Unit 6 Day 2
1.3 and 1.4
More Group Ranking Methods
And Approval Voting

Warm Up Day 2

Determine the Plurality, Borda, Runoff, and Sequential Runoff winners. *majority,*
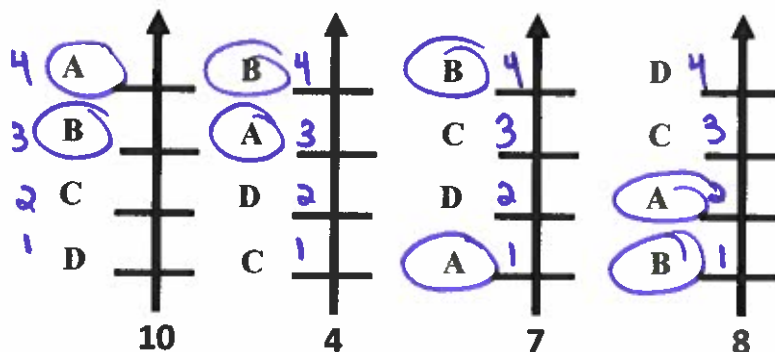
☑ Add majority

A	B	B	D
B	A	C	C
C	D	D	A
D	C	A	B
10	4	7	8

List the following drinks in your order of preference:
Coke, Diet Dr. Pepper, Sprite, Pepsi, Water

Warm Up Day 2

Determine the Plurality, Borda, Runoff, and Sequential Runoff winners.

4	A		4	B		4	B		4	D
3	B		3	A		3	C		3	C
2	C		2	D		2	D		2	A
1	D		1	C		1	A		1	B
	10			4			7			8

List the following drinks in your order of preference:
Coke, Diet Dr. Pepper, Sprite, Pepsi, Water

Total: 29 votes

- Plurality: B
  $(4 + 7 = 11)$
- Majority: None
  $(11 \text{ is} < \frac{1}{2}(29))$
- Runoff : A
  → A & B have most 1st place
  → A: 18
     B: 11

• Borda: A: $10(4) + 4(3) + 7(1) + 8(2) = 75$
B wins
B: $10(3) + 4(4) + 7(4) + 8(1) = 82$ ← B wins
C: $10(2) + 4(1) + 7(3) + 8(3) = 69$
D: $10(1) + 4(2) + 7(2) + 8(4) = 64$

Warm Up Day 2 ANSWERS

Determine the Plurality, Borda, Runoff, and Sequential Runoff winners.

A		B		B		D
B		A		C		C
C		D		D		A
D		C		A		B
10		4		7		8
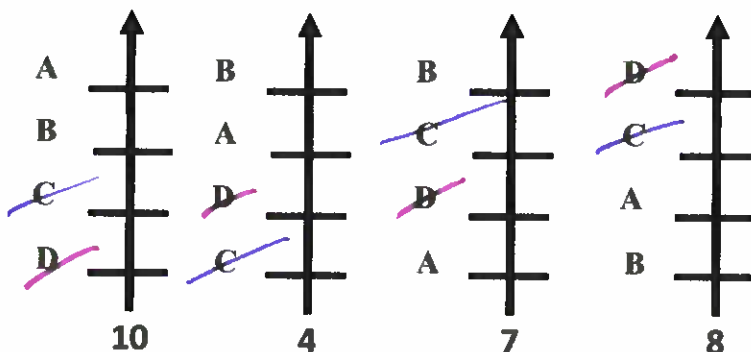
Plurality: B   Borda: B   Runoff: A   Seq. Runoff: A

• Sequential Runoff : A
→ Remove C with no 1st place votes
→ Remove D with only 8 1st place votes
→ Remove B with just 11 1st place votes

HW Questions?

Homework Day 2

- Packet p. 4 - 5
- Front Side of Worksheet ("Worksheet p21-24")

Notes 1.3:
More Group Ranking
Methods and Paradoxes

Pairwise Voting

- Once all of the ballots are submitted, we consider all of the different pairings of two candidates against one another

- If there are three candidates, there are three pairings: A vs. B, A vs. C, and B vs. C

- If there are four candidates, there are six pairings: A&B, A&C, A&D, B&C, B&D, C&D

Condorcet

The Marquis de Condorcet was a friend of Jean-Charles de Borda. He believed that a choice that could obtain a *head-to-head* majority over every other choice should win (using pairwise voting)

8 5 6 7
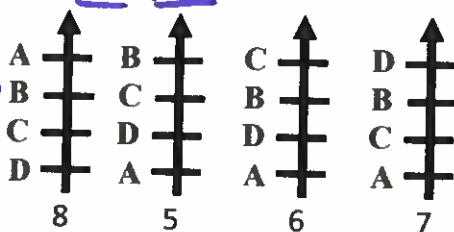
*Make an educated guess for the winner and compare with other candidates.

A vs B (B wins) 8 A vs 18 B
B vs C (B wins) 20 B vs 6 C
B vs D (B wins) 19 B vs 7 D

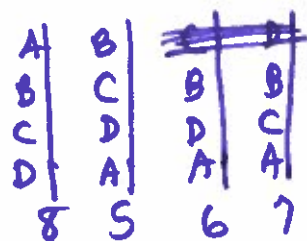Compare each choice with every other choice. Record the wins and losses in a table.

Since __B__ wins *head-to-head* over <u>every</u> other choice, it is the Condorcet winner.
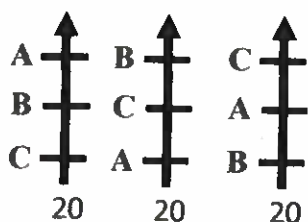
Handwritten margin notes (left):
* tip: when comparing 2 candidates, cover up (with fingers) others that you're not considering

Ex: for A vs B,

A | B | |
B | C | B | B
C | D | D | C
D | A | A | A
8 | 5 | 6 | 7

Handwritten margin notes (right):
can use table to record work

	A	B	C	D
A	X	B	C	D
B	X	X	B	B
C	X	X	X	C
D	X	X	X	X

The Condorcet method has a flaw.

Consider this set of preference schedules.

20 20 20

Example Head-to-head:
A vs B (A wins) 40 A vs 20 B
B vs C (B wins) 40 B vs 20 C
A vs C (C wins) 20 A vs 40 C

} No winner

Condorcet sometimes fails to produce a winner. This is known as a <u>Paradox</u>.

Another Example head-to-head:
A vs B (tie)
B vs C (tie)
C vs D (tie)

} No winner

Group ranking methods may violate the Transitive Property.
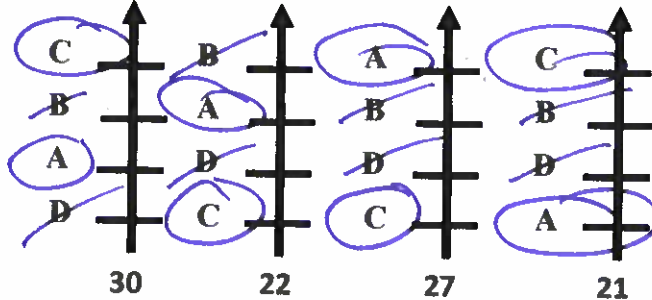
like

Handwritten at bottom:
A > B
B > C
but then A ≯ C

☑ Add majority

Total Votes = 100

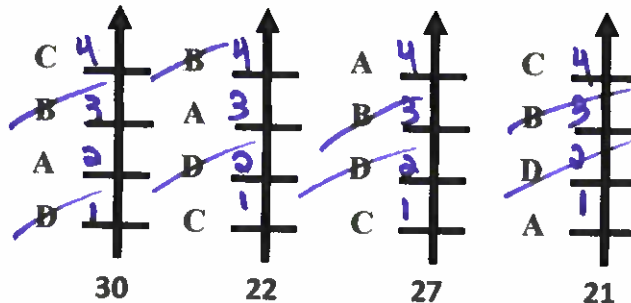**You Try!** Find the winner using **Condorcet,** majority, **plurality, runoff, sequential runoff** and **Borda:**

C	B	A	C
B	A	B	B
A	D	D	D
D	C	C	A
30	22	27	21

Condorcet: C

	A	B	C	D
A	X	B	C	A
B	X	X	C	B
C	X	X	X	C
D	X	X	X	X

B vs A
23   27

A vs C
49   51

A vs D
79   21

B vs C
49   51

B vs D
100   0

C vs D
51   49

C wins vs every other choice

- **Plurality: C**
  → C 51 has most 1st place

- **majority: C**
  → C has over ½ the votes

- **Runoff: C**
  → narrow to A and C with most 1st place & cross off others
  → C: 51
     A: 49

- **Sequential Runoff: C**
  → eliminate D with 0 1st place
  → eliminate B with only 22 1st place
  → eliminate A with only 49 1st place

Find the winner using **Condorcet, plurality, runoff, sequential runoff** and **Borda:**

C 4	B 4	A 4	C 4
B 3	A 3	B 3	B 3
A 2	D 2	D 2	D 2
D 1	C 1	C 1	A 1
30	22	27	21

Condorcet: C
Plurality: C
Runoff: C
Sequential runoff: C
Borda: B

Borda: B
A: 30(2) + 22(3) + 27(4) + 21(1) = 255
B: 30(3) + 22(4) + 27(3) + 21(3) = 322 ←
C: 30(4) + 22(1) + 27(1) + 21(4) = 253
D: 30(1) + 22(2) + 27(2) + 21(2) = 170

Majority: C

Classwork:
Textbook p. 21 – 23
Exercises 4, 5, 7, 8, 9, 10

These are problematic scenarios. Be prepared to describe and discuss with the class the problems that can arise from the various group ranking methods.

Notes 1.4:
Arrow's Conditions &
Approval Voting

Arrow's 5 Conditions Necessary for a Fair Group Ranking Method

Kenneth Arrow is an American economist and mathematician. He gained worldwide recognition for his mathematical applications to election theory.

The many paradoxes in election methods led Mr. Arrow to formulate a list of conditions he thought were necessary for a group ranking to be fair.
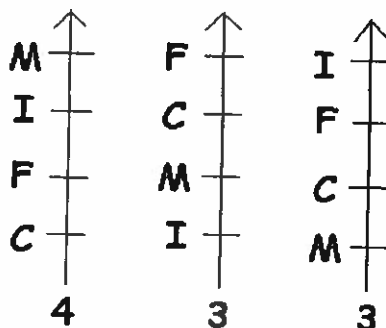

The Tech Established 1881 **ONLINE EDITION**

Take a few minutes to read this article.
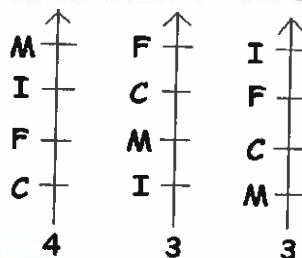
https://tinyurl.com/hex8ven

Ten representatives of the language clubs at Central High School are meeting to select a location for the clubs annual joint dinner. They must choose between a Chinese, French, Italian, or Mexican restaurant.

```
  M ↑      F ↑      I ↑
  I        C        F
  F        M        C
  C        I        M
    4        3        3
```

$I > F$ ←

Racquel suggests that because the last 2 dinners have been held at Mexican and Chinese restaurants, this year's dinner should be at either an Italian or French restaurant. They vote 7 to 3 in favor of the Italian restaurant.

$M > I$ ←

Martin doesn't like Italian food and says that the new Mexican restaurant is really good. He proposes that the group choose between Italian and Mexican. They voted 7 to 3 to hold the dinner at the Mexican restaurant.

$C > M$ ←

Sarah's parents own a Chinese restaurant and say that she can get a group discount. The group votes between the Mexican and Chinese restaurant and selects the Chinese restaurant by a 6 to 4 margin.

```
  M ↑      F ↑      I ↑
  I        C        F
  F        M        C
  C        I        M
    4        3        3
```

This is an example of **Pairwise Voting** and Mr. Arrow considers this group ranking method to be flawed.

* If we look back at their original preferences, we see that French food was preferred to Chinese food in every case, yet they voted for Chinese food.

chinese BUT   F > C on all preference schedules

⟹ PARADOX

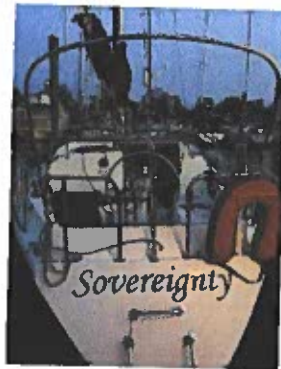# Arrow's 5 Conditions Necessary for a Fair Group Ranking Method

1. Non-Dictatorship

- The preference of a single individual should not become the group ranking without considering the preferences of others.

2. Individual Sovereignty

- Each individual should be allowed to order the choices in any way and to indicate ties.

3. Unanimity

- If everyone prefers one choice over another, then the group ranking should do the same.
- Example:
  - If every voter ranks candidate A higher than candidate B, then the final ranking should place candidate A higher than candidate B.

"Then we are agreed nine to one that we will say our previous vote was unanimous!"
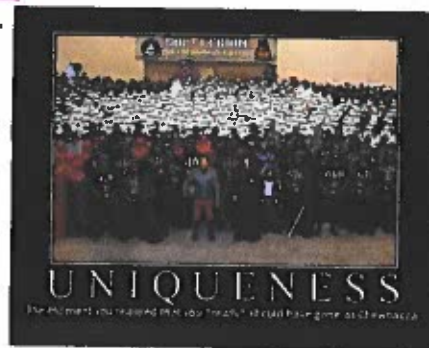
4. Freedom from Irrelevant Alternatives

- The winning choice should still win if one of the other choices is removed.
- The choice that is removed is known as an irrelevant alternative.

5. Uniqueness of the Group Ranking

- The method of producing the group ranking should give the same result whenever it is applied to a given set of preferences.

UNIQUENESS

Exercise 1

- Your teacher decides to order drinks for the class based on the vote just conducted. In doing so, she selects Hannah's preference schedule because she likes the drinks she chose. Which of Arrow's conditions are violated by this method of determining a group ranking?

Non-Dictatorship

Exercise 2

- Instead of selecting the preference schedule of a single student, your teacher places all of the individual preferences in a hat and draws one. If this method were repeated, would the same group ranking result? Which of Arrow's conditions does this violate?

Uniqueness of the Group Ranking

we would not get same result each time

Exercise 3

- Do any of Arrow's condition's require that the voting mechanism include a secret ballot? Is a secret ballot desirable in all group ranking situations? Explain why or why not.

No.

Ex: You might want to know other's thoughts so your vote isn't "wasted" by picking a choice very few would pick.

Approval Voting:

- Kenneth Arrow proved that **no method**, known or unknown, could always obey **all 5 conditions**.

  (Any group-ranking method will violate at least one of Arrow's conditions in certain situations)

- Although a perfect group ranking will never be found, current methods can still be improved.

- A new system is called **Approval Voting:**

Soft Drink Ballots

Do the soft drinks vote again, but this time use Approval Voting.

Your ballot still has these soft drinks listed.

Coke, Diet Dr. Pepper, Sprite, Pepsi, Water

Place an *X* beside each of the soft drinks you find acceptable.

Tally the drinks you approve on the board.

Determine the group ranking.

Was the winner the same as with any of the other group ranking methods from before?

Approval Voting

In Approval Voting,

you may vote for as many choices as you like, but

you do not rank them.

You mark all those of which you approve.

For example, if there are five choices, you may vote for

as few as none or as many as five.

Advantages of Approval Voting?

- It gives voters more flexible options
- It reduces negative campaigning
- It increases voter turnout
- It give minority candidates their proper due

What are some disadvantages?

- Approval voting forces voters to cast equally weighted votes for candidates they approve of.
- Voting for your second choice candidate can in some cases lead to the defeat of your favorite candidate.

Approval Voting Practice

The participants in a summer school recreation program decided to vote on which activity they preferred, Running Track, Softball, Badminton, or Swimming. The winning activity was determined by Approval Voting.

The following summarizes the responses of the participants:
12 participants voted for Swimming and Badminton.
5 participants voted for Badminton, Running Track, and Softball.
10 participants voted for Running Track and Badminton.
13 participants voted for Softball and Badminton.

1. How many total votes did **Swimming** receive?   **12**
2. How many total votes did **Badminton** receive?   **40**   (12+5+10+3)
3. How many total votes did **Running Track** receive?   **15**   (5+10)
4. How many total votes did **Softball** receive?   **18**   (5+13)
5. Which activity is selected by the summer school participants using Approval Voting?   **Badminton**

You Try! Frisbee Club members decided to let the participants vote on the color of the T-shirt, using Approval Voting. The possible colors are Steel Gray, Robin's Egg Blue, Eggshell, Candy Apple Red, and Sunflower Yellow.

Approval Voting Practice

Here is a summary of the results:

12 participants voted for Steel Gray.

7 participants voted for Steel Gray and Sunflower Yellow.

20 participants voted for Eggshell and Candy Apple Red.

18 participants voted for Robin's Egg Blue, Eggshell, and Candy Apple Red

23 participants voted for Sunflower Yellow and Robin's Egg Blue.

25 participants voted for Candy Apple Red.

Use Approval Voting to determine the color of the t-shirt.

Candy Apple Red wins with 63 votes

(handwritten:)
Eggshell: 38
(20+18)
Candy Apple Red: 63
(20+18+25)
Steel Gray: 19
(12+7)
Sunflower Yellow: 3̶
(7+23)
Robin's Egg Blue: 41
(18+23)

Example: Determine the winner by the Condorcet Method

Place	Number of votes received			
	390	360	300	450
1st	Shawn	Gail	Gail	Ricco
2nd	Twanda	Twanda	Twanda	Twanda
3rd	Ricco	Ricco	Shawn	Shawn
4th	Gail	Shawn	Ricco	Gail

(handwritten table:)

	S	T	R	G
S	X	T	R	S
T	X	X	T	T
R	X	X	X	R
G	X	X	X	X

Twanda ☺

(she wins versus every other choice)

Homework Day 2

- Packet p. 4 - 5
- Front Side of Worksheet ("Worksheet p21-24")

Classwork:

PBS Mathline
Activity 3: Pairwise Comparisons

(That's another term for the Condorcet Method)

Classwork:

PBS Mathline
Activity 4: Approval Voting