beat by opponent
Ex: E beat A
so $r_1c_5 = 0$
but $r_5c_1 = 1$

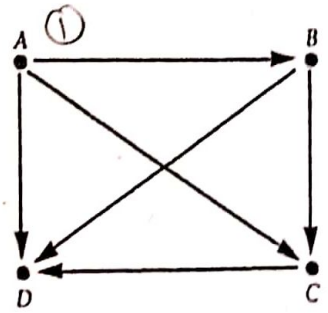
c. Find the sum of the numbers in the second row. What does that total indicate? 3 → B won 3 head-to-head matchups

d. Find the sum of the numbers in the second column. What does that total indicate? 2 → B lost 2 head-to-head matchups

9. Consider the set of preference schedules from Lesson 1.3:



The first preference schedule could be represented by the following tournament.



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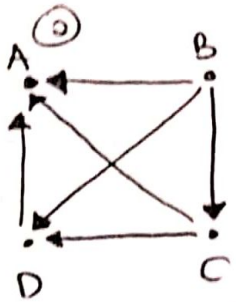
a. Construct tournaments for each of the three other preference schedules.



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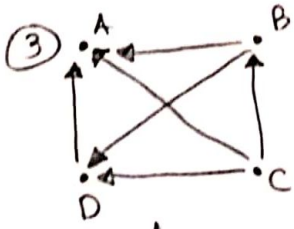

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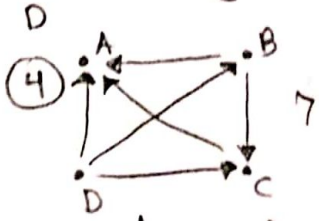

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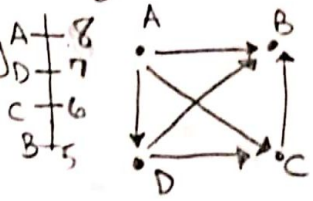
b. Construct a cumulative preference tournament that would show the overall results of the four individual preference schedules. Plurality



c. Is there a Condorcet winner in the election? (Recall from Lesson 1.3 that a Condorcet winner is one who is able to defeat each of the other choices in a one-on-one contest.) B is Condorcet winner

	A	B	C	D
A	x	B	C	D
B	x	x	B	B
C	x	x	x	C
D	x	x	x	x

A vs B → B wins
8 18

B vs C → B wins
20 6

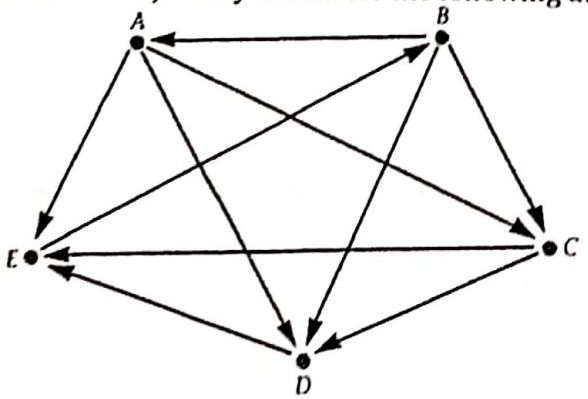
d. Find a Hamiltonian path for the cumulative tournament. What does this path indicate? A D C B } shows order of A winner

A: vertex with highest out degree → to

B: vertex with lowest out degree

D ↓ to
C
B loser

10. Construct an adjacency matrix for the following digraph, and call it M.



$$M = \begin{array}{c} \\ A \\ B \\ C \\ D \\ E \end{array} \begin{array}{c} \begin{array}{ccccc} A & B & C & D & E \end{array} \\ \begin{bmatrix} 0 & 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

row 1 shows A beat C, D, and E
row 5 shows E only beat B
column 2 shows B lost only to E