

## Unit 6 Day 4 Review

### Warm Up Day 4

1)

- Freshmen get 3 votes.
- Sophomores get 5 votes.
- Juniors get 7 votes.
- Seniors get 8 votes.
- A winning coalition must have 6 votes.

a) What are all the possible coalitions?

b) Of these coalitions which ones are winners?

c) What is the Power Index for each grade level?

Freshmen \_\_\_\_\_ Juniors \_\_\_\_\_  
Sophomores \_\_\_\_\_ Seniors \_\_\_\_\_

d) Who are the dictators or dummies?

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## Warm Up ANSWERS

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a) What are all the possible coalitions?

{F; 3}, {So; 5}, {Jr; 7}, {Sr; 8}, {F, So; 8}, {F, Jr; 10},  
 {F, Sr; 11}, {So, Jr; 12}, {So, Sr; 13}, {Jr, Sr; 15}, {F,  
 So, Jr; 15}, {F, So, Sr; 16}, {So, Jr, Sr; 20}, {F, Jr, Sr;  
 18}, {F, So, Jr, Sr; 23}, {none; 0}

b) Of these coalitions which ones are winners?

{Jr; 7}, {Sr; 8}, {F, So; 8}, {F, Jr; 10}, {F, Sr; 11}, {So, Jr; 12},  
 {So, Sr; 13}, {Jr, Sr; 15}, {F, So, Jr; 15}, {F, So, Sr; 16}, {So, Jr,  
 Sr; 20}, {F, Jr, Sr; 18}, {F, So, Jr, Sr; 23}

c) What is the Power Index for each grade level?

Freshmen 1 Juniors 3  
 Sophomores 1 Seniors 3

d) Who are the dictators or dummies?

None. No voter is essential to all winning coalitions or zero winning coalitions.

3) (Everyone wins) Consider the following set of preference lists:

# of Voters	3	1	1	1	1	1	1
1 <sup>st</sup>	A	A	B	B	C	C	D
2 <sup>nd</sup>	D	B	C	C	B	D	C
3 <sup>rd</sup>	B	C	D	A	D	B	B
4 <sup>th</sup>	C	D	A	D	A	A	A

Calculate the winner using:

- plurality voting
- Borda count method
- Sequential Runoff method
- Cordocet voting.

4) Consider the following set of preference lists:

# of Voters	2	2	1	1	1
1 <sup>st</sup>	C	D	C	B	A
2 <sup>nd</sup>	A	A	D	D	D
3 <sup>rd</sup>	B	C	A	A	B
4 <sup>th</sup>	D	B	B	C	C

Calculate the winner using:

- plurality voting
- Borda count method
- Sequential Runoff method
- Cordocet voting.

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5) Ten board members vote by approval voting on eight candidates for new positions on their board as indicated in the following table.

	1	2	3	4	5	6	7	8	9	10
A	X	X	X			X	X	X		X
B		X	X	X	X	X	X	X	X	
C			X					X		
D	X	X	X	X	X		X	X	X	X
E	X		X		X		X		X	
F	X		X	X	X	X	X	X		X
G	X	X	X	X	X			X		
H		X		X		X		X		X

- Which candidate is chosen for the board if just one of them is selected?
- Which candidates are chosen if the top four are selected?
- Which candidates are selected if 80% approval is necessary and at most four are elected?
- Which candidates are elected if 60% approval is necessary and at most four are selected?

### Practice: Day 2 (continued)

1) Find the winner of the election under the method of pairwise comparisons (Cordocet).

# of Voters	5	3	5	3	2	3
1 <sup>st</sup>	A	A	C	D	D	B
2 <sup>nd</sup>	B	D	E	C	C	E
3 <sup>rd</sup>	C	B	D	B	B	A
4 <sup>th</sup>	D	C	A	E	A	C
5 <sup>th</sup>	E	E	B	A	E	D

3) Consider the following set of preference lists:

	1	1	1	1	1	1	1
1 <sup>st</sup>	C	D	C	B	E	D	C
2 <sup>nd</sup>	A	A	E	D	D	E	A
3 <sup>rd</sup>	E	E	D	A	A	A	E
4 <sup>th</sup>	B	C	A	E	C	B	B
5 <sup>th</sup>	D	B	B	C	B	C	D

Calculate the winner using:

- Plurality
- Borda count
- Cordocet method
- Sequential Runoff

2) The 45 members of a school's football team vote on three nominees, A, B, and C by approval voting for the award of "most improved player" as indicated in the table. Who would win most improved player?

	7	8	9	9	6	3	1
A	X			X	X		X
B		X		X		X	X
C			X		X	X	X



3) (Everyone wins) Consider the following set of preference lists:

# of Voters	3	1	1	1	1	1	1
1 <sup>st</sup>	A	A	B	B	C	C	D
2 <sup>nd</sup>	D	B	C	C	B	D	C
3 <sup>rd</sup>	B	C	D	A	D	B	B
4 <sup>th</sup>	C	D	A	D	A	A	A

Total: 9 voters

Calculate the winner using:

a) plurality voting **A** (1<sup>st</sup> place) A: 4 B: 2 D: 1

b) Borda count method

A:  $4(3) + 4(1) + 1(1) + 2(1) + 1(1) + 1(1) + 1(1) = 22$   
 B:  $2(3) + 3(1) + 4(1) + 4(1) + 3(1) + 2(1) + 2(1) = 24$   
 C:  $1(3) + 2(1) + 3(1) + 3(1) + 4(1) + 4(1) + 3(1) = 22$   
 D:  $3(3) + 1(1) + 2(1) + 1(1) + 2(1) + 3(1) + 4(1) = 22$

**B** with 24 points

c) Sequential Runoff method

**C**

- Eliminate D with 1 1<sup>st</sup> place
- Eliminate B with 2 1<sup>st</sup> place
- A: 4 1<sup>st</sup> place now (3+1)  
C: 5 1<sup>st</sup> place now (1+1+1+1+1)

d) Cordocet voting.

None

no one wins versus everyone else

	A	B	C	D
A	X	B	C	A
B	X	X	B	D
C	X	X	X	C
D	X	X	X	X

4) Consider the following set of preference lists:

# of Voters	2	2	1	1	1
1 <sup>st</sup>	C	D	C	B	A
2 <sup>nd</sup>	A	A	D	D	D
3 <sup>rd</sup>	B	C	A	A	B
4 <sup>th</sup>	D	B	B	C	C

Total: 7 voters

Calculate the winner using:

a) plurality voting **C** with 3 1<sup>st</sup> place votes

b) Borda count method

A:  $3(2) + 3(2) + 2(1) + 2(1) + 4(1) = 20$   
 B:  $2(2) + 1(2) + 1(1) + 4(1) + 2(1) = 13$   
 C:  $4(2) + 2(2) + 1(1) + 1(1) + 1(1) = 19$   
 D:  $1(2) + 1(2) + 3(1) + 3(1) + 3(1) = 19$

c) Sequential Runoff method

- Eliminate A + B with 1 1<sup>st</sup> place
- C: 3 1<sup>st</sup> place now
- D: 4 1<sup>st</sup> place now ← wins

**D**

d) Cordocet voting.

	A	B	C	D
A	X	A	A	D
B	X	X	C	D
C	X	X	X	D
D	X	X	X	X

**D** D wins versus everyone head-to-head

5) Ten board members vote by approval voting on eight candidates for new positions on their board as indicated in the following table.

	1	2	3	4	5	6	7	8	9	10
A	X	X	X			X	X	X		X
B		X	X	X	X	X	X	X	X	
C			X					X		
D	X	X	X	X	X		X	X	X	X
E	X		X		X		X		X	
F	X		X	X	X	X	X	X		X
G	X	X	X	X	X		X	X		
H		X		X		X		X		X

- 7 A
- 8 B
- 2 C
- 9 D
- 5 E
- 8 F
- 6 G
- 5 H

	A	B	C	D	E
A	X	A	A	A	E
B	X	X	B	D	B
C	X	X	X	C	C
D	X	X	X	X	D
E	X	X	X	X	X

ex: A: 13 vs B: 8 → A wins  
 A: 11 vs C: 10 → A wins  
 A: 11 vs D: 10 → A wins  
 A: 10 vs E: 11 → E wins

Practice: Day 2 (continued)

1) Find the winner of the election under the method of pairwise comparisons (Cordocet).

# of Voters	5	3	5	3	2	3
1 <sup>st</sup>	A	A	C	D	D	B
2 <sup>nd</sup>	B	D	E	C	C	E
3 <sup>rd</sup>	C	B	D	B	B	A
4 <sup>th</sup>	D	C	A	E	A	C
5 <sup>th</sup>	E	E	B	A	E	D

see work above

none... no one wins versus all candidates head-to-head

2) The 45 members of a school's football team vote on three nominees, A, B, and C by approval voting for the award of "most improved player" as indicated in the table. Who would win most improved player?

	7	8	9	9	6	3	1
A	X			X	X		X
B		X		X		X	X
C			X		X	X	X

total votes  
 23 for A  
 21 for B  
 19 for C

A with 23 total votes

3) Consider the following set of preference lists:

	1	1	1	1	1	1	1
5 1 <sup>st</sup>	C	D	C	B	E	D	C
4 2 <sup>nd</sup>	A	A	E	D	D	E	A
3 3 <sup>rd</sup>	E	E	D	A	A	A	E
2 4 <sup>th</sup>	B	C	A	E	C	B	B
1 5 <sup>th</sup>	D	B	B	C	B	C	D

Calculate the winner using:

a) Plurality **C** with 3 first place votes

b) Borda count  
 A:  $4(1) + 4(1) + 2(1) + 3(1) + 3(1) + 3(1) + 4 = 23$   
 B:  $2(1) + 1(1) + 1(1) + 5(1) + 1(1) + 2(1) + 2(1) = 14$   
 C:  $5(1) + 2(1) + 5(1) + 1(1) + 2(1) + 1(1) + 5(1) = 21$   
 D:  $1 + 5 + 3 + 4 + 4 + 5 + 1 = 23$   
 E:  $3 + 3 + 4 + 2 + 5 + 4 + 3 = 24$

c) Cordocet method

**None** No one wins vs all others.

d) Sequential Runoff

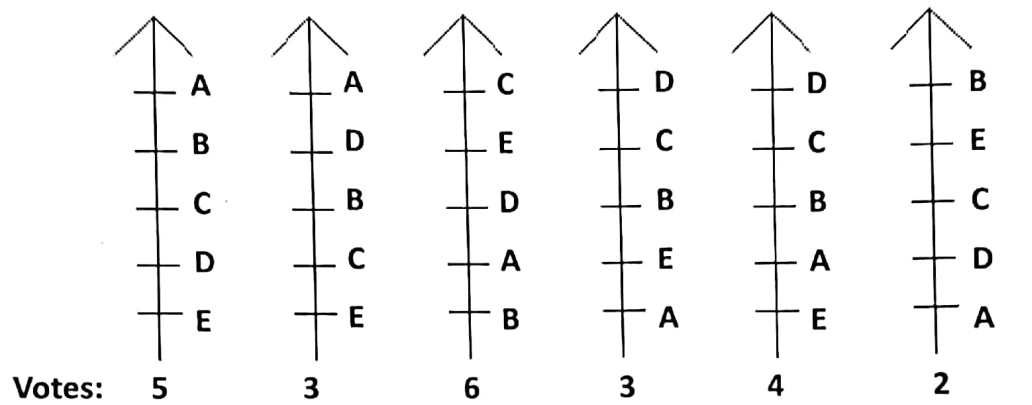
- Eliminate A with 0 1<sup>st</sup>s
- Eliminate B with 1 1<sup>st</sup>
- Eliminate E with 1 1<sup>st</sup>

	A	B	C	D	E
A	X	A	A	D	A
B	X	X	C	D	E
C	X	X	X	D	E
D	X	X	X	X	E
E	X	X	X	X	X

## Review

1) Plurality, Majority, Runoff, Sequential, Borda , Condorcet, Approval

For approval, the top 3 votes were approved.



Plurality:

Majority:

Borda:

Runoff:

Seq. Runoff:

Condorcet:

Approval:

### Review

1) Plurality, Majority, Runoff, Sequential, Borda, Condorcet, Approval

For approval, the top 3 votes were approved.

	↑	↑	↑	↑	↑	↑
	A	A	C ✓	D	D	B ✓
	B ✓	D	E ✓	C ✓	C ✓	E ✓
	C ✓	B ✓	D	B ✓	B ✓	C ✓
	D	C ✓	A	E ✓	A	D
	E ✓	E ✓	B ✓	A	E ✓	A
Votes:	5	3	6	3	4	2

Plurality: **A** (with 8 1st place)  
 Majority: None (no one with > 50%)  
 Borda: **C** (85 pts) → see next page  
 Runoff: **D**  
 Seq. Runoff: **C** (with 15 1st place) ↓ see next page  
 Approval: **C** with 20 votes → see 3rd page

Total votes:  
23

~~Fix~~

Runoff: Narrow to top 2 at the start

① Narrow down to A with 8 and D with 7 votes first place at start

② Cross off everyone else

③ Check new first place totals: A → 8 (5+3)  
D → 15 (6+3+4+2)

**D** wins with 15 1st place votes

Sequential Run off: Eliminate one with least 1st place one by one.

① Eliminate E with 0 1st place

② Reassess current 1st place votes. Eliminate B with only 2 1st place votes now.

③ Reassess current 1st place votes. Eliminate D with only 7 1st place now.

④ Reassess current 1st place votes.

A: 8 (5+3)

C: 15 (6+3+4+2)

**C** wins with 15 1st place votes



### Review

1) Plurality, Majority, Runoff, Sequential, Borda, Condorcet, Approval

For approval, the top 3 votes were approved.

	$\begin{array}{c} \nearrow \\ 5 - A \\ 4 - B \\ 3 - C \\ 2 - D \\ 1 - E \end{array}$	$\begin{array}{c} \nearrow \\ 5 - A \\ 4 - D \\ 3 - B \\ 2 - C \\ 1 - E \end{array}$	$\begin{array}{c} \nearrow \\ 5 - C \\ 4 - E \\ 3 - D \\ 2 - A \\ 1 - B \end{array}$	$\begin{array}{c} \nearrow \\ 5 - D \\ 4 - C \\ 3 - B \\ 2 - E \\ 1 - A \end{array}$	$\begin{array}{c} \nearrow \\ 5 - D \\ 4 - C \\ 3 - B \\ 2 - A \\ 1 - E \end{array}$	$\begin{array}{c} \nearrow \\ 5 - B \\ 4 - E \\ 3 - C \\ 2 - D \\ 1 - A \end{array}$
Votes:	5	3	6	3	4	2

Plurality:                      Majority:                      Borda: **C with 85 pts**                      Runoff:

Seq. Runoff:                      Condorcet:                      Approval:

Borda: **C** with 85 points

$$A: 5(5) + 3(5) + 2(6) + 1(3) + 2(4) + 1(2) = 65$$

$$B: 4(5) + 3(3) + 1(6) + 3(3) + 3(4) + 5(2) = 66$$

$$C: 3(5) + 2(3) + 5(6) + 4(3) + 4(4) + 3(2) = 85$$

$$D: 2(5) + 4(3) + 3(6) + 5(3) + 5(4) + 2(2) = 79$$

$$E: 1(5) + 1(3) + 4(6) + 2(3) + 1(4) + 4(2) = 50$$

Condorcet: **C** because C wins versus everyone else

	A	B	C	D	E
A	X	A	C	D	A
B	X	X	C	D	B
C	X	X	X	C	C
D	X	X	X	X	D
E	X	X	X	X	X

- A vs B } A wins  
14 vs 9
- A vs C } C wins  
8 vs 15
- A vs D } D wins  
8 vs 15
- A vs E } A wins  
12 vs 11

**Review**

1) Plurality, Majority, Runoff, Sequential, Borda, Condorcet, Approval

For approval, the top 3 votes were approved.

5	A	3	A	6	C	3	D	4	D	2	B
5	B	3	D	6	E	3	C	4	C	2	E
5	C	3	B	6	D	3	B	4	B	2	C



**Tonight's HW =**  
**Quiz Review on Packet p. 7-8**  
(Section 1.1-1.5)



**Group Ranking Methods**

1. Plurality
2. Majority
3. Runoff
4. Sequential Runoff
5. Borda
6. Condorcet
7. Approval

