Unit 7 Day 5
Notes: graph coloring, Graph theory review & Quiz

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Warm-Up

- Phones OFF & in Blue Pockets!
- Get out paper for notes!
- Agenda
  - Notes first,
  - Then do practice and HW questions
  - Quiz at the end

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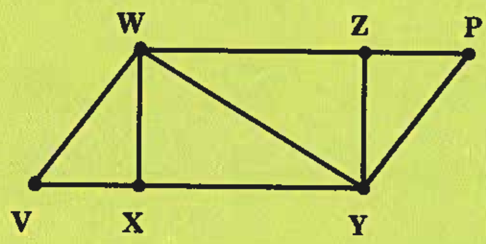
GRAPH THEORY PRACTICE

Determine if each exists.
If it exists, write the path or circuit.
If it doesn't exist, explain why not.

1. Euler Path?



2. Euler Circuit?
3. Hamiltonian Path?
4. Hamiltonian Circuit?

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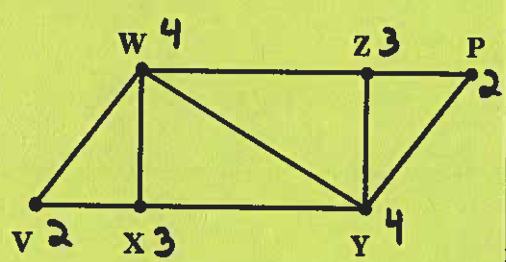
GRAPH THEORY PRACTICE

Determine if each exists.
If it exists, write the path or circuit.
If it doesn't exist, explain why not.

1. Euler Path? Yes
Ex: XVWXYWZYPZ



2. Euler Circuit?
   No because not all even degrees
3. Hamiltonian Path?
   Yes. Ex: WZPYXV
4. Hamiltonian Circuit?
   Yes. Ex: WZPYXVW

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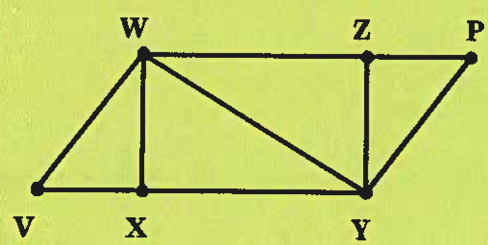
GRAPH THEORY PRACTICE ANSWERS

Determine if each exists.
If it exists, write the path or circuit.
If it doesn't exist, explain why not.

1. Euler Path?
   Yes! ZPYZWYXWVX
2. Euler Circuit?
   No, there is an Euler path and these events are mutually exclusive
3. Hamiltonian Path?
   Yes! WVXYPZ
4. Hamiltonian Circuit?
   Yes! WVXYPZW



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Tonight's Homework Day 5

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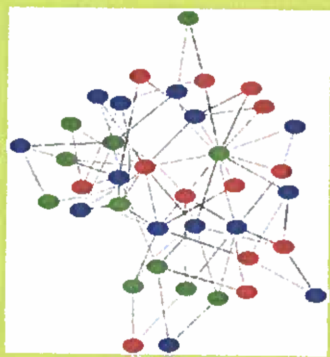
Notes: Graph Coloring
Section 4.6



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Explore This

Here is a table of clubs at Central High School and students who hold offices in these clubs.

	Math Club	Honor Club	Science Club	Art Club	Pep Club	Spanish Club
Matt	X	X	X	—	—	—
Marty	X	—	—	X	X	—
Kim	—	X	—	—	—	X
Lois	X	—	X	—	—	—
Dot	X	—	—	—	X	—

Each club at Central High wants to meet once a week. Since several students hold offices in more than one organization, it is necessary to arrange the meeting days so that no students are scheduled for more than one meeting on the same day. Is it possible to create such a schedule? What is the minimum number of days needed?

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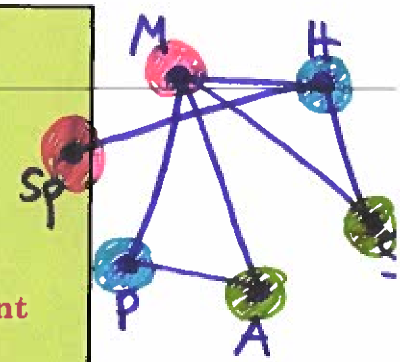
(1st) Draw diagram with vertices and edges for all conflicts



	Math Club	Honor Club	Science Club	Art Club	Pep Club	Spanish Club
Matt	X	X	X	—	—	—
Marty	X	—	—	X	X	—
Kim	—	X	—	—	—	X
Lois	X	—	X	—	—	—
Dot	X	—	—	—	X	—

1. Construct a graph in which the vertices represent the clubs and the edges represent CONFLICTS.

Ex: Since Matt is in 3 clubs, those 3 clubs have conflicts with each other.

2. Then assign the vertices(clubs) days of the week to meet. Use a different color to represent different days.

3. What is the fewest number of days we can use without having a conflict? Remember, adjacent clubs must have different days, because they are in conflict.

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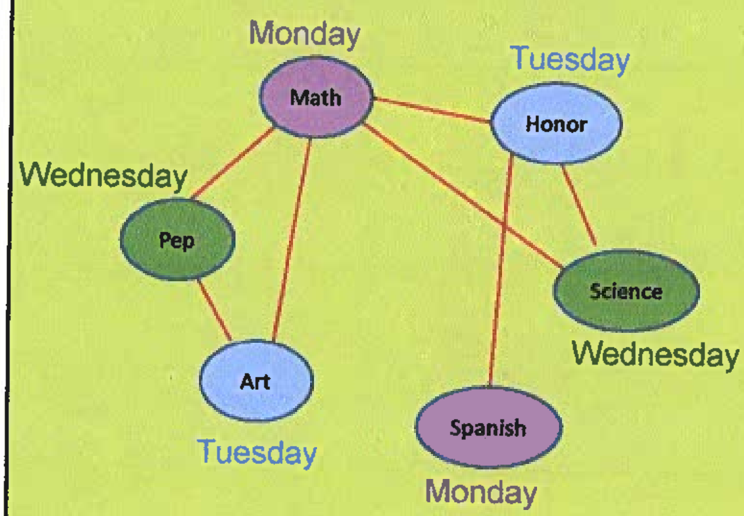
- draw △ between math, honor, and science to show Matt's conflicts;
- draw △ between math, art, pep, to show Marty's conflicts
- etc

Answer



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(2nd) For coloring, start with highest degree vertex. You can use the same color for nonadjacent vertices.

Ex: use pink for math & Spanish, blue for honor + pep, green for art + Sci

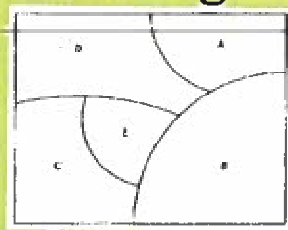
⇓

Mon: math + Spanish clubs

Tues: honor + pep clubs

Wed: art + Science clubs

Graph Coloring Example



Color the map using <u>four or fewer</u> colors. Each region must not touch the same color.

- Represent the map with a graph in which each vertex represents a region of the map.

- Draw edges between vertices if the regions on the map have a common border.

- Label the graph using a minimum number of colors. This number is the graphs <u>"chromatic number."</u>

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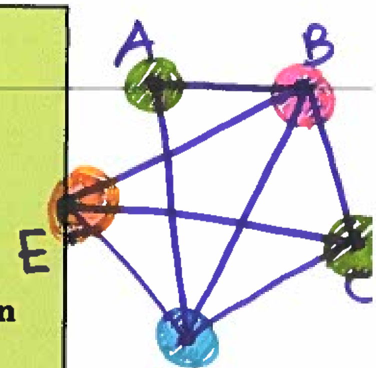

B
D
E
A,C
4 colors needed

Chromatic number is 4 for this graph

Graph Coloring Example ANSWERS

Color the map using <u>four or fewer</u> colors. Each region must not touch the same color.

- Represent the map with a graph in which each vertex represents a region of the map.

- Draw edges between vertices if the regions on the map have a common border.

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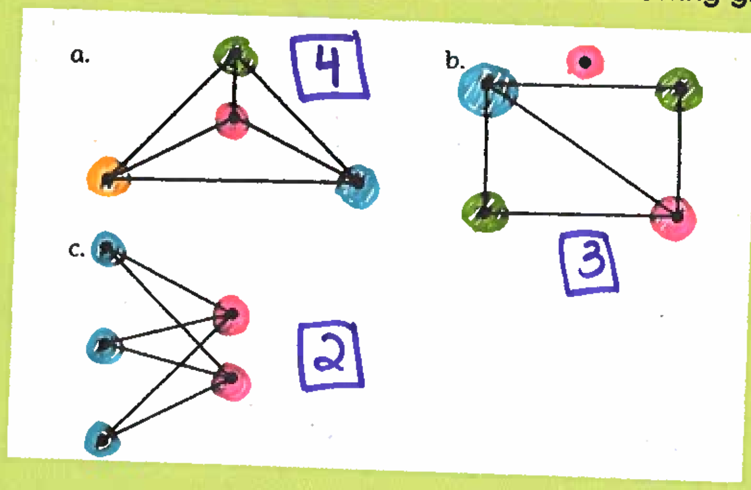
Practice

Find the chromatic number for each of the following graphs.



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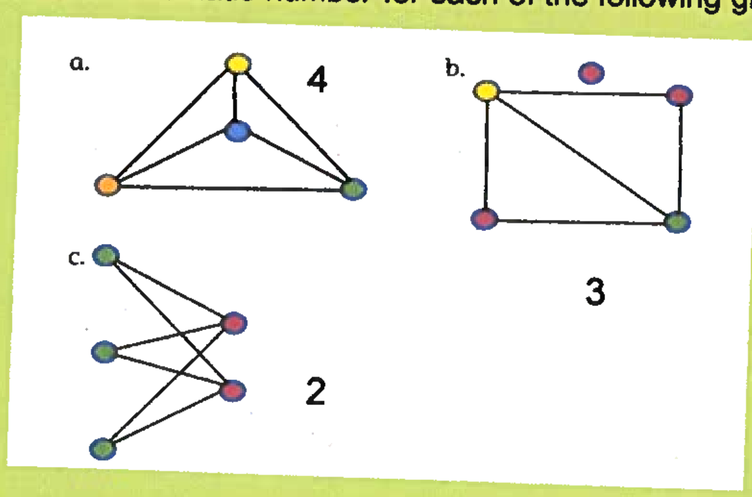
Practice ANSWERS

Find the chromatic number for each of the following graphs.



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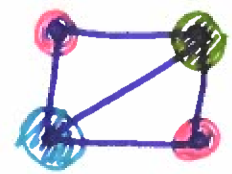
Practice

2. A) Draw a graph that has four vertices and a chromatic number of 3.



B) Draw a graph that has four vertices and a chromatic number of 1.

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Practice ANSWERS

2. A) Draw a graph that has four vertices and a chromatic number of 3.

B) Draw a graph that has four vertices and a chromatic number of 1.

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Practice

3. As the number of vertices in a graph increases, a systematic method of labeling (coloring) the vertices becomes necessary. One way to do this is to create a coloring algorithm.

   A) It is possible to begin the coloring process in several different ways, but one way is to color first the vertices with the conflict. How can the vertices be ranked from those with the most conflict to those with the least?
   B) After having colored the vertex with the most conflict, which other vertices can receive that same color?
   C) Which vertex would then get the second color? Which other vertices could get that same second color?
   D) When would the coloring process be complete?
   E) Refer back to parts A and D of this exercise and create an algorithm that colors a graph.

This is "Exercise 3" that your HW will refer to.

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a) highest Degree to lowest Degree
b) nonadjacent vertices
c) next highest degree vertex & its nonadjacent vertices
d) when all vertices are colored

Up Next…Graph Theory Practice

- Review for Quiz ☺

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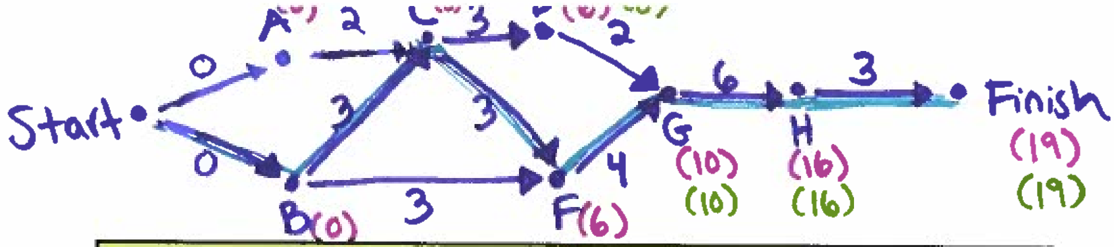

= EST
= LST

Graph Theory Practice: Draw a directed graph to represent the tasks below...

TASK	TIME (in days)	PREREQUISITE
Start	0	----------------
A	2	none
B	3	none
C	3	A,B
D	2	C
F	4	B,C
G	6	D,F
H	3	G

- Determine the Minimum Project Time. 19 days
- State the Critical Path. Start – BCFGH – Finish
- What is the Latest Start Time for D 8 days

(Finish – H – G – D = 8)
19 – 3 – 6 – 2

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Draw a directed graph to represent the tasks below... ANSWERS

TASK	TIME (in days)	PREREQUISITE
Start	0	----------------
A	2	none
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- Determine the Minimum Project Time. 19 days
- State the Critical Path. Start – BCFGH-Finish
- What is the Latest Start Time for D 8 days

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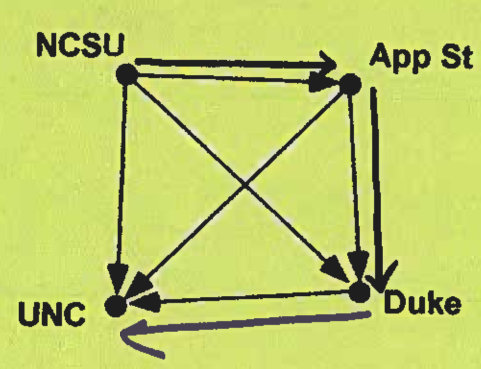
GRAPH THEORY PRACTICE

Does this digraph have exactly one Hamilton Path? Is this complete?

If so, rank the Teams.



Yes

NCSU
App
Duke
UNC

*Tip: Start at ones with highest outdegree & end with lowest outdegree

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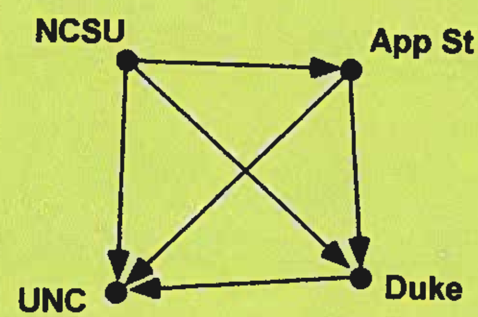
GRAPH THEORY PRACTICE ANSWERS

Does this digraph have exactly one Hamilton Path? Is this complete?

YES

If so, rank the Teams.

1st: NCSU
2nd: App State
3rd: Duke
4th: UNC



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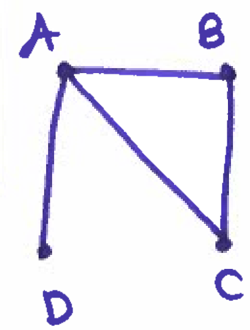
GRAPH THEORY PRACTICE

Construct a graph for the adjacency matrix…

$$\begin{matrix} A \\ B \\ C \\ D \end{matrix}\begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$



- What is the degree of vertex A? 3 vertex C? 2
- Is the graph complete? Explain using def. NO not all vertices are adjacent.
- Is the graph connected? Explain using def.
  Yes. You can eventually get from any vertex to any other vertex.

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GRAPH THEORY PRACTICE ANSWERS

Construct a graph for the adjacency matrix…

$$\begin{matrix} A \\ B \\ C \\ D \end{matrix}\begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

- What is the degree of vertex A? vertex C? degA = 3, degC = 2
- Is the graph complete? Explain using def.
  This graph is NOT complete since every vertex does not connect to each other vertex.
- Is the graph connected? Explain using def.
  This graph is connected since there is a path to all vertices.

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Homework Day 4

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Quiz Time

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