

KEY

Unit 6 Day 1

1.1 and 1.2

Plurality, Borda Method, Runoff Method, & Sequential Runoff

Grab a Unit 6 Election Theory Packet!

Write down tonight's HW:
Packet p. 1-3



Warm-Up: Homecoming King and Queen Elections

You have been chosen to serve on the committee that decides who this year's Homecoming King and Queen will be. As a committee, you have already determined the three sets of finalists to be, in no particular order, Alan and Alice, Bob and Betty, and Carl and Cathy. Please note that all finalists are seniors. Furthermore, you have already held elections through class meetings and have collected the following results:

	<u>Freshmen</u>	<u>Sophomores</u>	<u>Juniors</u>	<u>Seniors</u>
1st	Alan/Alice	Bob/Betty	Carl/Cathy	Carl/Cathy
2nd	Bob/Betty	Alan/Alice	Bob/Betty	Bob/Betty
3rd	Carl/Cathy	Carl/Cathy	Alan/Alice	Alan/Alice
class size	60 students	50 students	40 students	30 students

You'll look at how the couples should be ranked as an individual and a small group, then as a class....

Warm-Up: Homecoming King and Queen Elections

	<u>Freshmen</u>	<u>Sophomores</u>	<u>Juniors</u>	<u>Seniors</u>
1st	Alan/Alice	Bob/Betty	Carl/Cathy	Carl/Cathy
2nd	Bob/Betty	Alan/Alice	Bob/Betty	Bob/Betty
3rd	Carl/Cathy	Carl/Cathy	Alan/Alice	Alan/Alice
class size	60 students	50 students	40 students	30 students

On your own:

1. In your opinion, which couple should be Homecoming King and Queen? Who would finish 2nd and 3rd? Justify your answer.

In your group:

2. Compare the results within your group. Does everyone have the same result? Discuss your reasoning. Were there reasons that you did not take into account? Do you feel that these reasons are valid?

3. As a group, come to consensus as to which couple should finish 1st, 2nd, and 3rd. Explain, in detail, the method your group used in determining this order.

Decision Making is an important part of life.

You will make many important individual decisions.

But, in our society we make many decisions as a group.

So, how are the wishes of many individuals combined to yield a single result?

Examples of Group Decision Making:

1. Political Offices
2. Nielson TV Ratings
3. Heisman Trophy
4. Olympics Venue

Are Group Decisions FAIR?

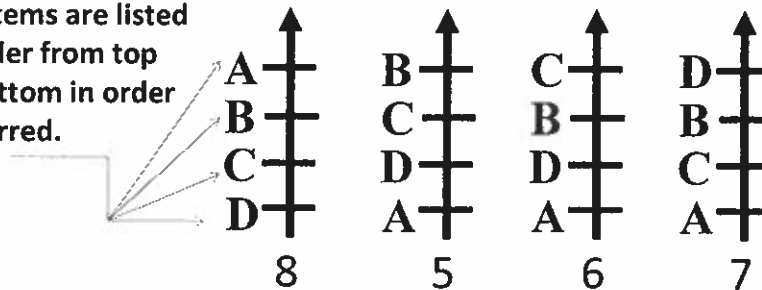
Unit 6 Notes Day 1:
Election Methods

Preference Schedules

- A way to represent the preferences of one or more individuals.

- Ex.

The items are listed in order from top to bottom in order preferred.



$$\text{Total \# of voters} = 8 + 5 + 6 + 7 = 26$$

Preference Schedules

When your class members voted, they ranked the couples from first through third.

However, voters in most U.S. elections do not get to rank candidates. Do you think allowing voters to rank candidates would be a good practice? Explain.

Preference Schedules

- How many preference schedules are possible if there are 4 choices?

$4!$ or $4 \times 3 \times 2 \times 1 =$
24 total preference schedules

- If there are 5 choices? 6 choices? 7 choices?

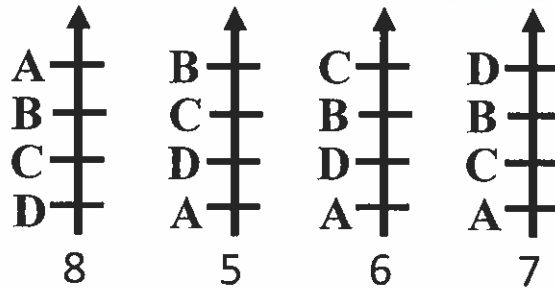
$5! = 120$ $6! = 720$ $7! = 5040$

1.2: Group Ranking Methods and Algorithms



Plurality

Winner is determined by who has the most 1st place votes



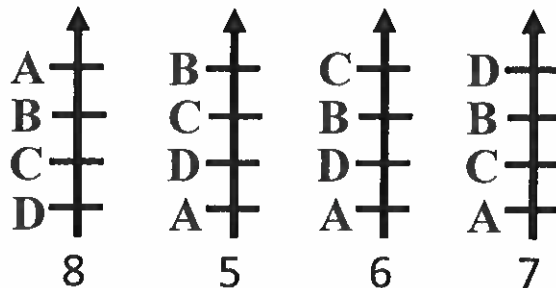
The Plurality winner is A with 8 first-place votes.

Notice that's only 30.8% of the votes (8 out of 26).

Majority

Candidate with over $\frac{1}{2}$ the 1st place votes wins

– There is not always a majority winner



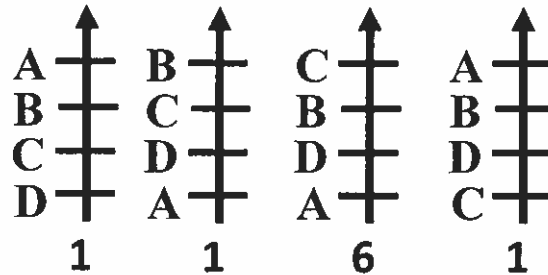
majority
winner
here!
None

– How many votes would be needed for there to be a majority winner?

14

(26 total
so 13 is only 50%...
need > 50%)

Example:



9 total voters

C is a plurality winner and majority winner.

Borda Method

Assigning points to develop a ranking is called the BORDA Method or BORDA Count.

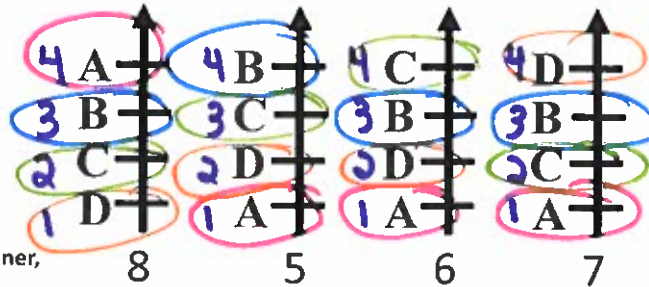
It is named for Jean-Charles de Borda, a French cavalry officer, naval captain, mathematician and scientist. He preferred a method that assigned points to rank individuals because he was dissatisfied with the plurality method.

- Determine the winner by assigning point values to 1st, 2nd, 3rd, and 4th place votes.
- With 4 places it will look like this:
 - 1st place vote → 4 points
 - 2nd place vote → 3 points
 - 3rd place vote → 2 points
 - 4th place vote → 1 point



Heisman winners are determined with a method like the Borda method

Ex 1: Borda Count

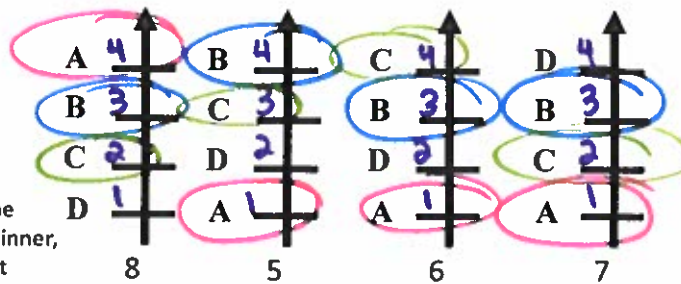


Notice: The plurality winner, A, does not bode well here...

$$\begin{aligned}
 \text{A: } & 4(8) + 1(5) + 1(6) + 1(7) = 30 \\
 \text{B: } & 3(8) + 4(5) + 3(6) + 3(7) = 83 \\
 \text{C: } & 2(8) + 3(5) + 4(6) + 2(7) = 69 \\
 \text{D: } & 1(8) + 2(5) + 2(6) + 4(7) = 58
 \end{aligned}$$

B wins with Borda method

Ex 1 ANSWERS: Borda Count



Notice: The plurality winner, A, does not bode well here...

$$\begin{aligned}
 \text{A: } & 8(4) + 5(1) + 6(1) + 7(1) = 50 \\
 \text{B: } & 8(3) + 5(4) + 6(3) + 7(3) = 83 \\
 \text{C: } & 8(2) + 5(3) + 6(4) + 7(2) = 69 \\
 \text{D: } & 8(1) + 5(2) + 6(2) + 7(4) = 58
 \end{aligned}$$

B wins with Borda method

EX. 2 YOU TRY!

Place	Number of votes received			
	390	360	300	450
1 st	4 Shawn	4 Gail	4 Gail	4 Ricco
2 nd	3 Twanda	3 Twanda	3 Twanda	3 Twanda
3 rd	2 Ricco	2 Ricco	2 Shawn	2 Shawn
4 th	1 Gail	1 Shawn	1 Ricco	1 Gail

votes total = 1500

Using the Borda Count Method, determine the total number of points awarded to the following candidates in this election

- Shawn $4(390) + 1(360) + 2(300) + 2(450) = 3420$
- Gail $1(390) + 4(360) + 4(300) + 1(450) = 3480$
- Twanda $3(390) + 3(360) + 3(300) + 3(450) = 4500$
- Ricco $2(390) + 2(360) + 1(300) + 4(450) = 3600$

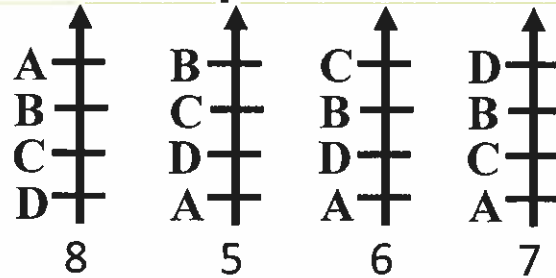
- Using the Borda Count Method, who wins this election? Twanda with 4500 pts
- Who is the plurality winner? Why? Gail; she has 660 1st place votes → the most 1st place votes wins with plurality method
- Who is the majority winner? Why?

None. Gail has 660 1st place votes, but need over half the 1st place votes to win majority, so someone needs >750 votes to win by majority.

Runoff

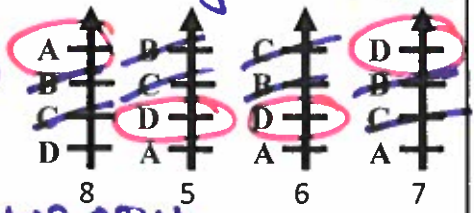
- Often used when there is no majority winner.
- Many elections require a majority winner. If there is no majority winner, a run-off election between the top two candidates is held.
- To conduct a runoff, determine the number of firsts for each choice.
- Then narrow the selection to the top TWO candidates.
- **Negative Aspects:**
 - Time consuming and costly.
 - Lower voter turnout the second time around.

Example: Runoff



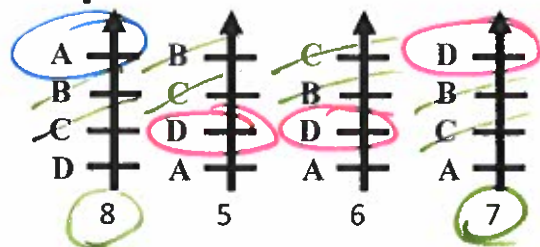
26 voters

- Is there a majority winner? **NO. No one got > 1/2 the votes.**
- Who are the top two candidates? **A (8^{1st} place) + D (7^{1st} place)**
- Eliminate the other candidates and compare again! **Cross off B and C... then see who is on top now.**
- Now A has 8 1st place votes and D has 18 1st place votes, so the winner is **D**!



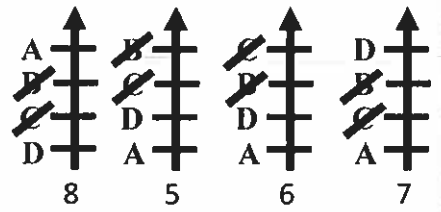
↑
5+6+7

Example ANSWERS: Runoff



A: 8 1st place
D: 18 1st place

- Is there a majority winner? **No**
- Who are the top two candidates? **A with 8 first place votes and D with 7 first place votes**
- Eliminate the other candidates and compare again!
- Now A has 8 1st place votes and D has 18 1st place votes, so the winner is **D**!



USE EX. 2 FROM EARLIER...

Place	Number of votes received			
1 st	Shawn	Gail	Gail	Ricco
2 nd	Tyanda	Tyanda	Tyanda	Tyanda
3 rd	Ricco	Ricco	Shawn	Shawn
4 th	Gail	Shawn	Ricco	Gail

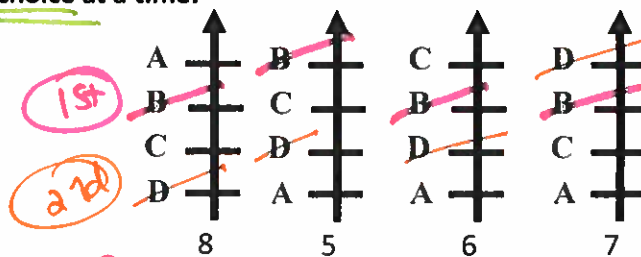
FIND THE RUNOFF WINNER

At start, Gail has 660 1st place + Ricco has 450 1st place
 ↳ eliminate Shawn + Tyanda.

See who is now on top in each preference schedule.
 → Ricco now has 840 (390+450) 1st place votes
 → Gail still has only 660 (360+300) 1st place votes
 Ricco wins with 840 votes!

Sequential Runoff

Some elections, such as the voting to determine the site for the Olympic Games, are conducted by a variation of the runoff method that eliminates one choice at a time.



1st
2nd

Eliminate B because it has the least amount of 1st place votes. Then reevaluate.

B has only 5 1st place

Now D has the least amount of 1st place votes. (After B is eliminated, C has 11 1st place votes) So, eliminate D.
 Reevaluate again.

D with only 7 1st place votes

Lastly A has the fewest 1st place votes. So, C is the winner.

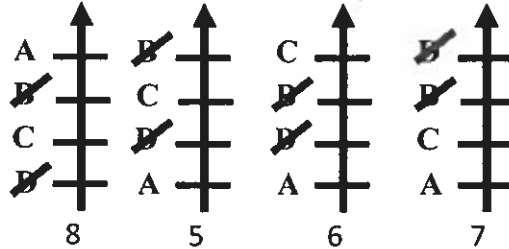
Fix 'D' →

only 8 1st place votes

18 (5+6+7) 1st place votes

Sequential Runoff ANSWER

Some elections, such as the voting to determine the site for the Olympic Games, are conducted by a variation of the runoff method that eliminates one choice at a time.



Eliminate B because it has the least amount of 1st place votes. Then reevaluate.

Now D has the least amount of 1st place votes. (After B is eliminated, C has 11 1st place votes) So, eliminate D.
Reevaluate again.

Lastly A has the fewest 1st place votes. So, C is the winner.

~~Fix~~

FIND THE SEQUENTIAL RUNOFF WINNER

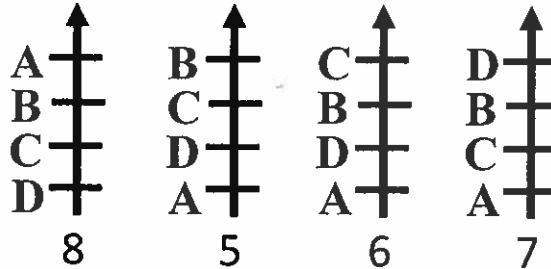
	Number of votes received			
Place	130	120	150	100
1 st	Grizzlies	Indians	Broncos	Jets
2 nd	Hoosiers	Broncos	Hoosiers	Grizzlies
3 rd	Broncos	Jets	Grizzlies	Hoosiers
4 th	Indians	Grizzlies	Jets	Indians
5 th	Jets	Hoosiers	Indians	Broncos

Be careful

- 1st Eliminate Hoosiers with 0 1st place votes
 - 2nd Eliminate Jets with 100 1st place votes
 - 3rd Eliminate Indians with just 120 1st place votes
 - 4th Eliminate Grizzlies with just 230 1st place votes
- Broncos win with 270 votes!

Bringing it all together...

So who was the *real* winner?



- Plurality Winner: A
- Majority Winner: None
- Borda Winner: B
- Runoff Winner: D
- Sequential Runoff Winner: C
- ????



Do
* Kahoot practice here!

**HOMEWORK:
PACKET P.1-3**