

## Unit 7 Day 1

### Task Graphs and Critical Paths

Remember that College  
Tour Activity  
is due at the beginning of  
class tomorrow!!

## Notes Day 1

### Task Graphs and Critical Paths

## GRAPH APPLICATIONS

- > How does a building contractor organize all of the jobs needed to complete a project?
- > How do your parents manage to get all the food ready for Thanksgiving dinner at the same time?

### PLANNING!!

- > Everybody plans. Planning your day-to-day activity seems like second nature, but it's not formal or organized.
- > For people who work in the business world, this haphazard method of planning is not an efficient or cost effective way to complete a job.
- > A scientific, more organized method must be used.

## GRAPH APPLICATIONS

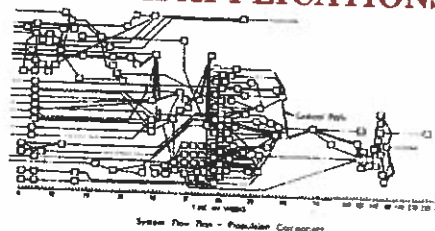
What do you know about graphs?

1. Line graphs
2. Linear equations, Quadratic equations
3. Bar graphs
4. Circle graphs, Pie charts

The graphs we will examine look different from graphs you are used to thinking of.

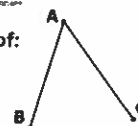
But, they still provide a way of displaying and organizing information.

## GRAPH APPLICATIONS



This new kind of graph is made up of:

- Vertices (points)
- Edges (connecting lines)



This graph has 3 vertices & 2 edges

The Central High yearbook staff has only **16** days left before the deadline for completing their book. They are running behind schedule and still have several tasks to finish. The remaining tasks and times that it takes to complete each task are listed in the following table.

Is it possible to complete the yearbook on time ?

| Task                       | Time (days) | Prerequisite Task |
|----------------------------|-------------|-------------------|
| Start                      | 0           | ---               |
| A. Buy film                | 1           | none              |
| B. Load cameras            | 1           | A                 |
| C. Take photos of clubs    | 3           | B                 |
| D. Take sports photos      | 2           | C                 |
| E. Take photos of teachers | 1           | B                 |
| F. Develop film            | 2           | D, E              |
| G. Design layout           | 5           | D, E              |
| H. Print and mail pages    | 3           | F, G              |

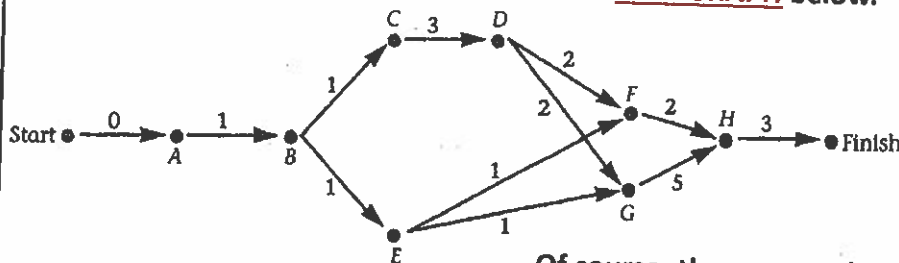


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You may have noticed, some of the yearbook jobs can be done simultaneously, while several of them cannot be started until others have been completed.

Assume these prerequisites. Now, how soon can the project be completed ?

Drawing a graph of this information makes it easier to see the relationships among the tasks. Consider the **TASK GRAPH** below.



| Task                       | Time (days) | Prerequisite Task |
|----------------------------|-------------|-------------------|
| Start                      | 0           | ---               |
| A. Buy film                | 1           | none              |
| B. Load cameras            | 1           | A                 |
| C. Take photos of clubs    | 3           | B                 |
| D. Take sports photos      | 2           | C                 |
| E. Take photos of teachers | 1           | B                 |
| F. Develop film            | 2           | D, E              |
| G. Design layout           | 5           | D, E              |
| H. Print and mail pages    | 3           | F, G              |

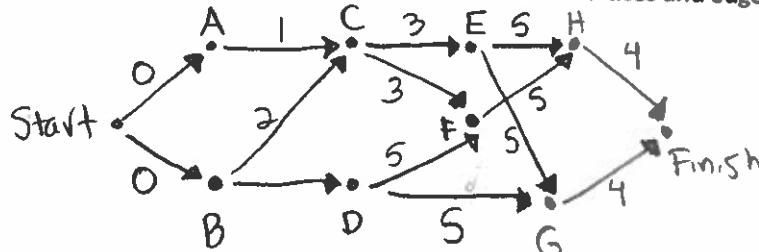
Of course, these examples are more simple than constructing a submarine, but the concept is the same.

Examine the construction of the graph based on the information in the table.

- Prerequisites
- Task Times

**For now, let's practice drawing the graphs!**

Use the task table to draw a task graph with labeled vertices and edges.



\* Draw arrows with colored in triangles not  $\rightarrow$ , so it's clearer

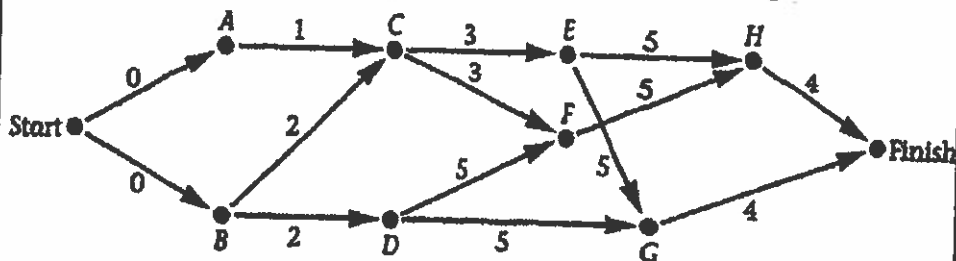
| Task   | Time | Prereqs |
|--------|------|---------|
| Start  | 0    | ---     |
| A      | 1    | none    |
| B      | 2    | none    |
| C      | 3    | A, B    |
| D      | 5    | B       |
| E      | 5    | C       |
| F      | 5    | C, D    |
| G      | 4    | D, E    |
| H      | 4    | E, F    |
| Finish |      |         |

TIPS \* place the times AFTER the vertices are done  
 \* Look at prerequisite tasks to help you know where to put the vertices (dots)  
 Ex)  $\rightarrow$  A + B have no prereq's, so put them on top of each other

$\rightarrow$  C + D both have B as a prereq: so put them on top of each other

**ANSWER: For now, let's practice drawing the graphs!**

Use the task table to draw a task graph with labeled vertices and edges.



| Task   | Time | Prereqs |
|--------|------|---------|
| Start  | 0    | ---     |
| A      | 1    | none    |
| B      | 2    | none    |
| C      | 3    | A, B    |
| D      | 5    | B       |
| E      | 5    | C       |
| F      | 5    | C, D    |
| G      | 4    | D, E    |
| H      | 4    | E, F    |
| Finish |      |         |

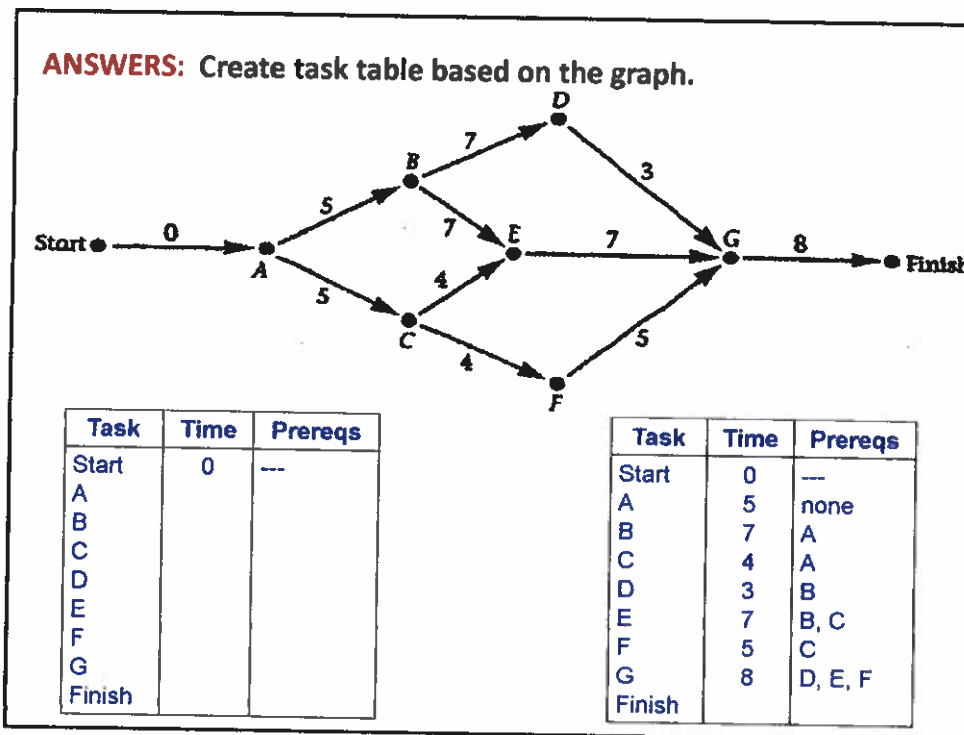
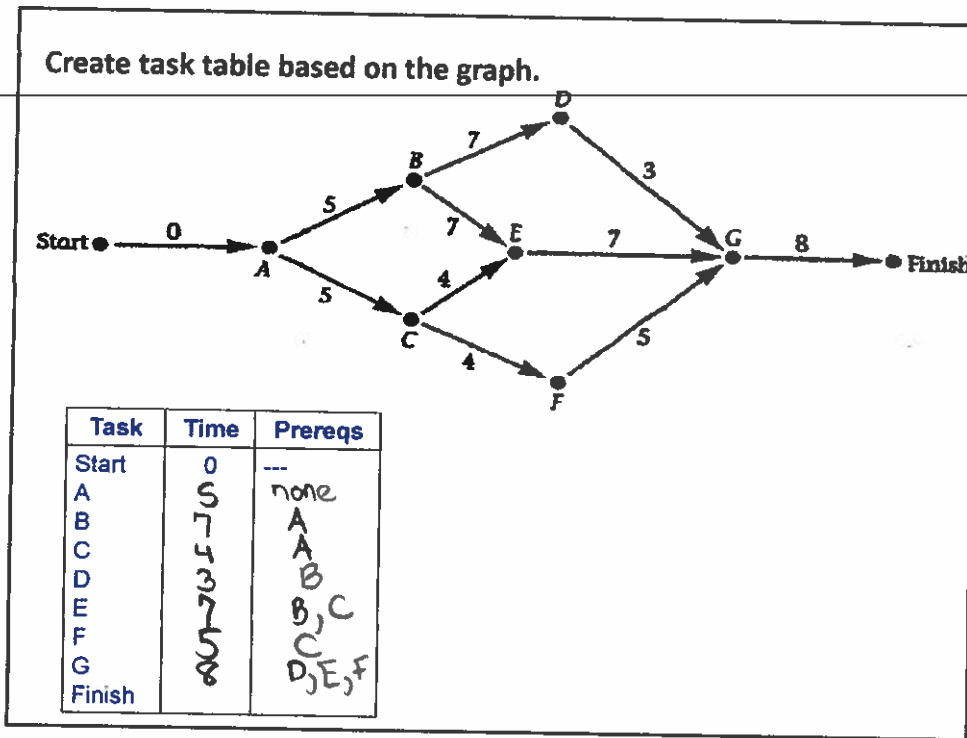
You Try!

| Task   | Time | Prereqs |
|--------|------|---------|
| Start  | 0    | ---     |
| A      | 5    | none    |
| B      | 6    | A       |
| C      | 4    | A       |
| D      | 4    | B       |
| E      | 8    | B, C    |
| F      | 4    | C       |
| G      | 10   | D, E, F |
| Finish |      |         |

Use the task table to draw a task graph with labeled vertices and edges.

| Task   | Time | Prereqs |
|--------|------|---------|
| Start  | 0    | ---     |
| A      | 5    | none    |
| B      | 6    | A       |
| C      | 4    | A       |
| D      | 4    | B       |
| E      | 8    | B, C    |
| F      | 4    | C       |
| G      | 10   | D, E, F |
| Finish |      |         |

**ANSWERS:** Use the task table to draw a task graph with labeled vertices and edges.



~~You Try: Packet #1 #7  
(if time allows)~~

# Critical Paths Section 4.2



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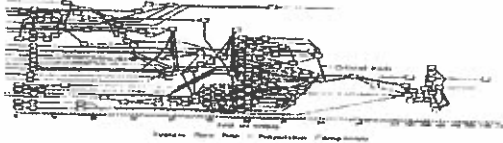
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## Task Graphs - Critical Paths

> It is relatively simple to find the shortest time needed to complete a project if the project consists of only a few activities. As the tasks increase in number, the problem becomes more difficult to solve by inspection alone.



> In the 1950s the government was faced with the need to complete very complex systems such as the US Navy Polaris Submarine project. In order to do this efficiently, a method was developed called

**PERT (Program Evaluation & Review Technique)**

in which those tasks which were critical to the earliest completion of the project were targeted.

The path of targeted tasks from the start to the finish of the project became known as the **CRITICAL PATH**.

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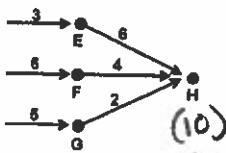
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## Task Graphs - Critical Paths

> We need a systematic way to identify the critical path of a project.

> An **Earliest Start Time (EST)** must be found for each task/activity. (the earliest that an activity can begin if all the activities preceding it begin as early as possible)

Consider this portion of a task graph:  
(times are in minutes)



E to H is a total of 9 mins.

F to H is a total of 10 mins.

G to H is a total of 7 mins.

For H to start, ALL of its prerequisites have to finish.

So, the EST for H is 10 mins.

marking

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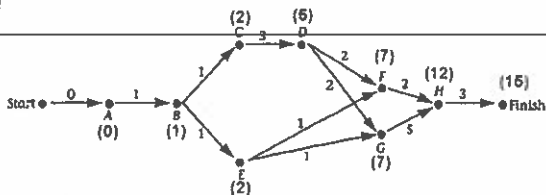
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### Task Graphs - Critical Paths Earliest Start Time (EST)

Recall the Yearbook example from section 4.1.



To calculate the EST for each task, begin at the start and label each vertex with the smallest possible time that is needed before the task can begin.

The EARLIEST TIME in which the Yearbook project can be completed is 15 days. It seems paradoxical, but the least amount of time it takes to complete all the tasks in the project is the LONGEST PATH through the graph.

That longest path is the CRITICAL PATH.

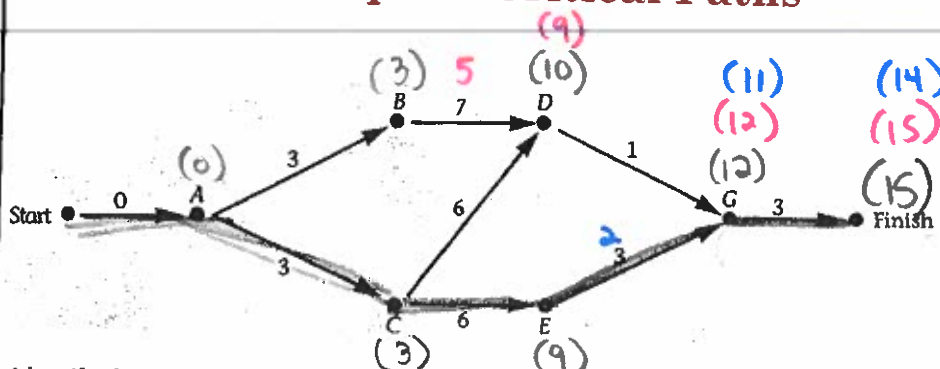
ave  
PS  
-21  
graphs  
2 → see root

You Try: Packet p1 #76 (Find ESTs + Critical Path)

### Tips:

- Add <sup>prev</sup> EST and time on branch to get the next EST
- If 2 edges lead to a vertex, pick the EST from the longest path (otherwise, prerequisites will not all be done before that task)

### Task Graphs - Critical Paths



Identify the EST ( Earliest Start Time ) for each task. *see above*

Identify the Critical Path. *Start - ACEG - Finish*

Trace through the path backwards.

Which path did each EST come from? *the critical path*

If Task B took 5 minutes instead of 7, would that shorten the completion time? **NO**

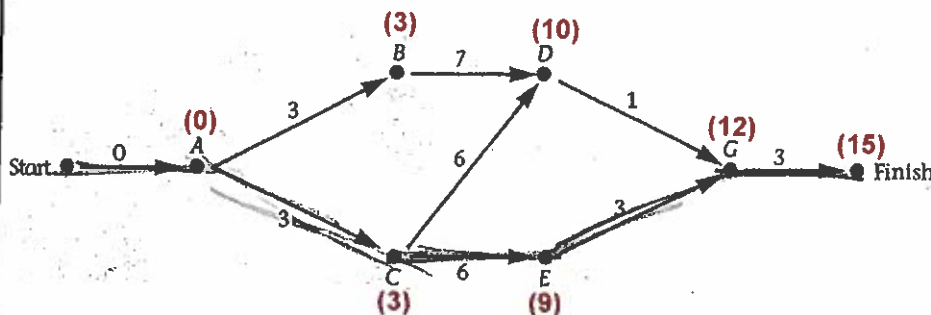
If Task E took 2 minutes instead of 3, would that shorten the completion time?

*Yes. It would cut the project*

*time to 14 minutes because E is on the critical path. AND we'd have two*

*because task B is NOT on the critical path (see pink tweaks) above*

### ANSWERS: Task Graphs - Critical Paths



Identify the EST ( Earliest Start Time ) for each task. *see above*

Identify the Critical Path. *Start - ACEG - Finish*

Trace through the path backwards.

Which path did each EST come from? **Start-ACEG-Finish** *(the critical path)*

If Task B took 5 minutes instead of 7, would that shorten the completion time? **No**

If Task E took 2 minutes instead of 3, would that shorten the completion time?

**Yes, it would cut the project time to 14 minutes.**

*because task B is NOT on the critical path*

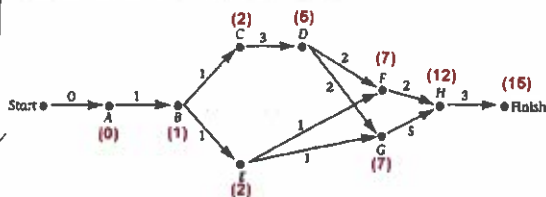
*critical paths now!!*



Do this before another example

### Task Graphs - Critical Paths Earliest Start Time (EST)

Recall the Yearbook example from section 4.1.



To calculate the EST for each task, begin at the start and label each vertex with the smallest possible time that is needed before the task can begin. The EARLIEST TIME in which the Yearbook project can be completed is 15 days. It seems paradoxical, but the least amount of time it takes to complete all the tasks in the project is the same as the LONGEST PATH through the graph.

That longest path is the **CRITICAL PATH**.

You Try! Packet p1 #7b (Find ESTs + Critical Path)

### Latest Start Time (LST)

If an activity is not on the critical path, it is possible for it to start later than its earliest start time and not delay the project. The latest a task can begin without delaying the project's minimum completion time is known as the **LATEST START TIME (LST)** for the task.

Tips:  
 • Add prior EST and time on branch to get the next EST  
 • If 2 edges lead to a vertex, pick the EST from the longest path (otherwise, prerequisites will not all be done before that task)

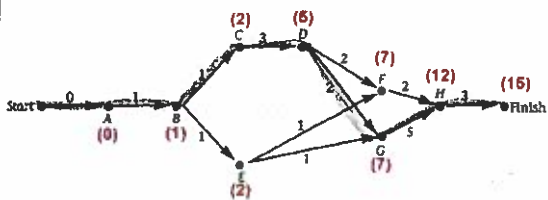
\* If multiple edges lead back to a path, pick smallest time for LST

Tips for critical paths:  
 EST

• Find numbers that add to your prior EST + the prior branch → trace that path

### Task Graphs - Critical Paths

To figure the CRITICAL PATH of the Yearbook example, trace through the path backwards.



The Critical Path for the Yearbook project is Start - A - B - C - D - F - H - Finish

Find the Latest Start Time for task F.  $10$  (do  $12 - 2 = 10$ )  
 Find the Latest Start Time for Task D.  $5$  ← same as EST because it is on the Critical Path (do  $7 - 2$ )

↑  
 $7 - 2 = 5$   
 G's EST + time for D

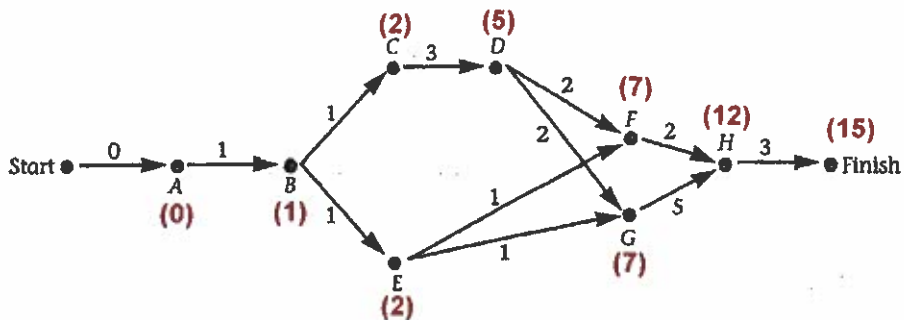
H - Finish  
 $12 - 2 = 10$

### Tips for LSTs:

• Find EST after, it on critical path and subtract task time(s) to get back to the task<sup>2</sup>

### Task Graphs - Critical Paths

To figure the CRITICAL PATH of the Yearbook example, trace through the path backwards.



*oops... slide here twice*

*See last page for work + tips*

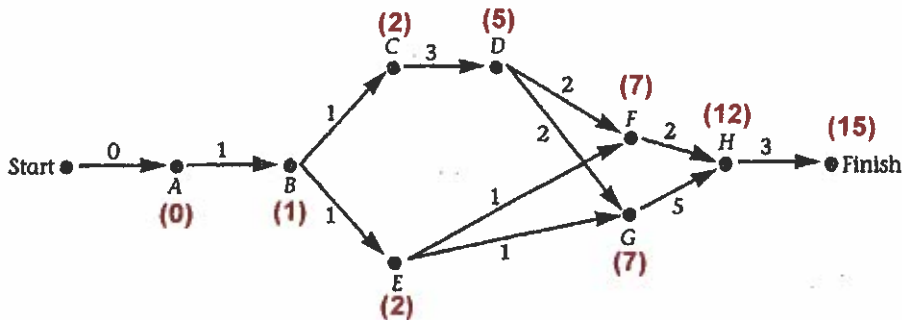
The Critical Path for the Yearbook project is

- Find the Latest Start Time for task F.
- Find the Latest Start Time for Task D.

← same as EST because it is on the Critical Path

### ANSWERS: Task Graphs - Critical Paths

To figure the CRITICAL PATH of the Yearbook example, trace through the path backwards.

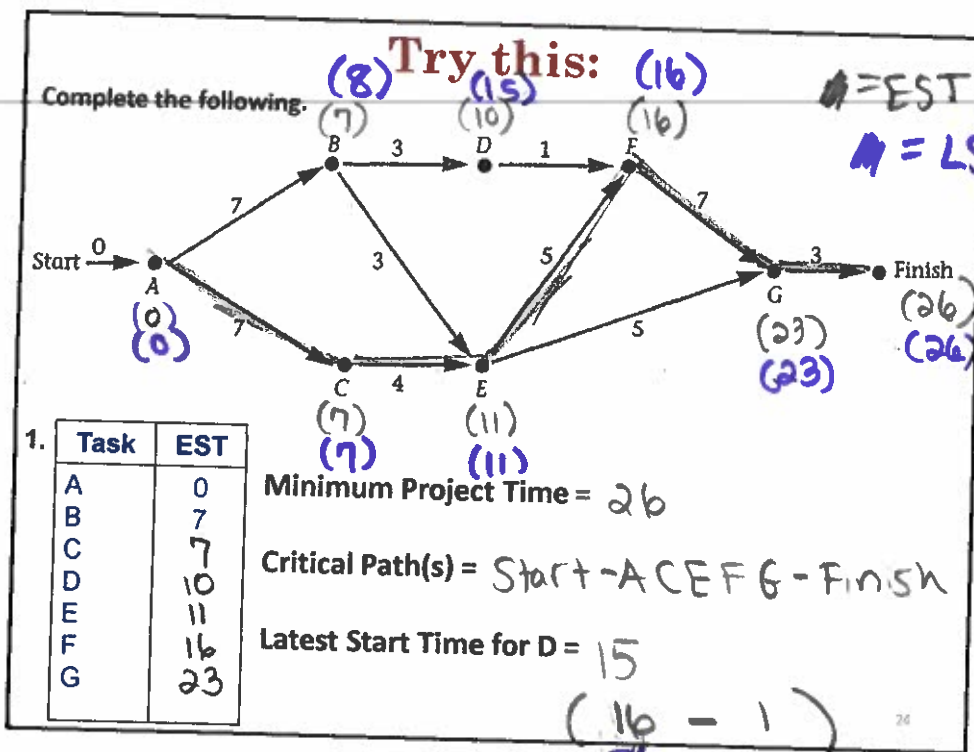


The Critical Path for the Yearbook project is **Start-ABCDGH-Finish**.

Find the Latest Start Time for task F. **10**

Find the Latest Start Time for Task D. **5** ← same as EST because it is on the Critical Path

You  
Try

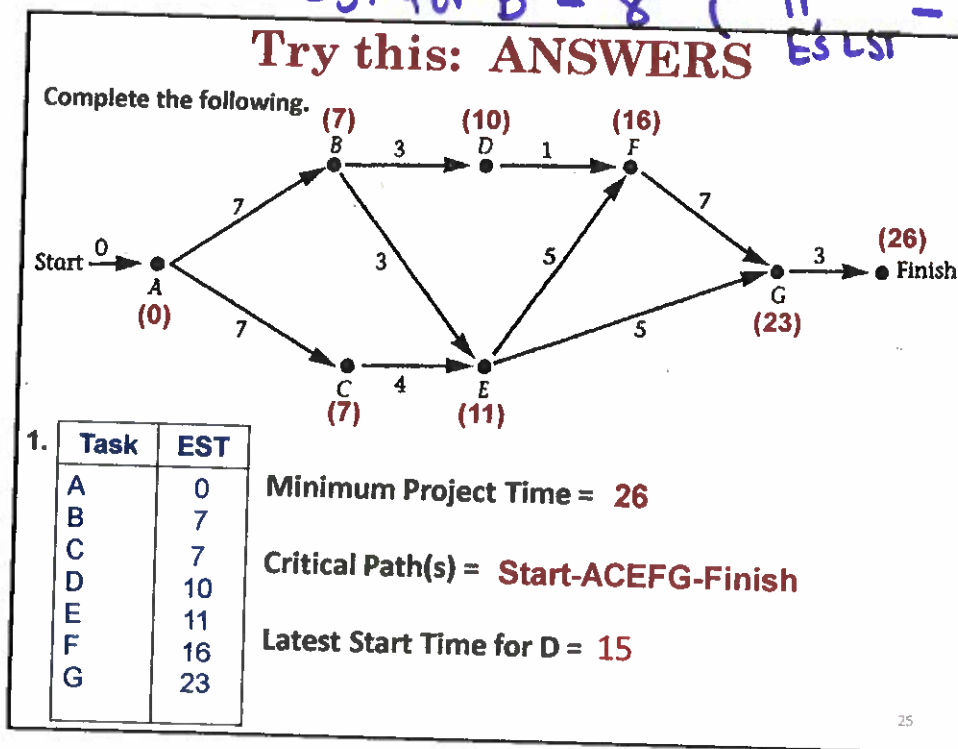


\* LST = EST on critical path

FS LST time for D  $(16 - 1)$

LST For B = 8  $(11 - 3)$

\* If multiple options for LST, pick smaller one



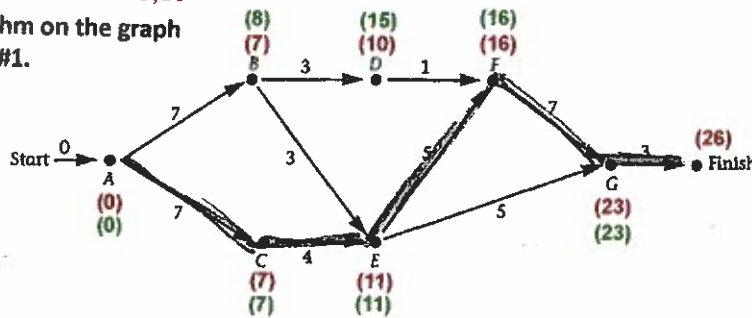
### LATEST START TIME Algorithm ☺

The general algorithm for finding the LST for each task in a graph is :

1. Begin with Finish and use the minimum project time.
2. Subtract from the minimum project time the time that it takes to complete the task(s) preceding it. This is the LST for the preceding tasks. Label the vertex with its LST.
3. If the time of a preceding task is dependent on more than one edge, choose the earliest time. *so project finishes on time*
4. Continue until all vertices are labeled.

Section 4.2 - Exercise #9,10

Try the algorithm on the graph from exercise #1.

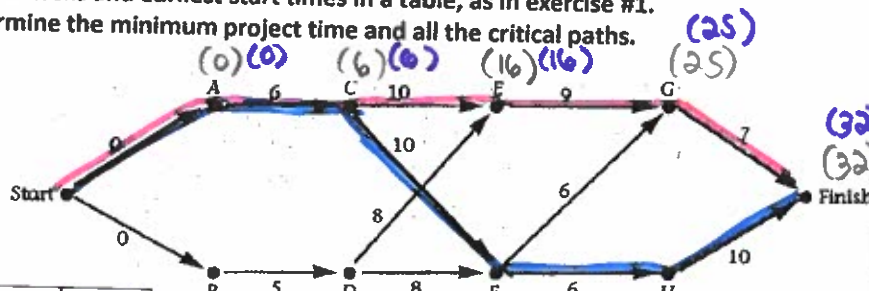


\* Same as graph from last slide \*

on critical path, LST = EST

### And this...

List the tasks and earliest start times in a table, as in exercise #1. Determine the minimum project time and all the critical paths.



# = EST  
# = LST

2.

| Task | EST |
|------|-----|
| A    | 0   |
| B    | 0   |
| C    | 6   |
| D    | 5   |
| E    | 16  |
| F    | 16  |
| G    | 25  |
| H    | 22  |

Minimum Project Time = 32

Critical Path(s) = Start-A-C-F-H-Finish  
Start-A-C-E-G-Finish

you try

□ Add in

Find LST for F = 16 (LST = EST b/c on critical path)  
and for D = 8 (Finish Time - F Time - D Time)  
32 - 10 - 6 - 8