Key

Unit 7 Day 3
Section 4.4

- Euler Circuits and Paths

Warm Up ~ Day 3

1. Create a Task Graph based on the Task Table shown.
2. List earliest start times in a table AND mark them on your graph.

Activity	Time	Depends on
A	3	----
B	5	----
C	2	A
D	3	A
E	3	B, D
F	5	C, E
G	1	C
H	2	F, G

Task	EST
A	
B	
C	
D	
E	
F	
G	
H	

3. Minimum Project Time =

4. Critical Path(s) =

5. What is the Latest Start time for D?

6. What is the Latest Start time for C?

Warm Up ~ Day 3

1. Create a Task Graph based on the Task Table shown.
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Activity	Time	Depends on
A	3	----
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C	2	A
D	3	A
E	3	B, D
F	5	C, E
G	1	C
H	2	F, G

Task	EST
A	0
B	0
C	3
D	3
E	6
F	9
G	5
H	14

3. Minimum Project Time =

16 days

4. Critical Path(s) =

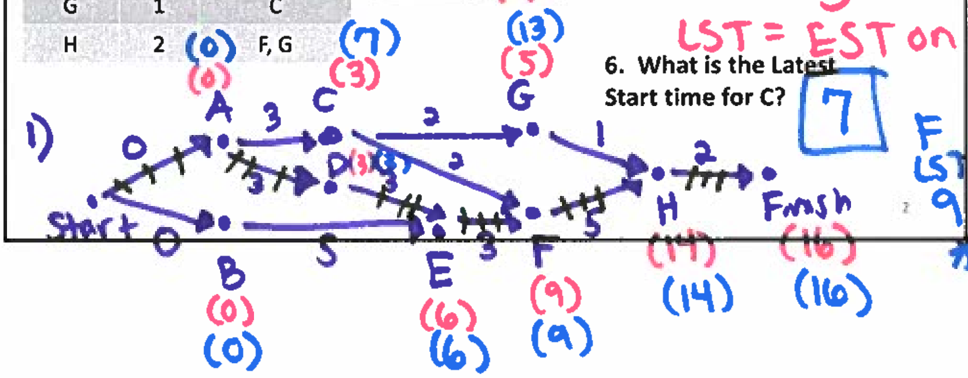
Start - ADEFH - Finish

5. What is the Latest Start time for D? 3

LST = EST on critical path

6. What is the Latest Start time for C? $\boxed{7}$

1)



F LST 9 - 2

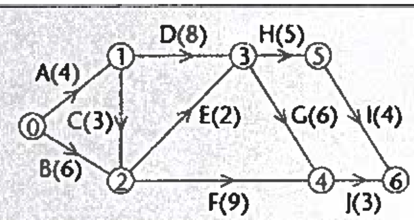
Time for C = 7

pick smaller for LST if multiple options

Warm Up ~ Day 3 ANSWERS

List the tasks and earliest start times in a table, as in exercise #1.
Determine the minimum project time and all the critical paths.

Determine the critical activities and the length of the critical path for this network.



A, D, C and I, 21

Finish H G C
16 - 2 - 1 - 2
= 11

bigger one does NOT allow time to get other projects on time.

Activity	Time	Depends on
A	3	----
B	5	----
C	2	A
D	3	A
E	3	B, D
F	5	C, E
G	1	C
H	2	F, G

Task	EST
A	
B	
C	
D	
E	
F	
G	
H	

Minimum Project Time =

Critical Path(s) =

What is the Latest Start time for D?

What is the Latest Start time for C?

Unit 7 Day 3
Section 4.4-4.5

- Euler Circuits and Paths

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Euler Circuits and Paths

Section 4.4

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Tonight's Homework

In your HW packet...

Packet p. 4 #10

and Packet p. 5

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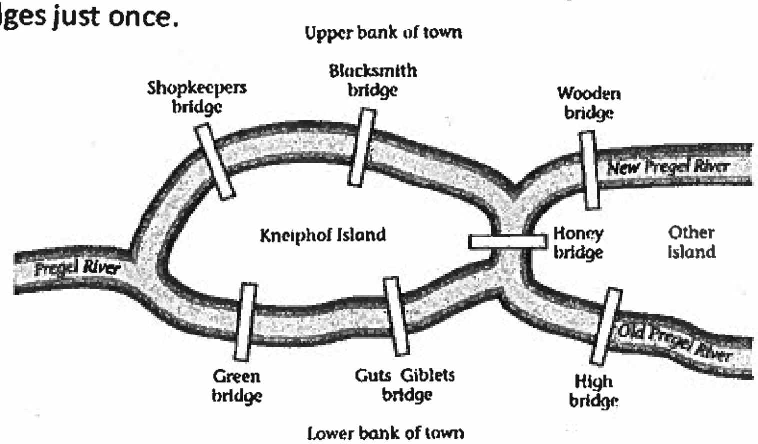
The Seven Bridges of Königsberg

The medieval town of Königsberg has a river running through it. There is an island and a fork in the river that together divide the city into four separate land areas. At the time, seven bridges connected the four land areas. The puzzle asked whether it was possible for a stroller to take a walk around the town, crossing each of the seven bridges just once.



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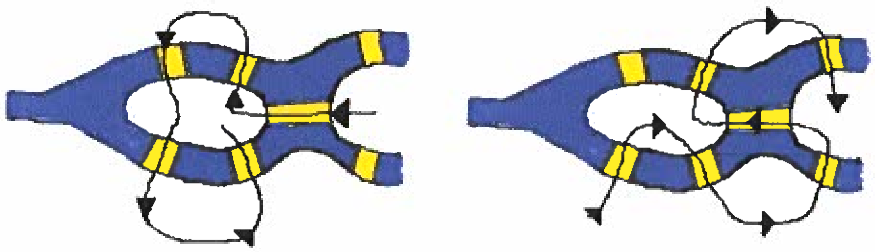
The Seven Bridges of Königsberg
Solution

- Having trouble? That's okay, so did Euler. It doesn't seem possible to cross every bridge exactly once. In fact it isn't.

Failed Attempts ☺



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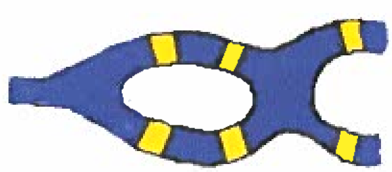
Königsberg Problem #2

- Suppose they had decided to build one fewer bridge in Konigsberg, so that the map looked like this:



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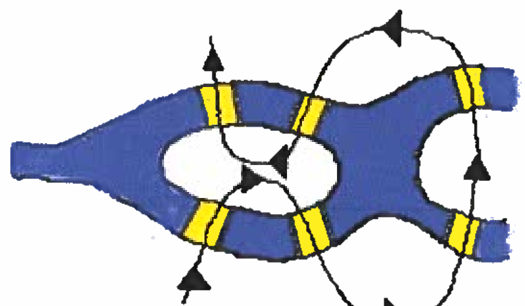
Problem #2 Solution

- What makes this one different from the 'real' Konigsberg problem? (Hint: How many bridges lead to each piece of land? Why is having an odd number of bridges leading to a single piece of land problematic?)



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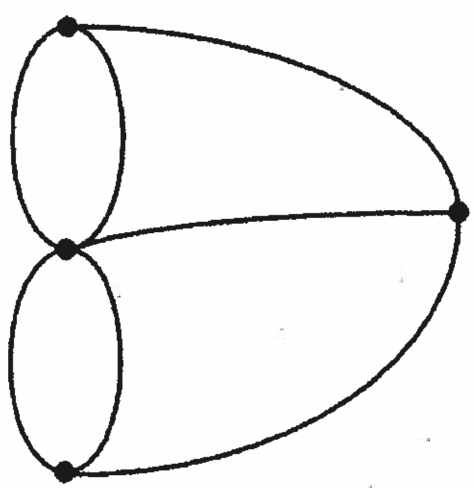
- Need even number of bridges to each piece of land.
- If odd number of bridges to one land area, then example...
  1st) Go to land
  2nd) Go off land
  3rd) Go back on to land AND stuck !!

Attempt to draw this figure without lifting your pencil from the page and without tracing any of the lines more than once.



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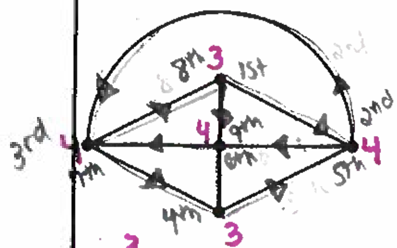
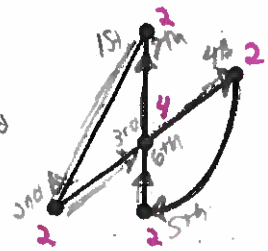
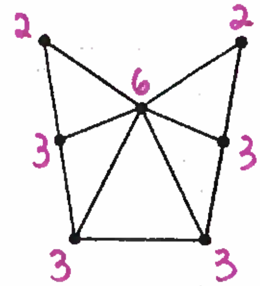
Try to reproduce the following figures without lifting your pencil or tracing the lines more than once. Also, write the degree of each vertex (number of vertices adjacent to it).



a.



b.



c.

1. When can you draw the figures without retracing any edges and still end up at your starting point? b's figure
2. When can you draw the figure without retracing and end up at a point other than the one from which you began? figure a
3. When can you not draw the figure without retracing? figure c

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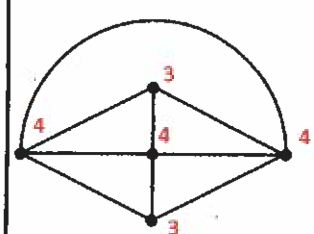
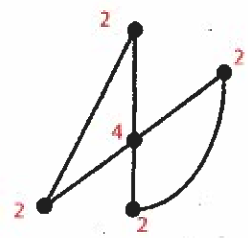
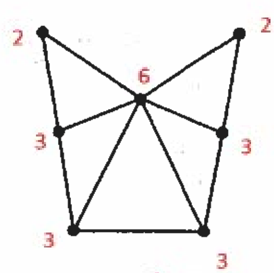
ANSWERS Try to reproduce the following figures without lifting your pencil or tracing the lines more than once. Also, write the degree of each vertex (number of vertices adjacent to it).



a.



b.



c.

1. When can you draw the figures without retracing any edges and still end up at your starting point?
2. When can you draw the figure without retracing and end up at a point other than the one from which you began?
3. When can you not draw the figure without retracing?

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Euler: Trace each edge once
+ don't pick up pencil

Our Findings (Hopefully)...

*Write this down!!

You can draw it and you end up where you started:	You can draw it, but you don't end where you started:
ALL DEGREES ARE EVEN	EXACTLY TWO OF THE VERTICES HAVE AN ODD DEGREE
This is an **EULER CIRCUIT**	This is an **EULER PATH**
Ex: Figure b from last example	Ex: Figure a from last example

*Tip: start at odd degree vertex + end at other odd degree vertex

Can't draw it:

NEITHER OF THE PREVIOUS CASES

Not Possible

Ex: Figure C from last example

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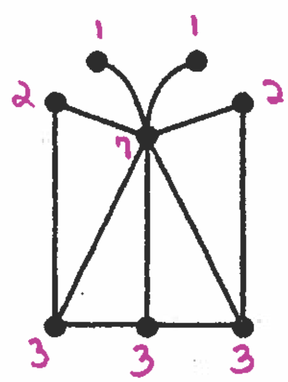
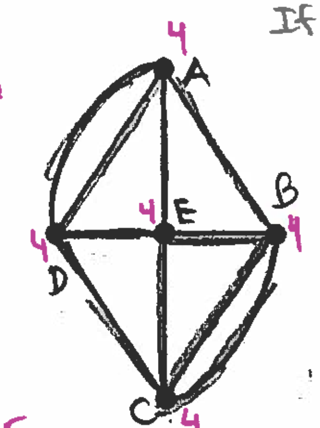
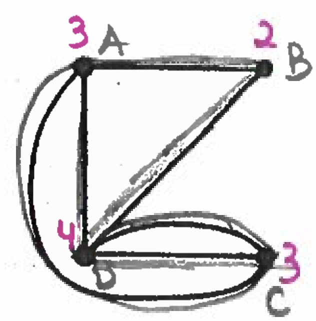
Determine if there is an Euler Circuit, Euler Path, or neither. If one exists, find a circuit or path.







A Neither (not all even, not 2 odd)

B Euler Circuit (all even degree)

C Euler Path (exactly 2 odd degree)

Euler Circuit: all degrees are even
Euler Path: exactly 2 vertices have an odd degree

A B D C A D C

start and end at different ones... at the odd degree vertices

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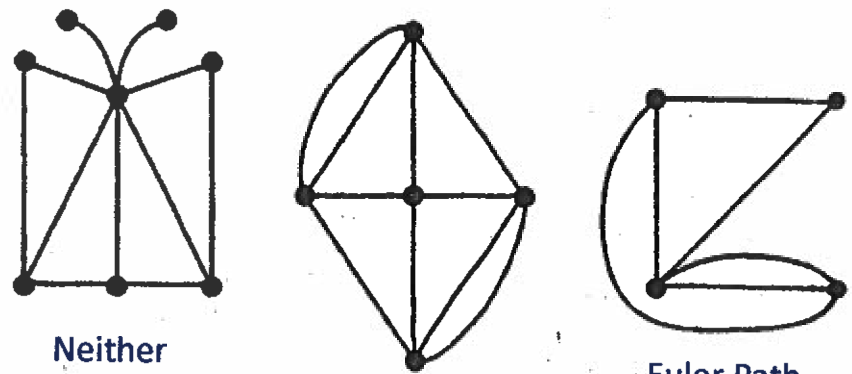
Example: A B C B E D C E A D A

start and end are same

* there are many possible Euler circuits *

Determine if there is an Euler Circuit, Euler Path, or neither.



Neither — A

Euler Circuit — B

Euler Path — C

Euler Circuit: all degrees are even
Euler Path: exactly 2 vertices have an odd degree

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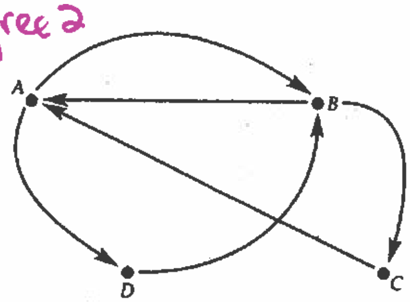
A "tidbit" on Digraphs

Directional Graphs **or** **DIGRAPHS** **are graphs with edges that have direction.**

Many applications of graphs require that the edges have direction.

- **A city with one-way streets**
- **A business model with buyers and sellers**
- **Water flow through a filtration system**

A: Indegree 2 + outdegree 2



D: Indegree 1 + outdegree 1

Indegree – **# of edges coming into a vertex.**

Outdegree – **# of edges going out of a vertex.**

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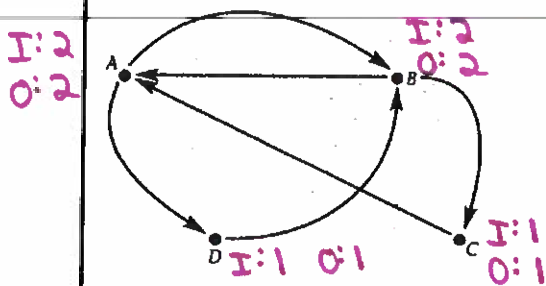
A few more "tidbits" on Digraphs



Indegree – # of edges coming into a vertex.

Outdegree – # of edges going out of a vertex.

Vertices – { A, B, C, D }

Ordered Edges – { AB, BA, BC, CA, DB, AD }

Notice how the edges are ordered. ... Put starting vertex 1st, then ending one and (one with arrow 2nd)

* **If the Indegree and Outdegree are equal at every vertex then the Digraph has an Euler Circuit ("Directed" Euler Circuit)** like the diagram above Ex: ABCADBA

If this represented a competition, who would win, A vs C? C (arrow starts at C) or C throws punch at A with the arrow pointing at A

A vs D? A (arrow starts at A) you can think of the arrow like a "punch" someone is throwing

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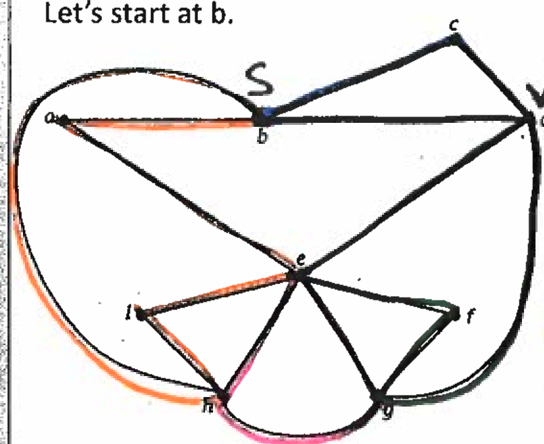
When a graph is small, it's easy to find an Euler Circuit by trial & error, but when the graph is bigger you need an algorithm.

Euler Circuit Algorithm

1. Pick any vertex, and label it S.
2. Construct a circuit, C, that begins and ends at S.
3. If C is a circuit that includes all edges of the graph, go to step 8.
4. Choose a vertex, V, that is in C and has an edge that is not in C.
5. Construct a circuit C' that starts and ends at V using edges not in C.
6. Combine C and C' to form a new circuit. Call this new circuit C.
7. Go to step 3.
8. Stop. C is an Euler circuit for the graph.

Let's start at b.



bcdb

dgfed

bcdgfedb

gheg

bcdgheg fedb

hieabh

Euler **Circuit** - cycle with no repetitions of ~~vertices or~~ edges, ~~other than~~ and you must have ~~the~~ repetition of the starting and ending vertex

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*TIP: Start with highest degree vertex and/or start with small circuit

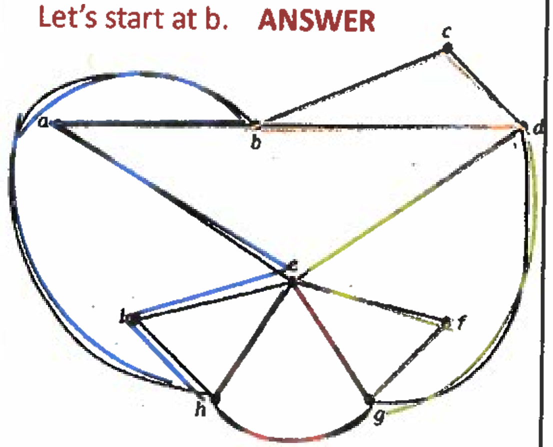
$\boxed{\text{bcdghieabhegfedb}}$

When a graph is small, it's easy to find an Euler Circuit by trial & error, but when the graph is bigger you need an algorithm.

Euler Circuit Algorithm

1. Pick any vertex, and label it *S*.
2. Construct a circuit, *C*, that begins and ends at *S*.
3. If *C* is a circuit that includes all edges of the graph, go to step 8.
4. Choose a vertex, *V*, that is in *C* and has an edge that is not in *C*.
5. Construct a circuit C_1 that starts and ends at *V* using edges not in *C*.
6. Combine *C* and C_1 to form a new circuit. Call this new circuit *C*.
7. Go to step 3.
8. Stop. *C* is an Euler circuit for the graph.

Let's start at b. **ANSWER**



Circuit - cycle with no repetitions of vertices or edges, other than the repetition of the starting and ending vertex

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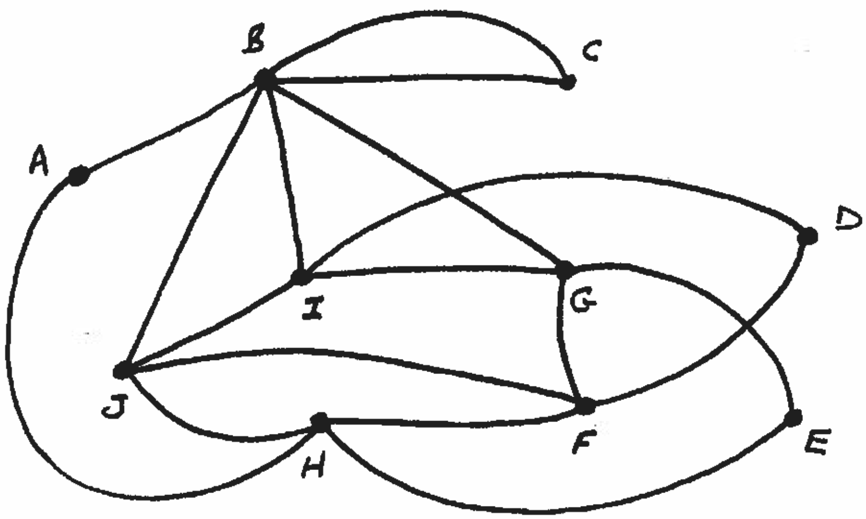
Euler Circuits Example

Identify an Euler Circuit by the algorithm.



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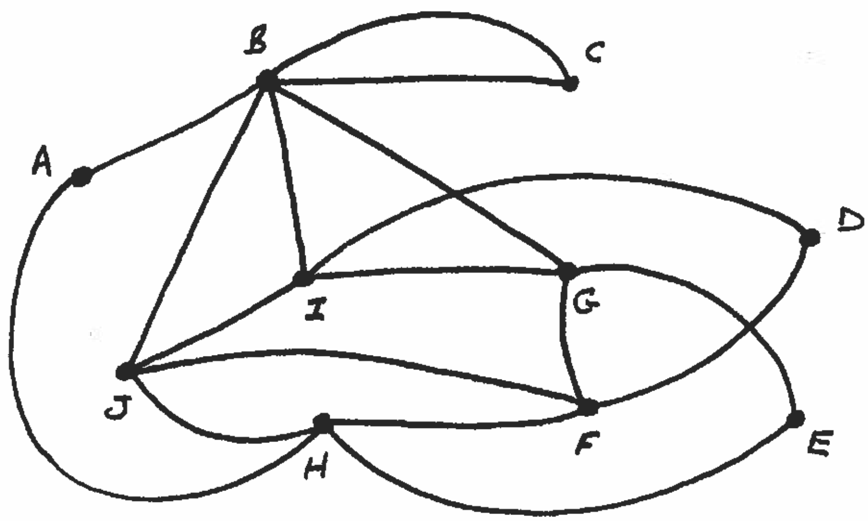
Euler Circuits Example ANSWER

Identify an Euler Circuit by the algorithm.



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Homework

In your HW packet...

Packet p. 4 #10

and Packet p. 5

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