Unit 7 Day 6
Graph Theory

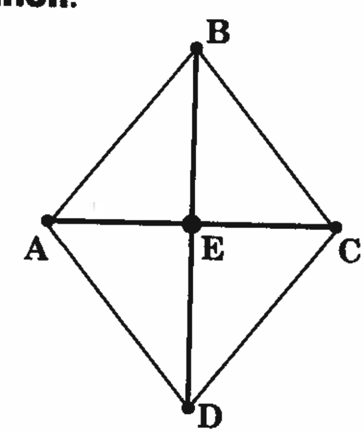
Section 5.1 and 5.2

1

Warm-Up Day 6

Determine if the below exist or not. If it exists, write it. If it doesn't exist, explain why using the definition.

- Euler Path?
- Euler Circuit?
- Hamiltonian Path?
- Hamiltonian Circuit?



- What is the chromatic number for the graph?

2

Warm-Up Day 6

Determine if the below exist or not. If it exists, write it. If it doesn't exist, explain why using the definition.

- Euler Path?

  No - not exactly 2 odd degree vertices.

- Euler Circuit?

  No - not all even degree vertices.

- Hamiltonian Path?

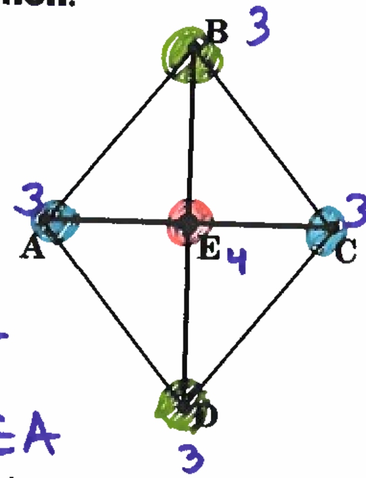
  Yes. Example: ABCDE

- Hamiltonian Circuit?

  Yes. Example: ABCDEA

- What is the chromatic number for the graph?

  3 - see diagram



2

Warm-Up Day 6 ANSWERS

Determine if the below exist or not. If it exists, write it. If it doesn't exist, explain why using the definition.

- Euler Path?

  No, because more than two have odd vertices

- Euler Circuit?

  No, because not all of the vertices have an even degree

- Hamiltonian Path?

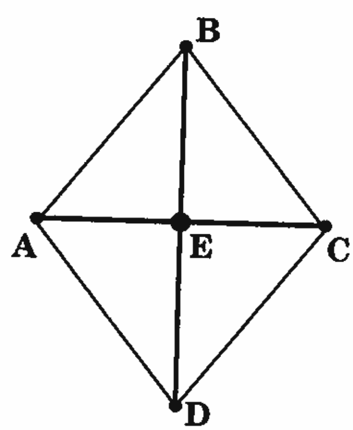
  Yes! BAEDC

- Hamiltonian Circuit?

  Yes! BAEDCB

- What is the chromatic number for the graph?

  3



3

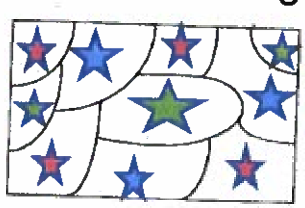
HW Discussion

Answers up next...

4

Homework Answers p.196

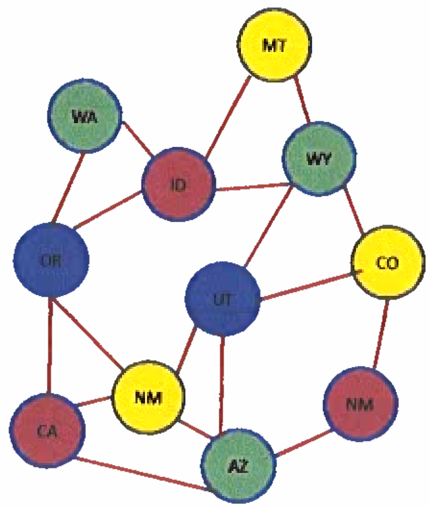
5. a. K_2: 2, K_3: 3, K_4: 4, K_5: 5
   b. The colors needed is equal to the number of vertices because each vertex connects to every other vertex.

8. Minimum of 3 cars
9. Minimum of 4 fish tanks
10. Minimum of 3 storage facilities
11. 

5

Homework Answers p.196

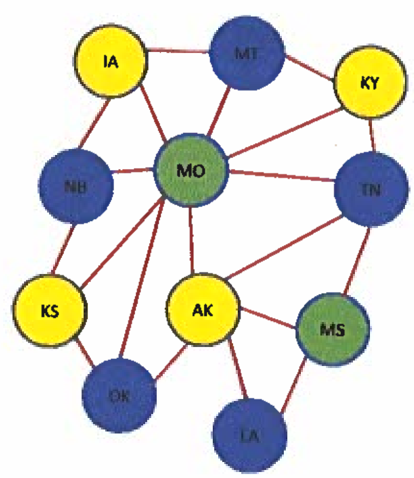
12. a.



6

Homework Answers p. 196

12. b.



7

Planarity of Graphs

NOTES
Section 5.1

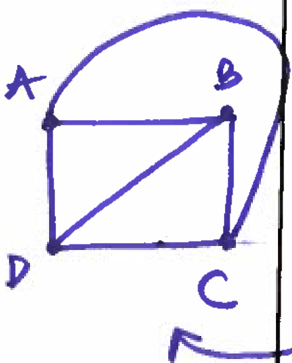
9

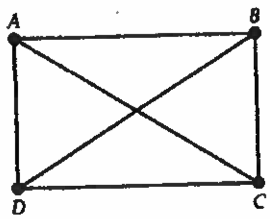
Because of modern technology, elaborate communication networks span the country and most of the earth. These networks affect the way we work, the way we learn, and the way we are entertained.

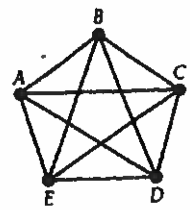
How can we construct communication networks at the lowest possible cost? How do we find the most efficient route between locations in a network? What about routes for airplanes and automobiles? Graph Theory plays an important role in solving these and many other problems that are important in our ever-changing world.

10

- The Four-Color Theorem states that any map that can be drawn on the surface of a sphere can be colored with at most 4 colors.
- So, why do some graphs require more than 4 colors?
- Try to redraw the following graphs so that their edges intersect only at the vertices.







- The K_4 graph can be moved to have no crossing edges. The K_5 cannot.

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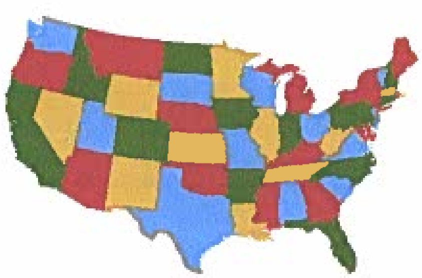
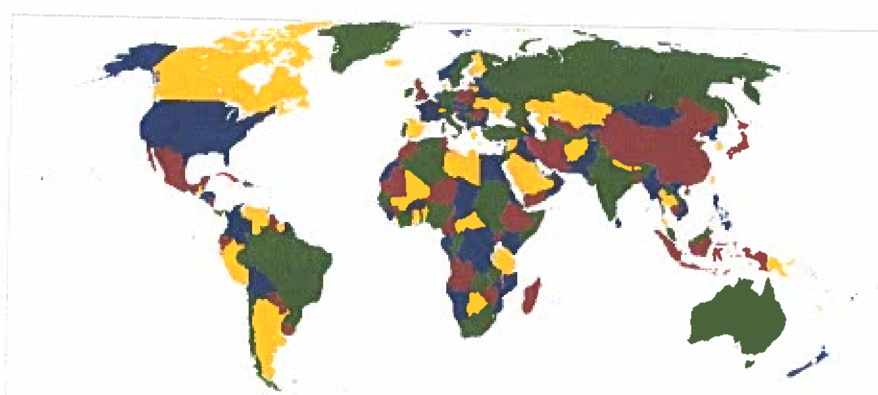


USA or
World
in 4 colors!



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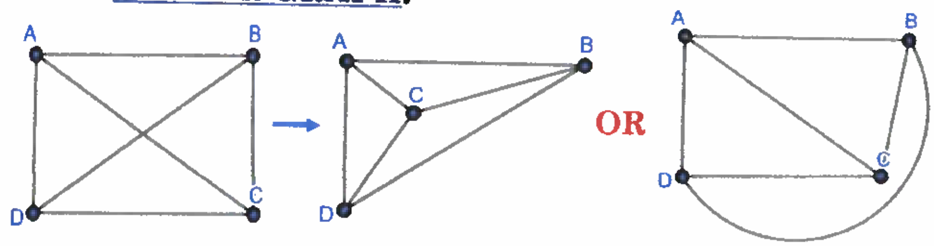
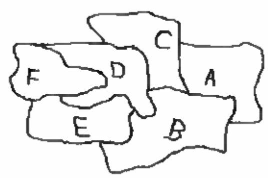
- If a graph can be drawn with no crossing edges, it is a PLANAR GRAPH.



- A graph resulting from a map is always PLANAR.
- Every PLANAR graph has a chromatic number less than or equal to 4. If it is not planar, we do not know how many colors it will take!
- Note that no one said that the *converse* of this statement is true!



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*Statement does NOT go both ways... So if chromatic # ≤ 4, the graph may or may not be planar

Planar Theorem

How can you verify that a graph is planar?

- **To see if you can draw it without edges crossing, then check by using Euler's Formula:**

regions + vertices = edges + 2

Let r = # regions (outside of graph counts as 1 region)
v = # of vertices
e = # edges

$r + v = e + 2$

Before you count regions, IT IS VERY IMPORTANT TO FIRST DRAW A PLANAR GRAPH SO THAT NO EDGES CROSS!

- **If this statement is true, then the graph is planar! Otherwise, it is nonplanar.**

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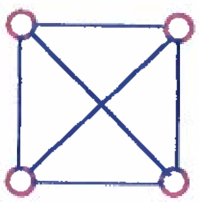
Use Euler's Formula to verify if the graphs are planar.

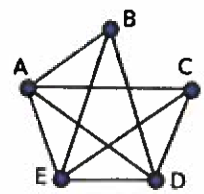
regions + vertices = edges + 2

Before you count regions, IT IS VERY IMPORTANT TO FIRST DRAW A PLANAR GRAPH SO THAT NO EDGES CROSS!

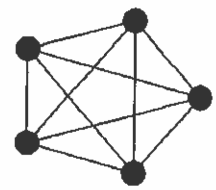
a.

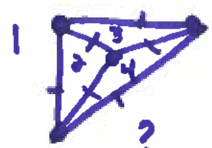


b.



c.

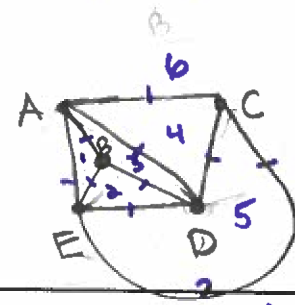




$r + v \stackrel{?}{=} e + 2$

$\checkmark\ 4 + 4 = 6 + 2$

→ $\boxed{\text{Planar}}$



$r + v \stackrel{?}{=} e + 2$

$\checkmark\ 6 + 5 = 9 + 2$

$\boxed{\text{Planar}}$

Non planar

A K_5 graph is non planar & can't be redrawn to have no crossing edges

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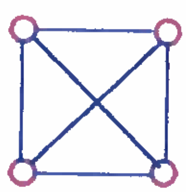
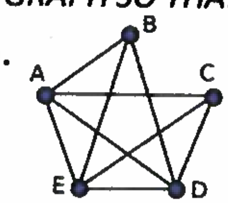
ANSWERS: Use Euler's Formula to verify if the graphs are planar. **regions + vertices = edges + 2**

Before you count regions, IT IS VERY IMPORTANT TO FIRST DRAW A PLANAR GRAPH SO THAT NO EDGES CROSS!

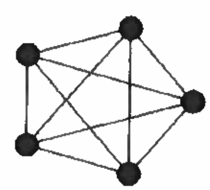
a.

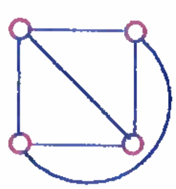


b.



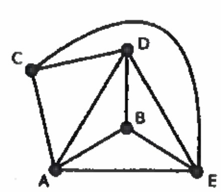
c.





4 + 4 = 6 + 2 ✓

Planar



6 + 5 = 9 + 2 ✓

Planar

Nonplanar

A K_5 graph cannot be drawn without edges crossing!

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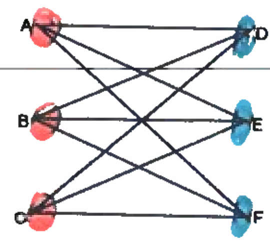
What is the chromatic number?
Can the graph be changed to have no crossing edges?



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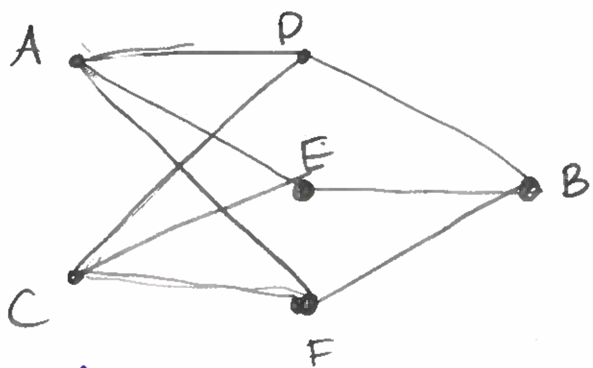
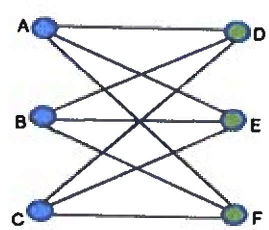


No. Can't redraw it with no crossing edges.

17

What is the chromatic number?
Can the graph be changed to have no crossing edges?



2

This is a <u>BIPARTITE</u> Graph.

<u>Bipartite</u> – **When the vertices of a graph can be divided into two distinct sets so that each edge has one vertex in each set.**

<u>Complete Bipartite</u> – **Contains <u>ALL possible edges</u> between the pairs of vertices in the two distinct sets.**

Complete Bipartite Notation – $K_{m,n}$
– where m,n are the number of vertices in the two sets.

So, this is a $K_{3,3}$ **graph.**

What are the two distinct sets of vertices?

{ A, B, C } { D, E, F }

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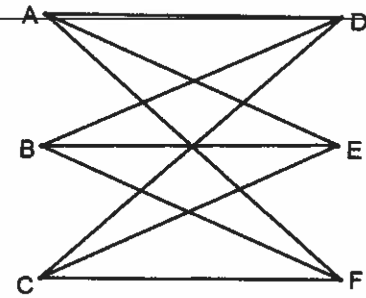
The $K_{3,3}$ shows a counterexample of the *converse* of the Four-Color Theorem.



A $K_{3,3}$ graph has a chromatic number of 2, *but it is not planar.*

So, if even a part of a bigger graph is a $K_{3,3}$ then we know that it is not planar.

Also, any complete graph (K_n) with 5 or more vertices *will not be planar.*

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If graph includes a $K_{3,3}$, it's not planar.

If complete graph with ≥ 5 vertices, it's not planar.

So...

- The chromatic number of any complete graph K_n is _*n*_ , because _Each vertex is connected to every other vertex._.
- The chromatic number of any bipartite graph is _2_.

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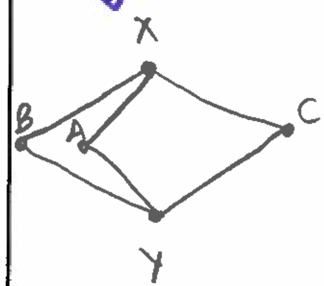
What about this example?

What is its chromatic number? 2

Can it be changed to have no crossing edges? Yes. Example



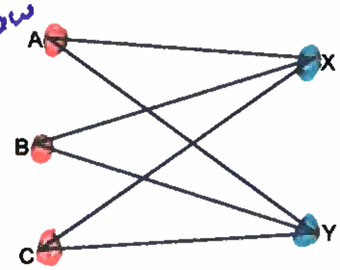
Is it complete bipartite? Notation? Yes $K_{3,2}$

Is it planar? Yes. Can redraw with no crossing edges

So, this is a $K_{\underline{3},\underline{2}}$ graph.

What are the two distinct sets of vertices?

{A, B, C} {x, y}



21

What about this example?
ANSWERS

What is its chromatic number? 2

Can it be changed to have no crossing edges? Yes

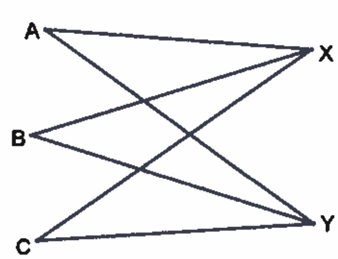
Is it complete bipartite? Notation? Yes. $K_{3,2}$

Is it planar? Yes

So, this is a $K_{3,2}$ graph.

What are the two distinct sets of vertices?

{ A, B, C } { X, Y }



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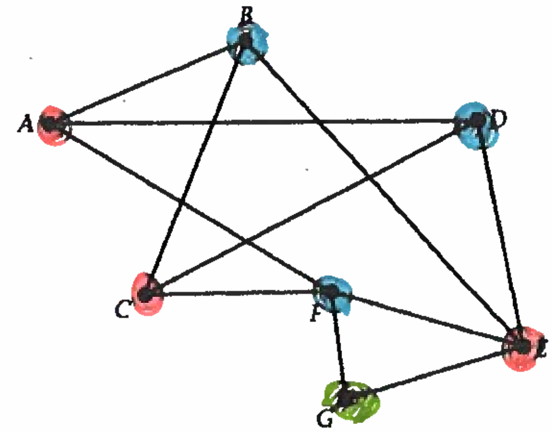
Determine whether the following graph is planar or nonplanar.



Nonplanar because a $K_{3,3}$ subgraph can be found.

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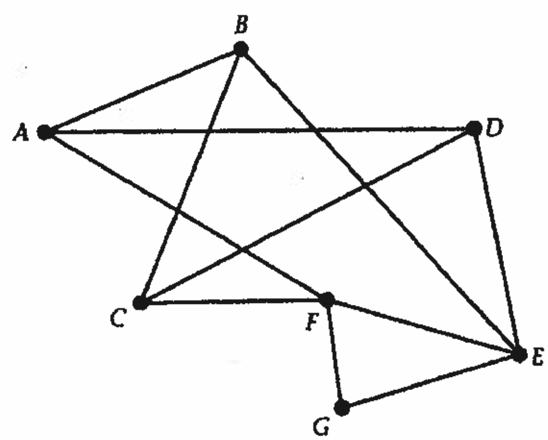
{A, C, E} and {B, D, F} make a $K_{3,3}$ subgraph

Determine whether the following graph is planar or nonplanar. **ANSWER**



A $K_{3,3}$ subgraph can be found. Thus, it is nonplanar.

{ A, C, E } and { B, D, F } create a $K_{3,3}$ subgraph

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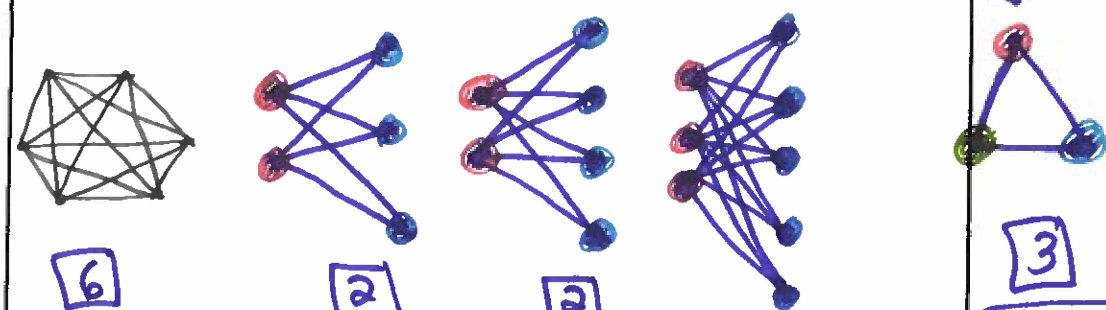
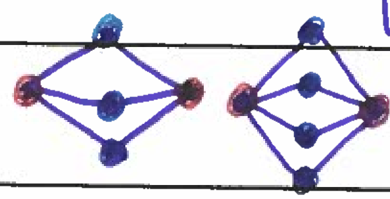
You Try! **Draw each of these graphs, identify its chromatic number, and identify if it is planar.**

K_6	$K_{2,3}$	$K_{2,4}$	$K_{3,5}$	K_3



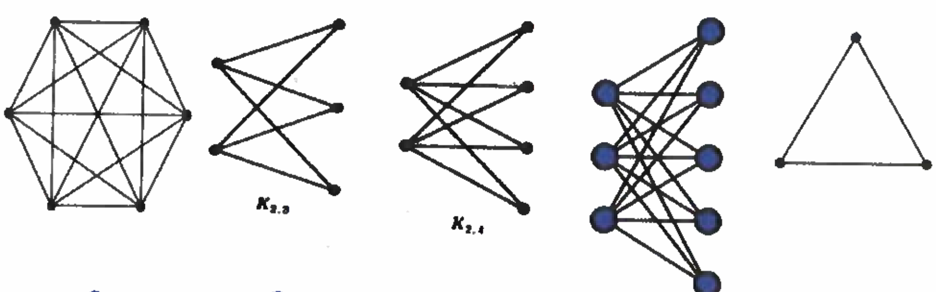
6	2	2	2	3
Nonplanar	Planar	Planar	Nonplanar	Planar
K_n graphs with $n \geq 5$ are nonplanar			(contains a $K_{3,3}$ subgraph)	



25

You Try! ANSWERS **Draw each of these graphs, identify its chromatic number, and identify if it is planar.**

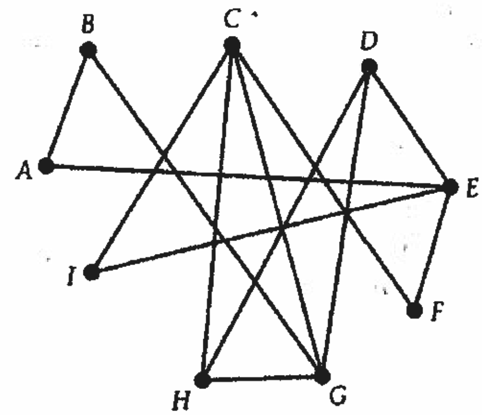
K_6	$K_{2,3}$	$K_{2,4}$	$K_{3,5}$	K_3



6	2	2	2	3
Nonplanar	Planar	Planar	Nonplanar (contains a $K_{3,3}$)	Planar

26

Example: The following graph is planar. Draw it without edge crossings.



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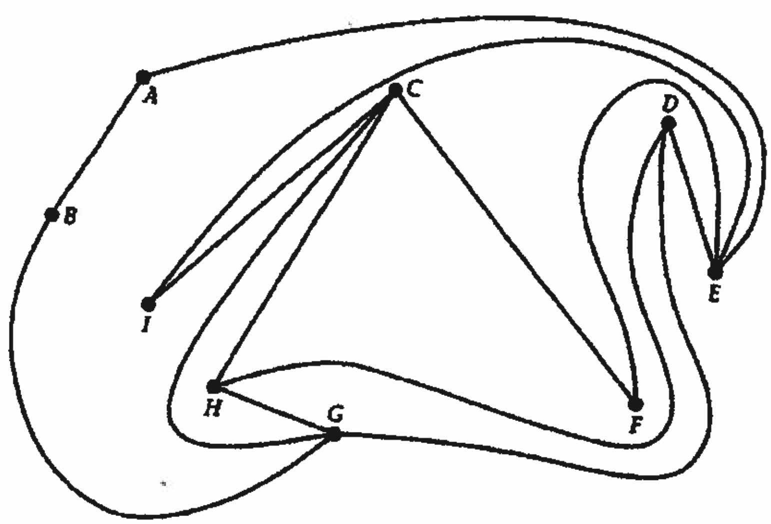
Example ANSWER



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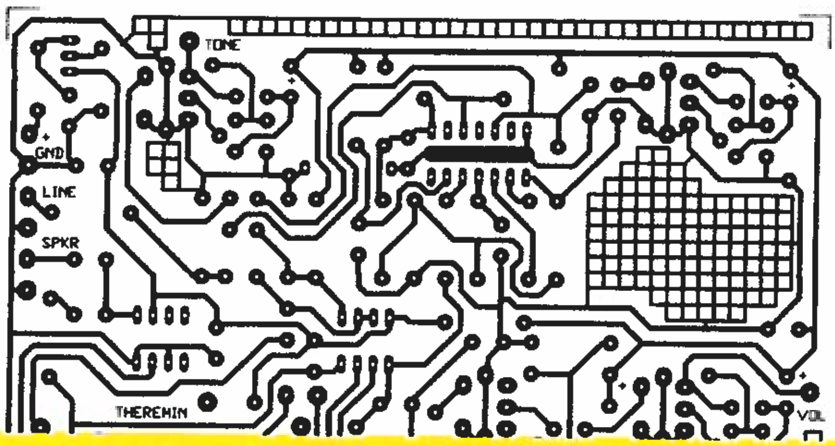
Circuit Boards

The concept of planarity is important to designing circuit boards for the electronics industry. Explain why.



Circuit boards must be planar.
Crossing the metal lines will short out the circuit.

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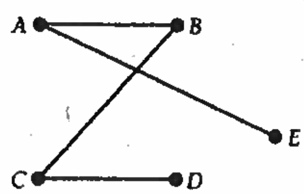
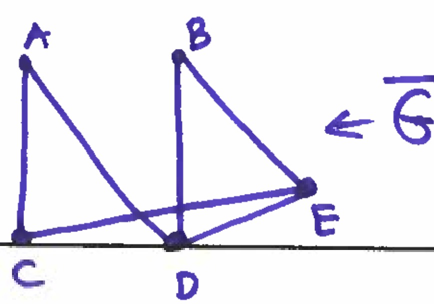
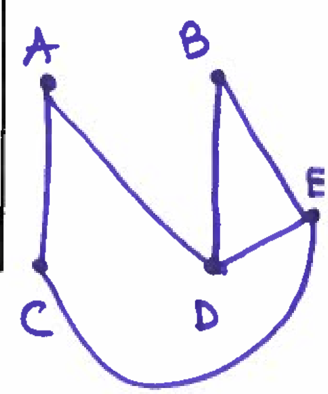
Graph Complements

The **complement** of a graph G is customarily denoted by $\bar{G}$. The complement $\bar{G}$ has the same vertices as G, but its edges are those not in G. The edges of G and $\bar{G}$ along with vertices from either set would make a complete graph. Draw the complement of the following graph.





OR



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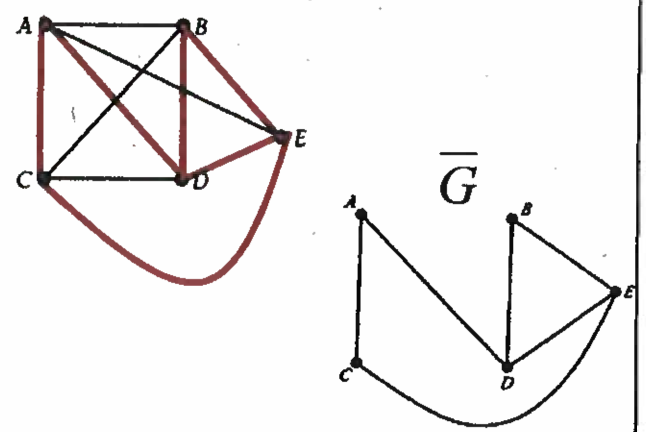
Graph Complements

The **complement** of a graph G is customarily denoted by $\overline{G}$. The complement $\overline{G}$ has the same vertices as G, but its edges are those not in G. The edges of G and $\overline{G}$ along with vertices from either set would make a complete graph. Draw the complement of the following graph.



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Planarity of Graphs – Section 5.1 – Review

1. How many edges are in a $K_{5,7}$ graph ? $5 \times 7 = \boxed{35 \text{ edges}}$
2. What is the chromatic number of a $K_{12,8}$ graph ? $\boxed{2}$ because it's bipartite
3. Is a $K_{12,8}$ graph planar? Explain your reasoning.
   $\boxed{\text{No}}$ because it contains a $K_{3,3}$ subgraph
4. Does a $K_{12,8}$ graph have an Euler circuit? Explain your reasoning. $\boxed{\text{Yes}}$ because each vertex will have an even degree since each set has an even number of vertices.
5. What is the chromatic number of a K_{14} graph ? $\boxed{14}$
6. Is a K_{14} graph planar? Explain your reasoning.
   $\boxed{\text{No}}$ Complete graphs with 5 or more vertices are nonplanar.
7. Does a K_{14} graph have an Euler circuit? Explain your reasoning.
   $\boxed{\text{No}}$ Each vertex will have degree 13 and Euler circuits need all even degree vertices.
8. What is the Four-Color Theorem?
   Any map can be colored with 4 or less colors.

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Any planar graph has chromatic number 4 or less.

Planarity of Graphs – 5.1 Review - ANSWERS

1. How many edges are in a $K_{5,7}$ graph ? $5 \times 7 = 35$ edges

2. What is the chromatic number of a $K_{12,8}$ graph ? 2

3. Is a $K_{12,8}$ graph planar? Explain your reasoning.
No. This graph will contain a $K_{3,3}$ as a subgraph. So, it is not planar.

4. Does a $K_{12,8}$ graph have an Euler circuit? Explain your reasoning. Yes. Since there is an even number of vertices in each group, each vertex will have an even degree. Thus, it will have an Euler circuit.

5. What is the chromatic number of a K_{14} graph ? 14

6. Is a K_{14} graph planar? Explain your reasoning. No.
Any complete graph with more than 4 vertices will not be planar.

7. Does a K_{14} graph have an Euler circuit? Explain your reasoning.
No. Each vertex will have a degree of 13. With vertices of odd degree, the graph can not have an Euler circuit.

8. What is the Four-Color Theorem?
Any map can be colored with 4 or fewer colors. Any planar graph has a chromatic number of 4 or less.

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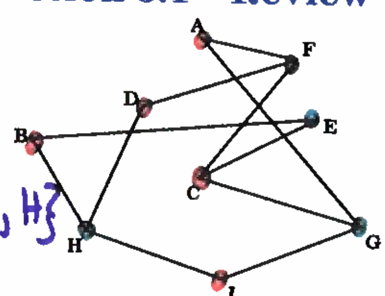
Planarity of Graphs – Section 5.1 – Review

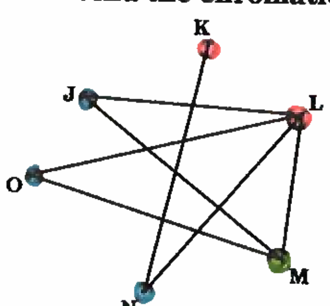
9. Is this graph bipartite? If it is, list the two distinct sets of vertices.



yes $K_{5,4}$

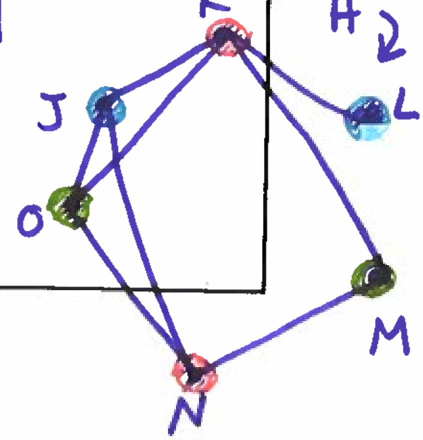
$\{A, B, C, D, I\}$ $\{E, F, G, H\}$

10. Graph H is shown below. Find its chromatic number. Draw $\overline{H}$. Find the chromatic number of $\overline{H}$.



$\boxed{3}$

$\overline{H}$



$\boxed{3}$

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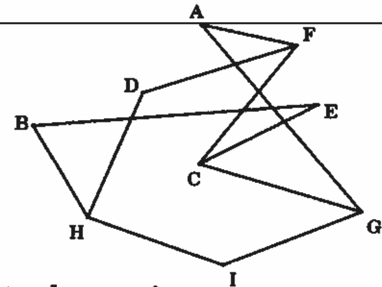
Planarity of Graphs – 5.1 Review - ANSWERS

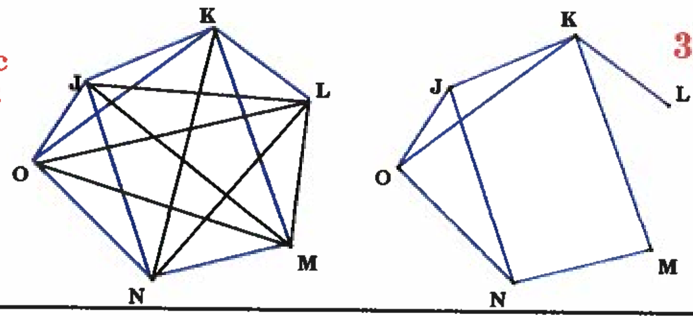
9. Is this graph bipartite? If it is, list the two distinct sets of vertices.



Yes.
{ A, B, C, D, I }
{ E, F, G, H }

10. Graph H is shown below. Find its chromatic number. Draw $\overline{H}$. Find the chromatic number of $\overline{H}$.

Chromatic Number: 3



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Tonight's HW

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