

Key
with some
work

Unit 7 Day 2 Section 4.3-4.5

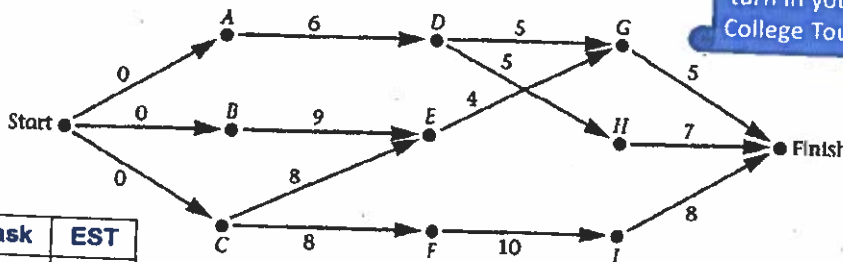
- Vocabulary & Graphical Representations
- Euler Circuits and Paths

1

Warm Up ~ Day 2

List the tasks and earliest start times in a table, as in exercise #1.
Determine the minimum project time and all the critical paths.

See board
for
directions to
turn in your
College Tour.



Task	EST
A	
B	
C	
D	
E	
F	
G	
H	
I	

Minimum Project Time =

Critical Path(s) =

What is the Latest Start time for D?

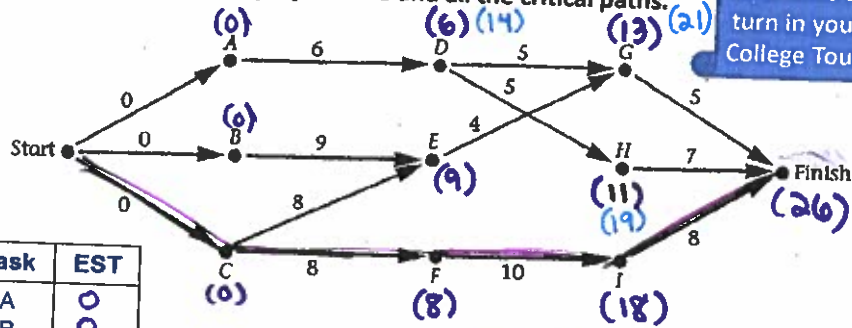
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2

Warm Up ~ Day 2

List the tasks and earliest start times in a table, as in exercise #1.
Determine the minimum project time and all the critical paths.

See board for directions to turn in your College Tour.



Task	EST
A	0
B	0
C	0
D	6
E	9
F	8
G	13
H	11
I	18

Minimum Project Time = 26

Critical Path(s) = Start - C - F - I - Finish

What is the Latest Start time for D? 14*

*If more than 1 EST, pick larger one so prerequisite are totally done before next task

= EST

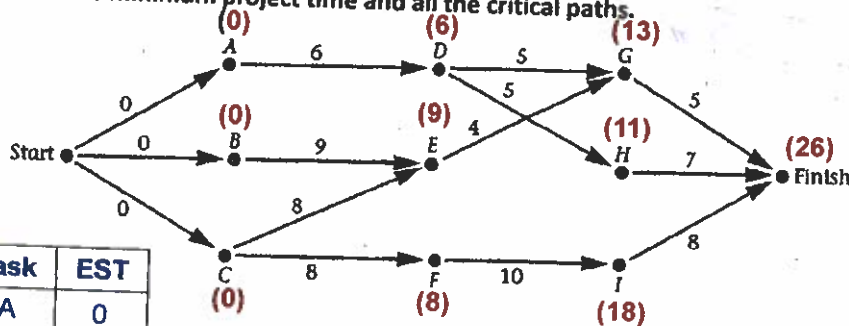
= LST

2 What is LST for F? 8

For LST, Finish time for H - 5 = 11 - 5 = 6
*If two paths, pick smaller
26 - 5 - 5 = 16
Finish time for G - 5 = 13 - 5 = 8

Warm Up ~ Day 2 ANSWERS

List the tasks and earliest start times in a table, as in exercise #1.
Determine the minimum project time and all the critical paths.



Task	EST
A	0
B	0
C	0
D	6
E	9
F	8
G	13
H	11
I	18

Minimum Project Time = 26

Critical Path(s) = Start - C - F - I - Finish

What is the Latest Start time for D? 14

What is LST for F? 8 same as EST b/c F is on critical path

4.1 HW ANSWERS

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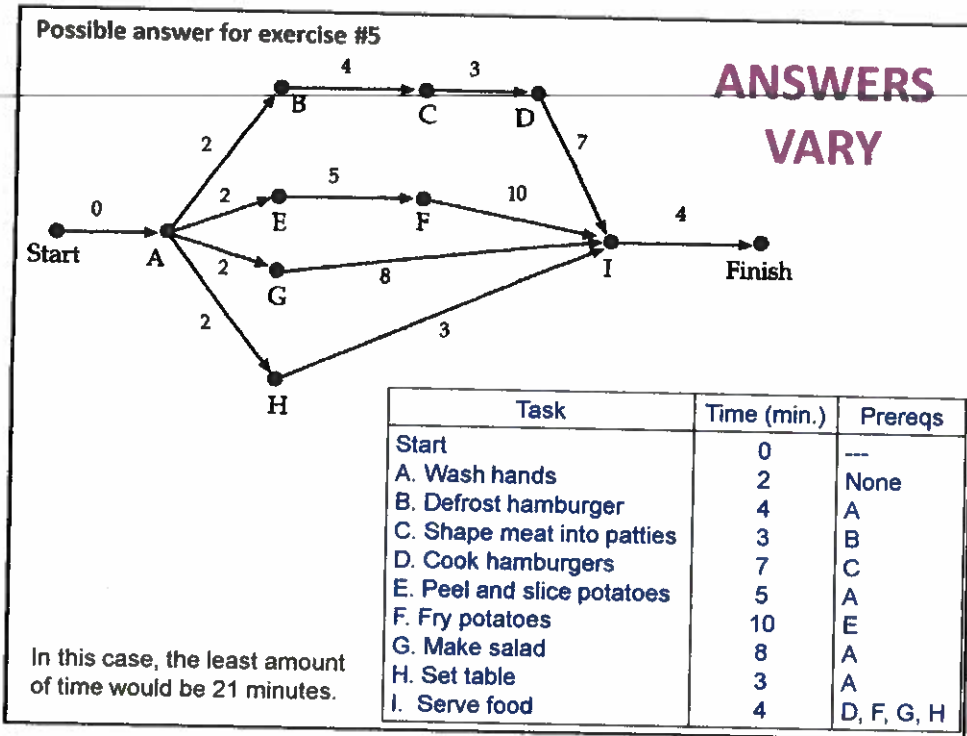
Section 4.1 – Exercise #5

To help organize the task of completing the family dinner, Mrs. Shu listed the following tasks.

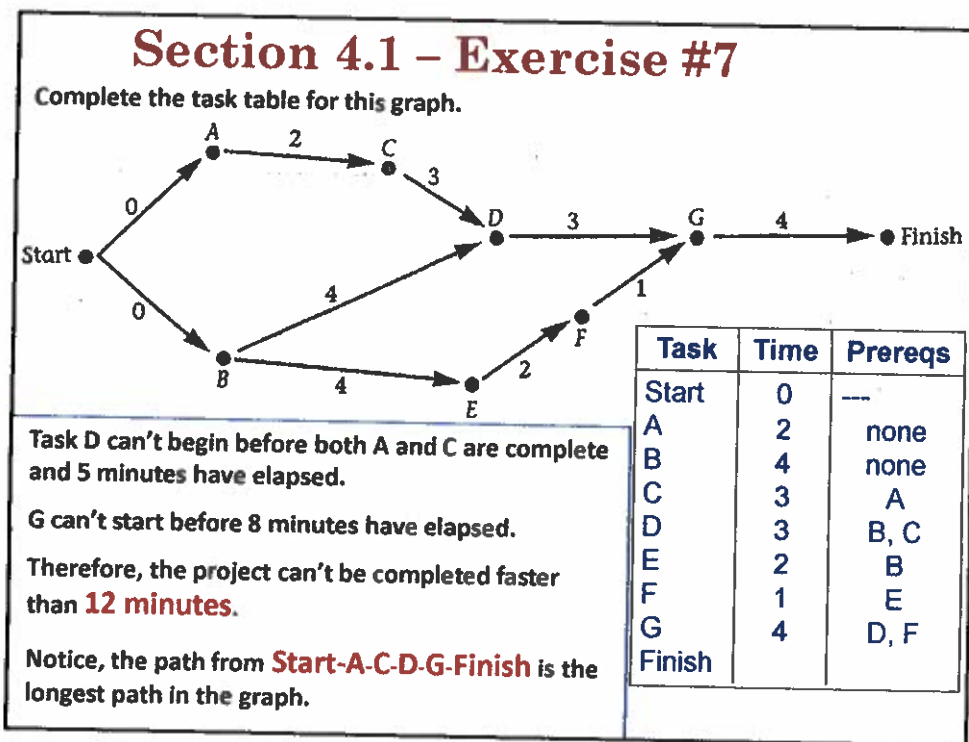
Task	Time (min.)	Prerequisite Task
Start	0	---
A. Wash hands B. Defrost hamburger C. Shape meat into patties D. Cook hamburgers E. Peel and slice potatoes F. Fry potatoes G. Make salad H. Set table I. Serve food	ANSWERS VARY	ANSWERS VARY

- Complete the table by making reasonable time estimates in minutes for each of these tasks and indicating the prerequisites.
- Construct a graph using the information from your table.
- Using this information what do you think is the least amount of time needed to prepare dinner?

7



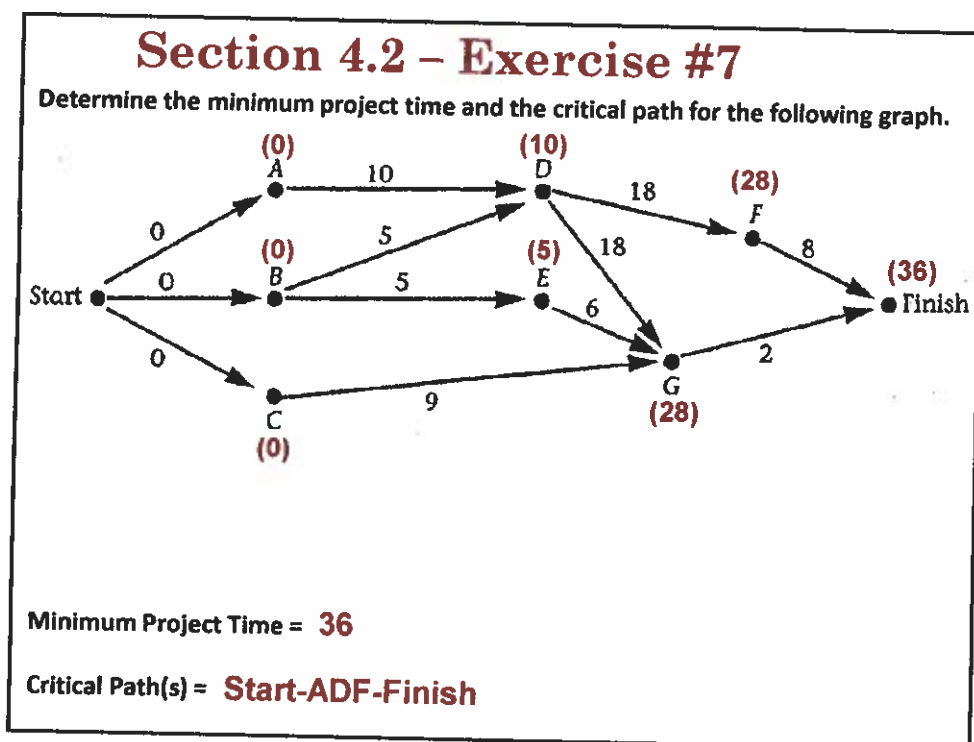
8



9

4.2 HW Answers

10



11

Section 4.2 – Exercise #8 LATEST START TIME

Task E can begin as early as day 9. If E begins on day 9, when will it be completed? **Day 16**

The entire project will still be completed on **DAY 20** → If E begins on day 10? **Day 17**

If E begins on day 11? **Day 18**

If E begins on day 12? **Task G will be delayed and the whole project will be delayed.**

What is the latest day on which task E can begin if task G is to begin on day 18? **Day 11**

If an activity is not on the critical path, it is possible for it to start later than its earliest start time and not delay the project. The latest a task can begin with out delaying the project's minimum completion time is known as the **LATEST START TIME (LST)** for the task. In this example, the *latest start time for task E is day 11.*

12

Section 4.2 – Exercise #8 LATEST START TIME

What about the LATEST START TIME for task C?

Task C effects the start times of both task D and E.

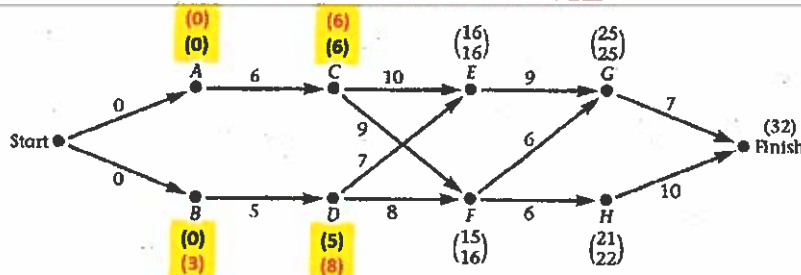
Task D is on the critical path, so the latest it can start is Day 10.

We determined already that the latest start time for task E is Day 11

So, what is the latest task C can begin without delaying the project? **Day 5**

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Section 4.2 - Exercise #9
LATEST START TIME



To find the LST for each task, begin at the Finish and use the Minimum Project Time. Work through the graph in reverse order.

Subtract from the Minimum Project Time the time it takes to complete the task preceding it.

For example: $32 - 10$ is 22, the LST for task H.
 $32 - 7$ is 25, the LST for task G. That's the same as its EST since it is on the critical path.

For task F use $22 - 6 = 16$, the LST for task F.

What about task D? It's a prerequisite for both task E and F.
 $E (16 - 7 = 9), F (16 - 8 = 8) \rightarrow$ Choose the earliest time.

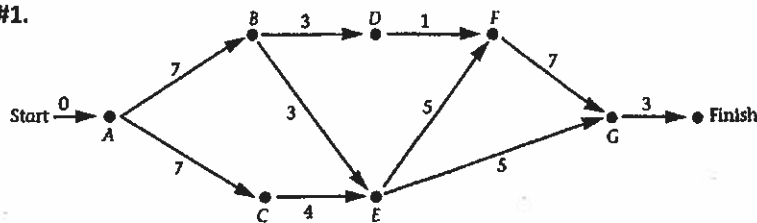
14

Section 4.2 - Exercise #9,10
LATEST START TIME Algorithm ANSWERS ☺

The general algorithm for finding the LST for each task in a graph is :

1. Begin with Finish and use the minimum project time.
2. Subtract from the minimum project time the time that it takes to complete the task(s) preceding it. This is the LST for the preceding tasks. Label the vertex with its LST.
3. If the time of a preceding task is dependent on more than one edge, choose the earliest time.
4. Continue until all vertices are labeled.

Try the algorithm on the graph from exercise #1.



Key
 — = EST
 — = LST

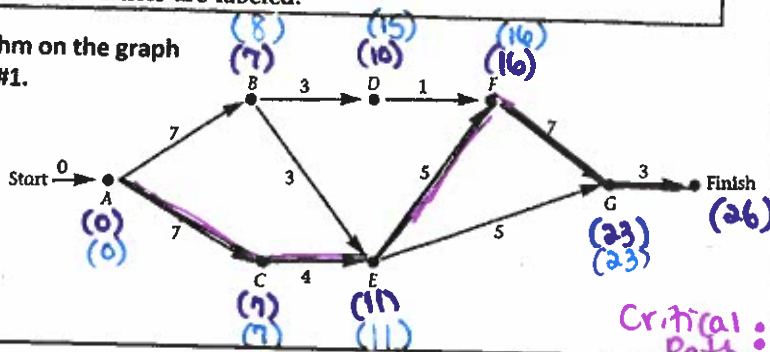
15

Section 4.2 - Exercise #9,10 LATEST START TIME Algorithm ANSWERS ☺

The general algorithm for finding the LST for each task in a graph is :

1. Begin with Finish and use the minimum project time.
2. Subtract from the minimum project time the time that it takes to complete the task(s) preceding it. This is the LST for the preceding tasks. Label the vertex with its LST.
3. If the time of a preceding task is dependent on more than one edge, choose the earliest time.
4. Continue until all vertices are labeled.

Try the algorithm on the graph from exercise #1.



4

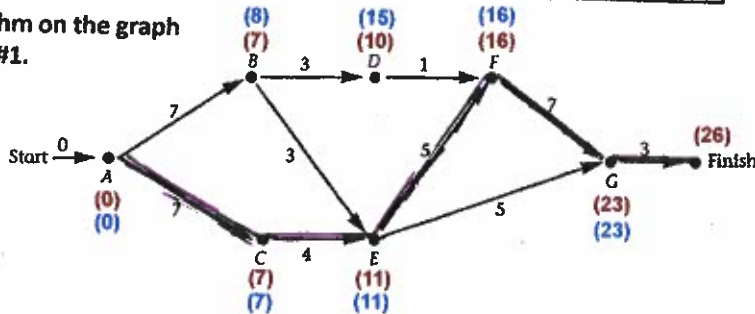
* Choose smaller of $23-5=18$ or $16-5=11$ *

Section 4.2 - Exercise #9,10 LATEST START TIME Algorithm ANSWERS ☺

The general algorithm for finding the LST for each task in a graph is :

1. Begin with Finish and use the minimum project time.
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Try the algorithm on the graph from exercise #1.



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Section 4.2 - Exercise #9,10

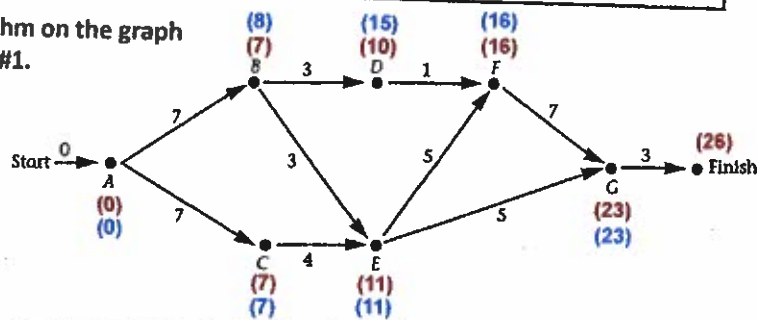
LATEST START TIME Algorithm ANSWERS ☺

The general algorithm for finding the LST for each task in a graph is :

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3. If the time of a preceding task is dependent on more than one edge, choose the earliest time.
4. Continue until all vertices are labeled.

Try the algorithm on the graph from exercise #1.

Key
 * = EST
 * = LST



The Vocabulary and Representations of Graphs

Section 4.3

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Vocabulary for Graphs

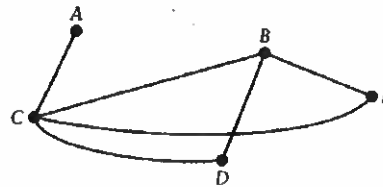
➤ Graphs show relationships between objects.

- Vertices are the objects

A, B, C, D, E

- Edges are the relationships

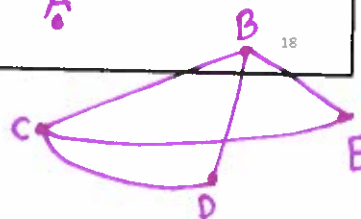
AC, BC, BD, BE,
CD, CE



The vertices in this graph represent the starting five players on high school basketball team.

1. Which player has only one friend?
2. How many friends does E have?
Who are they?
3. Redraw the graph so that A has no friends.

A
2
B, C
A

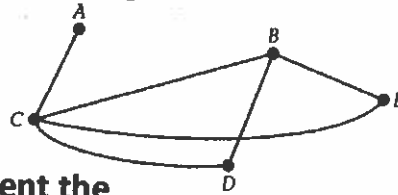


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Vocabulary for Graphs ANSWERS

➤ Graphs show relationships between objects.

- Vertices are the objects
- Edges are the relationships

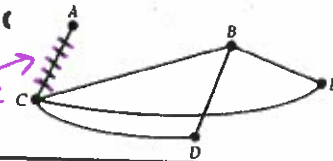


The vertices in this graph represent the starting five players on high school basketball team.

1. Which player has only one friend?
2. How many friends does E have? Who are they?
3. Redraw the graph so that A has no friends.

A
2
B, C

eliminate →

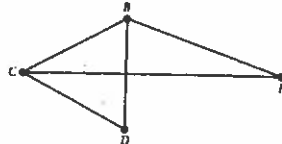
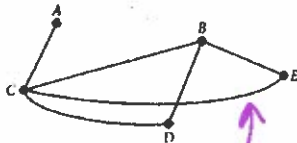


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The Vocabulary & Representations of Graphs

When two vertices are connected with an edge, they are said to be Adjacent.

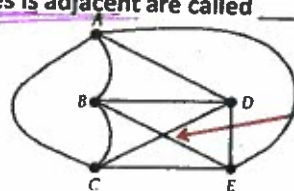
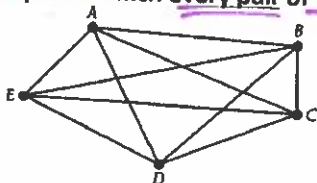
A + C are adjacent,
B + C are adjacent



A + B are not adjacent,
B + C are adjacent

A graph is Connected if there is a *path* between each pair of vertices. Every vertex is *reachable* from any other vertex.

Graphs in which every pair of vertices is adjacent are called Complete.



Notice, this is not a vertex even though the edges cross.

These two complete graphs have the same representation. Each vertex is adjacent to every other.

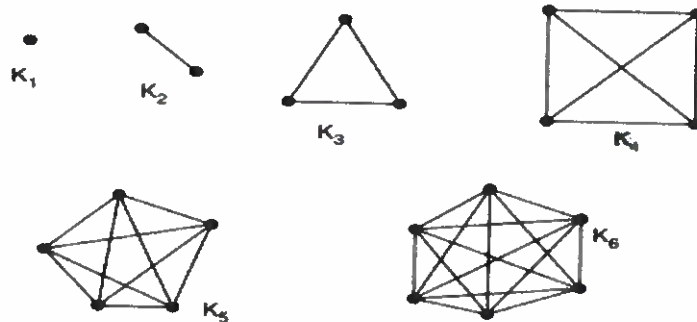
Complete graphs are denoted by K_n , where n is the number of vertices in the graph.

The two complete graphs above can be denoted as K_5 .

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Complete Graphs - Every vertex is connected to all other vertices.

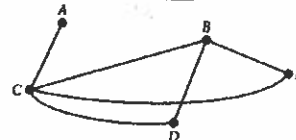
Complete Graphs



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Alternate Representations of Graphs

Graphs can be represented in several different ways. The diagram we have used so far is just one of those ways.



We can also represent a graph by listing the Set of Vertices and the Set of Edges.

The graph above can be represented like this:

Vertices = { A, B, C, D, E } **Edges = { AC, CB, CE, CD, BD, BE }**

A third way to represent this information is with an Adjacency Matrix.

	A	B	C	D	E
A	0	0	1	0	0
B	0	0	1	1	1
C	1	1	0	1	1
D	0	1	1	0	0
E	0	1	1	0	0

The entry in row 2, column 4 is a 1. That indicates that vertices B and D are adjacent. An edge exists between them.

You will be expected to represent graphs as:

- a Diagram
- Sets of Vertices and Edges
- an Adjacency Matrix

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Representations of Graphs

- Is this graph connected?
No. Every vertex isn't reachable from every other.
- Is this graph complete?
No. Every vertex isn't adjacent to every other.
- Name two vertices that are adjacent to C.
B, D
- Name a path from A to C of length 3.
A, B, D, C (takes 3 edges)
- How many vertices are adjacent to D?
(also known as the **DEGREE** of the vertex)
3 *E, B, C are adjacent to D*
- Represent this graph as sets of vertices and edges.
*Vertices: {A, B, C, D, E, F, G, H}
Edges: {AB, BC, BD, CD, DE, FG, GH, FH}*
- Represent this graph as an adjacency matrix.

	A	B	C	D	E	F	G	H
A	0	1	0	0	0	0	0	0
B	1	0	1	1	0	0	0	0
C	0	1	0	1	0	0	0	0
D	0	1	1	0	1	0	0	0
E	0	0	0	1	0	0	0	0
F	0	0	0	0	0	0	1	1
G	0	0	0	0	0	1	0	1
H	0	0	0	0	0	1	1	0

A is adjacent to B only

H is adjacent to F, G only (0 for all other entries to show they're non adjacent)

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Representations of Graphs ANSWERS

- Is this graph connected?
No
- Is this graph complete?
No
- Name two vertices that are adjacent to C.
D & B
- Name a path from A to C of length 3.
A, B, D, C
- How many vertices are adjacent to D?
(also known as the **DEGREE** of the vertex)
3 - E, B, C
- Represent this graph as sets of vertices and edges.
Vertices = { A, B, C, D, E, F, G, H }
Edges = { AB, ED, DC, DB, CB, FG, FH, GH }
- Represent this graph as an adjacency matrix.

	A	B	C	D	E	F	G	H
A	0	1	0	0	0	0	0	0
B	1	0	1	1	0	0	0	0
C	0	1	0	1	0	0	0	0
D	0	1	1	0	1	0	0	0
E	0	0	0	1	0	0	0	0
F	0	0	0	0	0	0	1	1
G	0	0	0	0	0	1	0	1
H	0	0	0	0	0	1	1	0

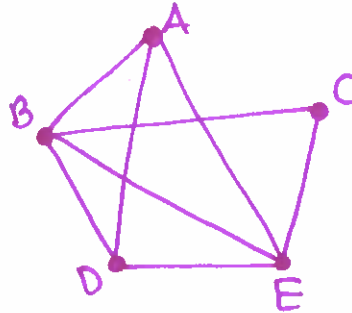
24

You Try

Representations of Graphs

Construct a diagram from this adjacency matrix.

	A	B	C	D	E
A	0	1	0	1	1
B	1	0	1	1	1
C	0	1	0	0	1
D	1	1	0	0	1
E	1	1	1	1	0



1. Is this graph connected?

Yes. Every vertex is reachable from every other vertex.

2. Is this graph complete?

No. Every vertex is not adjacent to every other vertex.

3. How many vertices are adjacent to E? (What is the degree of E?)

4: A, B, C, D

↓
4

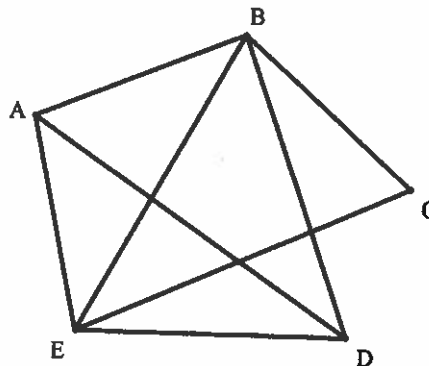
25

25

Representations of Graphs ANSWERS

Construct a diagram from this adjacency matrix.

	A	B	C	D	E
A	0	1	0	1	1
B	1	0	1	1	1
C	0	1	0	0	1
D	1	1	0	0	1
E	1	1	1	1	0



1. Is this graph connected?

Yes

2. Is this graph complete?

No

3. How many vertices are adjacent to E? (What is the degree of E?)

4 - A, B, C, D

26

26

Representations of Graphs

Consider these countries in South America and their borders.

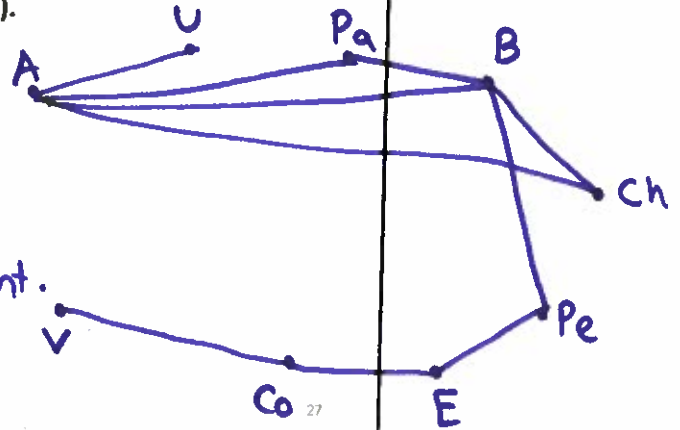
Argentina (A) borders Uruguay (U), Paraguay (Pa), Bolivia (B), and Chile (Ch).

Paraguay (Pa) borders Bolivia (B).

Bolivia (B) borders Chile (Ch) and Peru (Pe).

Ecuador (E) borders Peru (Pe) and Columbia (Co).

Venezuela (V) borders Columbia (Co).



Construct a graph representing these border relationships.

a) Is it a complete graph?

No. All vertices aren't adjacent.

b) Is it a connected graph?

Yes. All vertices are reachable from all the others.

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Representations of Graphs ANSWERS

Consider these countries in South America and their borders.

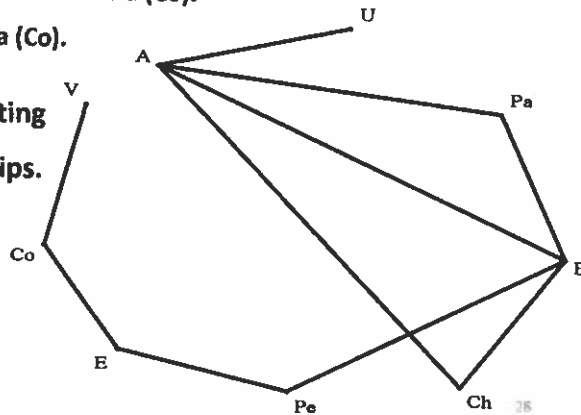
Argentina (A) borders Uruguay (U), Paraguay (Pa), Bolivia (B), and Chile (Ch).

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Venezuela (V) borders Columbia (Co).



Construct a graph representing these border relationships.

a) Is it a complete graph?

No

b) Is it a connected graph?

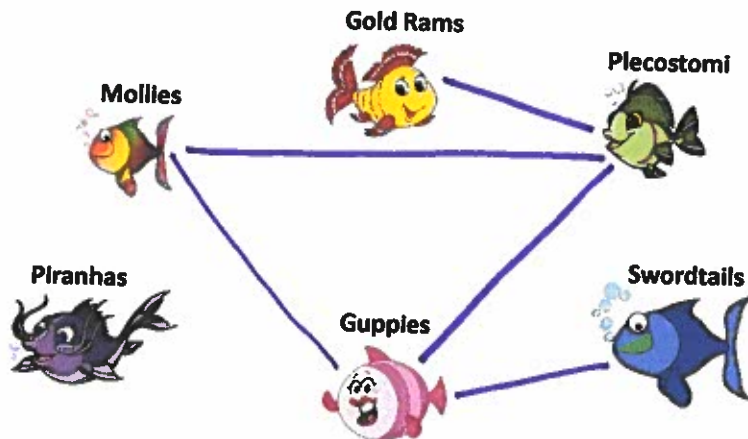
Yes

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Do this:

1. Mr. Butler bought six different types of fish. Some of the fish can live in the same aquarium, but others cannot. Guppies can live with Mollies, Swordtails can live with Guppies, Plecostomi can live with both Mollies and Guppies, Gold Rams can live only with Plecostomi, and Piranhas cannot live with any of the other fish. Draw a graph to illustrate this.

You Try

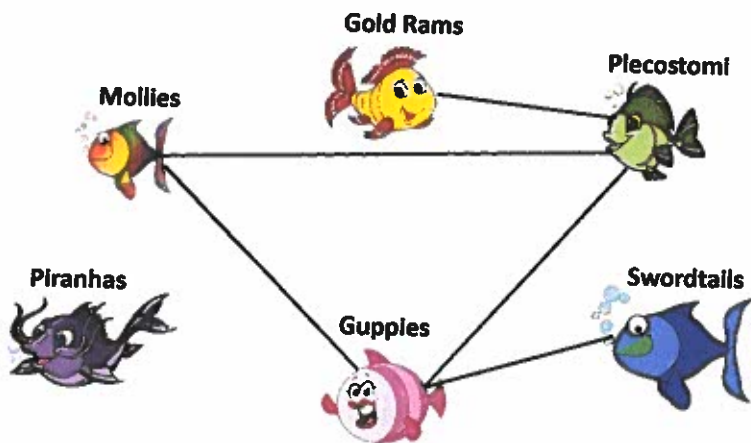


lines show compatible fish pairings

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Do this ANSWERS:

1. Mr. Butler bought six different types of fish. Some of the fish can live in the same aquarium, but others cannot. Guppies can live with Mollies, Swordtails can live with Guppies, Plecostomi can live with both Mollies and Guppies, Gold Rams can live only with Plecostomi, and Piranhas cannot live with any of the other fish. Draw a graph to illustrate this.



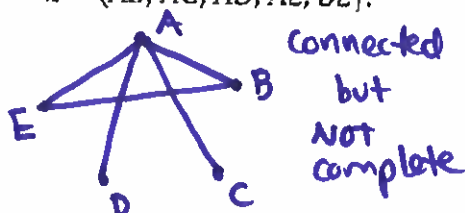
30

You Try

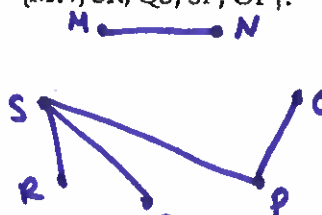
Another...

2. Construct a graph for each of the following sets of vertices and edges. Which of the graphs are connected? Which are complete?

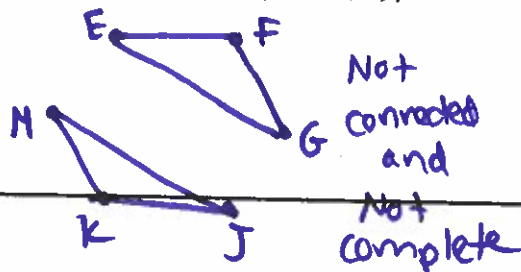
a. $V = \{A, B, C, D, E\}$
 $E = \{AB, AC, AD, AE, BE\}$



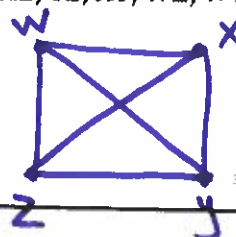
b. $V = \{M, N, O, P, Q, R, S\}$
 $E = \{MN, SR, QS, SP, OP\}$



c. $V = \{E, F, G, J, K, M\}$
 $E = \{EF, KM, FG, JM, EG, KJ\}$



d. $V = \{W, X, Y, Z\}$
 $E = \{WX, XZ, YZ, XY, WZ, WY\}$



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Another... ANSWERS

2. Construct a graph for each of the following sets of vertices and edges. Which of the graphs are connected? Which are complete?

a. $V = \{A, B, C, D, E\}$
 $E = \{AB, AC, AD, AE, BE\}$

a. Connected but NOT complete

b. $V = \{M, N, O, P, Q, R, S\}$
 $E = \{MN, SR, QS, SP, OP\}$

b. NOT connected and NOT complete

c. $V = \{E, F, G, J, K, M\}$
 $E = \{EF, KM, FG, JM, EG, KJ\}$

c. NOT connected and NOT complete

d. $V = \{W, X, Y, Z\}$
 $E = \{WX, XZ, YZ, XY, WZ, WY\}$

d. Connected and complete

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HW: Packet p3 + Packet p4 #8,9

Do Day 1 Exit Ticket here, if not yet.