Warm-up Day 3

1) Given these preference schedules, identify the Plurality, Borda, Runoff, Sequential Runoff, and Condorcet winners.

14	17	19	12	16	22
E	A	C	C	D	B
A	D	B	D	E	A
D	B	A	B	A	C
B	C	D	E	C	E
C	E	E	A	B	D

Plurality: C

Borda: A (see below)

Runoff: B

Seq. Runoff: A

Condorcet: See work on next page (NONE)

(handwritten annotations:)

with 31 first place votes (on middle two preference schedules)

Borda: remove E, D, A
then B vs C
B: 53 (14+17+22)
C: 47 (19+12+16)

Seq. Runoff: Remove
E: 14
D: 16
B: 22
→ A: 69 (14+17+16+22)
then A vs C
C: 31 (19+12)

2) If each voter approves of only the top three in his ranking, which is the Approval Voting winner?

14	17	19	12	16	22
5 E	5 A	5 C	5 C	5 D	5 B
4 A	4 D	4 B	4 D	4 E	4 A
3 D	3 B	3 A	3 B	3 A	3 C
2 B	2 C	2 D	2 E	2 C	2 E
1 C	1 E	1 E	1 A	1 B	1 D

See page 3 (5th slide) for work + explanation

Borda method:

A: 14(4) + 17(5) + 19(3) + 12(1) + 16(3) + 22(4) = 346

B: 14(2) + 17(3) + 19(4) + 12(3) + 16(1) + 22(5) = 317
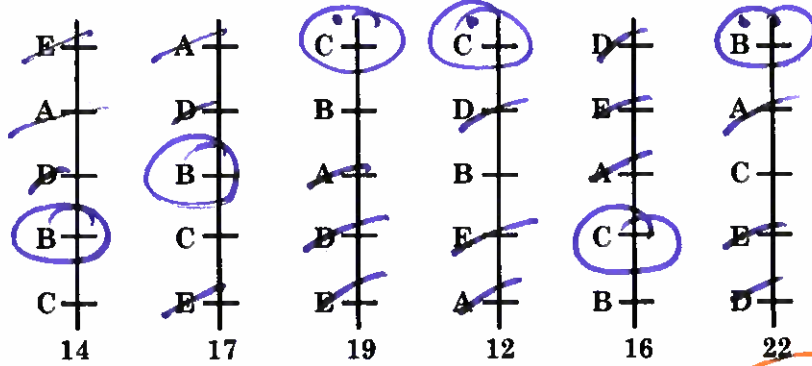
C: 14(1) + 17(2) + 19(5) + 12(5) + 16(2) + 22(3) = 301

D: 14(3) + 17(4) + 19(2) + 12(4) + 16(5) + 22(1) = 264

E: 14(5) + 17(1) + 19(1) + 12(2) + 16(4) + 22(2) = 236

Borda method: A wins with 346 points

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Warm-up Day 3

Given these preference schedules, identify the Plurality, Borda, Runoff, Sequential Runoff, and Condorcet winners.

E	A	C	C	D	B
A	D	B	D	E	A
D	B	A	D	A	C
B	C	D	E	C	E
C	E	E	A	B	D
14	**17**	**19**	**12**	**16**	**22**

Plurality:　　Borda:　　Runoff:　　Seq. Runoff:　　Condorcet:

(handwritten) none

(handwritten) B = 14 + 17 + 22
53

C = 19 + 12 + 16
47

(handwritten table)

	A	B	C	D	E
A	X	B	A	A	A
B	X	X	B	D	B
C	X	X	X	C	C
D	X	X	X	X	D
E	X	X	X	X	X

* MUST beat everyone else
(handwritten) Heads for Condorcet winners

Warm-up Day 3 ANSWERS

Given these preference schedules, identify the Plurality, Borda, Runoff, Sequential Runoff, and Condorcet winners.

E	A	C	C	D	B
A	D	B	D	E	A
D	B	A	B	A	C
B	C	D	E	C	E
C	E	E	A	B	D
14	**17**	**19**	**12**	**16**	**22**
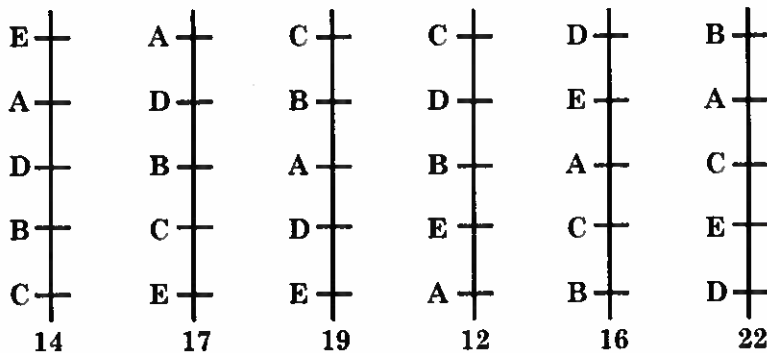
Plurality: C　　Borda: A　　Runoff: B　　Seq. Runoff: A　　Condorcet: None

(handwritten) Runoff → narrow down to C and B with most 1st place votes
↳ see work on top of page

1

2)

Warm-up Day 3

If each voter approves of only the top three in his ranking, which is the Approval Voting winner?

E 14	A 17	C 19	C 12	D 16	B 22
A 14	D 17	B 19	D 12	E 16	A 22
D 14	B 17	A 19	B 12	A 16	C 22
B	C	D	E	C	E
C	E	E	A	B	D
14	17	19	12	16	22

A: 14 + 17 + 19 + 16 + 22 = 88 votes
B: 17 + 19 + 12 + 22 = 70 votes

Approval: **A**

Warm-up Day 3 ANSWERS

If each voter approves of only the top three in his ranking, which is the Approval Voting winner?

E	A	C	C	D	B
A	D	B	D	E	A
D	B	A	B	A	C
B	C	D	E	C	E
C	E	E	A	B	D
14	17	19	12	16	22

Approval: A

Unit 6 Day ~~2~~ 3

~~1.3 and~~ 1.45

More Group Ranking Methods
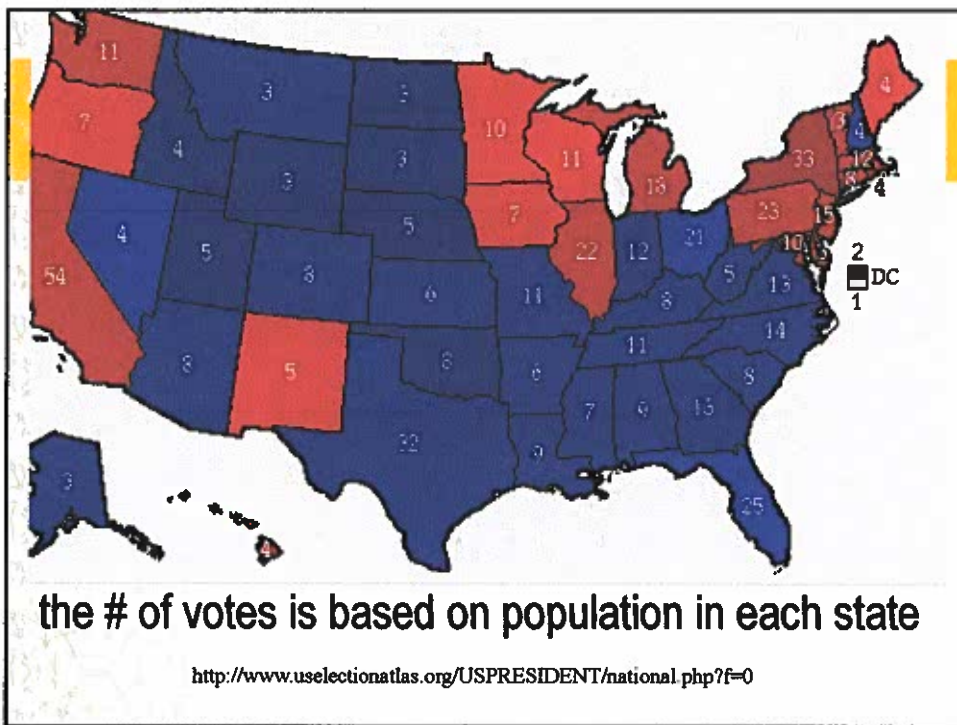### And Approval Voting

Weighted Voting + Voting Power

Weighted Voting and Voting Power

Section 1.5

WEIGHTED VOTING

☀ Definition: Method of voting when some members of a voting body have more votes than others.

☀ Why might it be fair for some members or groups to have more weight than others?

☀ Examples: Electoral College, Corporate stockholder meetings, Mom, etc.

the # of votes is based on population in each state

http://www.uselectionatlas.org/USPRESIDENT/national.php?f=0

Example:

A student council has 1 representative per class, but there are 500 sophomores, 300 juniors, and 300 seniors. How could votes be weighted to represent everyone fairly?

Every 100 students = 1 representative

How many votes would be needed to pass an issue? If not given, go for majority.

1100 total Students

11 total reps.

WINNING COALITIONS:

❧ Each collection is known as a **Coalition**.
  {So;5}, {Jr;3}, {So,Jr;8}, {Jr,Sr;6} ...

❧ Those with enough votes to pass an issue are known as **Winning Coalitions**. (A winning coalition is like an alliance in "Survivor.")

❧ A voter is **essential** when their vote is NECESSARY to win. (i.e. if you remove it, the winning coalition becomes a losing coalition.)

COALITIONS:

- Sophomores get 5 votes
- Juniors get 3 votes
- Seniors get 3 votes

Notation: {So;5}, {Jr;3}, {Sr;3}

There are a total of 11 votes
and they need 6 votes to win.
to get a majority

Could any group win
by themselves?

NO!!

So, join forces!

WINNING COALITIONS:
(like an alliance on Survivor)

- Sophomores get 5 votes.
- Juniors get 3 votes.
- Seniors get 3 votes.
- You need 6 votes to WIN so a winning coalition must have 6 votes.

What are all the
possible coalitions?

{none;0} {So;5} {Jr;3} {Sr;3}
{So,Jr;8} {So,Sr;8} {Jr,Sr;6}
{So,Jr,Sr;11}

Of these coalitions which
ones are winners?

{So,Jr;8} {So,Sr;8} {Jr,Sr;6}
{So,Jr,Sr;11}

"winning coalitions" have at least the 6 votes needed to win

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WINNING COALITIONS:
(like an alliance on Survivor)

$\{So,Jr;8\}$ $\{So,Sr;8\}$ $\{Jr,Sr;6\}$ $\{So,Jr,Sr;11\}$

Notice that the sophomore, junior, and senior representatives are each essential to 2 of the coalitions. *(would not with without them)*

This is a **PARADOX**: Although the votes have been distributed to give more power to the sophomores, the outcome is that all members have the same amount of power.

(handwritten left margin)
☐ Add into reminders slide 2
- Soph gets 5 votes
- Jrs get 3
- Srs get 3
Need 6 votes to pass

(handwritten right margin)
☐ Add ✓

Banzhaf Power Index:
determines the power of a member of a voting body

- ☙ **Step 1.** Make a list of all possible coalitions
- ☙ **Step 2.** Determine which of them are winning coalitions
- ☙ **Step 3.** In each winning coalition, determine which of the players are *essential* players
- ☙ **Step 4.** Count the total number of times each player is essential
- ☙ You just found the Banzhaf Power Index of each player!

Another Example

⚜ Consider the weighted voting situation:

Voter A – 15 votes;   Voter B – 12 votes
Voter C – 6 votes;   Voter D – 3 votes
20 *votes are needed to pass…*
List all of the possible coalitions:

{none;0} {A;15} {B;12} {C;6} {D;3}
{A,B;27} {A,C;21} {A,D;18}
{B,C;18} {B,D;15} {C,D;9}
{A,B,C;33} {A,B,D;30}
{A,C,D;24} {B,C,D;21}
{A,B,C,D;36}

List all of the winning coalitions:

{A,B;27} {A,C;21} {A,B,C;33} {A,B,D;30} {A,C,D;24} {B,C,D;21} {A,B,C,D;36}

SO….

A – 15 votes; B – 12 votes; C – 6 votes; D – 3 votes
20 *votes are needed to pass*

Winning coalitions:

{A,B;27} {A,C;21} {A,B,C;33} {A,B,D;30} {A,C,D;24} {B,C,D;21} {A,B,C,D;36}

Determine the POWER INDEX for each voter:

A is NEEDED for __5__ coalitions

B is NEEDED for __3__ coalitions

C is NEEDED for __3__ coalitions

D is NEEDED for __1__ coalitions

> **Power Index:**
> **Number of winning coalitions to which the voter is essential.**

(handwritten) A is not needed to pass with {A,B,C,D;36} because the group would have ≥20 votes anyway)

(handwritten) B is not needed to pass with {A,B,C;33} and {A,B,C,D;36}.)

(handwritten) C is not needed to get ≥20 votes for {A,B,C;33} and {A,B,C,D;36})

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One More Example

☙ Consider the weighted voting situation:

Voter A – 7 votes;   Voter B – 3 votes
Voter C – 3 votes;
7 votes are needed to pass…
List all of the possible coalitions:

{none ;0} {A;7} {B;3} {C;3}
{A,B;10} {A,C;10} {B,C;6}
{A,B,C;13}

List all of the winning coalitions:

{A;7} {A,B;10} {A,C;10} {A,B,C;13}

Power Indices: A **4**

                          B **0**

                          C **0**

Dummies and Dictators…

☙ **A Dictator** has all the "power" in a voting body. He is essential to EVERY winning coalition.

→ like A in the last example

☙ **A Dummy** has no power in a voting body. He is not essential to ANY winning coalitions.

→ like B and C in the last example

You Try

Consider a situation in which voters A, B, C, and D have 4, 3, 3, and 2 votes, respectively, and 7 votes are needed to pass an issue.

a. Identify all winning coalitions and their vote totals.

[handwritten] {A,B;7}, {A,C;7}, {A,B,C;10}, {A,B,D;9}, {A,C,D;9}, {B,C,D;8}, {A,B,C,D;12}

b. Find the power index for each voter.

[handwritten] A:5 B:3 C:3 D:1

c. Do the power indices reflect the distribution of votes?

[handwritten] No. A's power index is higher and D's is lower

d. Suppose the number of votes necessary to pass an issue is increased from 7 to 8. How does this change the power indices of the voters?

[handwritten] {A,B,C;10}, {A,B,D;9}, {A,C,D;9}, {B,C,D;8}, {A,B,C,D;12}
A:3 B:3 C:3 D:3

[handwritten] Each voter has equal power now.

You Try

Consider a situation in which voters A, B, C, and D have 4, 3, 3, and 2 votes, respectively, and 7 votes are needed to pass an issue.

a. Identify all winning coalitions and their vote totals.
A,B;7 A,C;7 A,B,C;10 A,B,D;9 A,C,D;9 B,C,D;8 A,B,C,D;12

b. Find the power index for each voter.
A:5, B:3, C:3, D:1

c. Do the power indices reflect the distribution of votes?
No, A's power is disproportionately high, while D's is low.

d. Suppose the number of votes necessary to pass an issue is increased from 7 to 8 . How does this change the power indices of the voters?
All voters now have equal power.

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Classwork

Packet p. 5 #1 and #2
Packet p. 6

Classwork

1.4-1.5 Practice: Approval Voting and Weighted Voting