Name: ☺ Notes Friday 3/1/19 ☺ Per: ___

Summary & Practice: Rational Functions

A rational function is an equation that can be written as a ratio (a fraction)

Types of Discontinuities of Rational Functions

1) Holes (,)

also called removable discontinuities

Step 1) Factor top and bottom

2) slash out any common factors

3) Find root of slashed factor (in other words, set slashed factor = 0 and solve)

gives x-value of hole

To find the y-value for the hole, substitute the x-value into the remaining equation (after factoring and crossing out shared factors)

2) Vertical Asymptotes

Are written as $x = \#$

Step 1) Factor top and bottom

2) slash out any common factors

2) Find root of denominator (in other words, set remaining denom. = 0 and solve)

* non-removable discontinuity

* infinite discontinuity

3) Horizontal Asymptotes

Are written as $y = \#$

There are 3 scenarios) 1st, Find degree of top and bottom

a) small degree / large degree -> y = 0

b) same degree -> y = ratio of leading coefficients

c) large degree / small degree -> none

You Try

Rational Functions Practice

Remember to show work with Algebra for credit! ☺

For each problem:

a) Find holes, vertical asymptotes, and horizontal asymptotes.

b) Find domain, x-intercept, and y-intercept.

1) $f(x) = \dfrac{x^2 - 4}{x - 2}$

2) $f(x) = \dfrac{(x^2 - 3x - 10)}{(3x^2 - 11x - 20)}$

You Try

3) $f(x) = \dfrac{x - 3}{x^2 - 9}$

$f(x) = \dfrac{\cancel{(x-5)}(x+2)}{(3x+4)\cancel{(x-5)}} \rightarrow y = \dfrac{x+2}{3x+4}$ } leftover

* use leftover to get x-int + y-int

Hole $\left(5, \frac{7}{19}\right)$ $\quad \dfrac{5+2}{3(5)+4} = \dfrac{7}{19}$

VA $x = -4/3$ $\quad 3x+4=0$, $3x=-4$

D: $(-\infty, -4/3) \cup (-4/3, 5) \cup (5, \infty)$

$-4/3$ $\quad 5$

$0 = \dfrac{x+2}{3x+4}$ $\quad 0 = x+2$

$(-2, 0)$ x-int

$y = \dfrac{0+2}{3(0)+4} = \dfrac{2}{4}$

$\left(0, \frac{1}{2}\right)$ y-int

Day 3

* Show work with Algebra for credit

4. Find the vertical asymptotes, if any, of the graph of the rational function.

$f(x) = \dfrac{3}{x^2 - 3x - 4}$

- x = 0
- no vertical asymptotes
- x = 4 and x = -1
- x = 4 and y = -1

5. Find the all the asymptotes, if any, of the graph of the rational function.

$f(x) = \dfrac{x^3 - 1}{x^2 - 9}$

- A. y = 0, x = 3, x = 0
- B. x = 3, x = -3
- C. y = x, y = 0
- D. y = x, x = 3, x = -3

6. Find the all the asymptotes, if any, of the graph of the rational function.

$f(x) = \dfrac{x^3 - 27}{x^2 - 9}$

- A. y = 0, x = 3, x = 0
- B. x = 3, x = -3
- C. x = 3
- D. x = -3

7. Find the location of all of the removable discontinuities, if any, of the graph of the rational function.

$f(x) = \dfrac{x^3 - 27}{x^2 - 9}$

- A. x = 3
- B. x = -3
- C. x = -27
- D. none

8. Find the horizontal asymptotes, if any, of the graph of the rational function.

$f(x) = \dfrac{2x^2}{x^2 + 4}$

- A. x = 2
- B. y = 0
- C. y = 2
- D. no horizontal asymptotes

Name: ____________ Per: ____

Summary & Practice: Rational Functions

A ________ ________ is an equation that can be written as a ________ (a fraction)

Types of Discontinuities of Rational Functions

1) Holes

also called

Step 1) ____________ top and bottom

2) ________ ________ any common factors

3) Find root of slashed factor
(in other words, set ________ ________ = 0 and solve)

To find the y-value for the hole, substitute the x-value into the remaining equation (after factoring and crossing out shared factors)

2) Vertical Asymptotes

Are written as

____ = #

Step 1) ____________ top and bottom

2) ________ ________ any common factors

2) Find root of denominator
(in other words, set remaining ________ = 0 and solve)

3) Horizontal Asymptotes

Are written as

____ = #

There are 3 scenarios) 1st, Find degree of top and bottom

a) small degree / large degree -> y = ____

b) same degree -> y = ________ ________

c) large degree / small degree -> ________

Rational Functions Practice Remember to show work with Algebra for credit! ☺

For each problem:

a) Find holes, vertical asymptotes, and horizontal asymptotes.
b) Find domain, x-intercept, and y-intercept.

1) $f(x) = \frac{x^2 - 4}{x - 2}$

Hole: (2,4)
VA: none
HA: none
D: $(-\infty,2)\cup(2,\infty)$
x-int: (-2,0) * not one at (2,0) because hole at (2,4)
y-int: (0,2)

Day 3

2) $f(x) = \frac{x^2 - 3x - 10}{3x^2 - 11x - 20}$

we'll learn about H.A. in a minute

3) $f(x) = \frac{x - 3}{x^2 - 9}$

Hole at (3, 1/6)
VA: x = -3
HA: y = 0
D: $(-\infty,-3)\cup(-3,3)\cup(3,\infty)$
x-int: none * not one at (3,0) because hole at (3, 1/6)
y-int: (0, 1/3)

*Show work with Algebra for credit

4. Find the vertical asymptotes, if any, of the graph of the rational function.

$f(x) = \frac{3}{x^2 - 3x - 4}$

- x = 0
- no vertical asymptotes
- x = 4 and x = -1
- x = 4 and y = -1

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$f(x) = \frac{x^3 - 1}{x^2 - 9}$

A. y = 0, x = 3, x = 0
B. x = 3, x = -3
C. y = x, y = 0
D. y = x, x = 3, x = -3

6. Find the all the asymptotes, if any, of the graph of the rational function.

$f(x) = \frac{x^3 - 27}{x^2 - 9}$

A. y = 0, x = 3, x = 0
B. x = 3, x = -3
C. x = 3
D. x = -3

7. Find the location of all of the removable discontinuities, if any, of the graph of the rational function.

$f(x) = \frac{x^3 - 27}{x^2 - 9}$

A. x = 3
B. x = -3
C. x = -27
D. none

8. Find the horizontal asymptotes, if any, of the graph of the rational function.

$f(x) = \frac{2x^2}{x^2 + 4}$

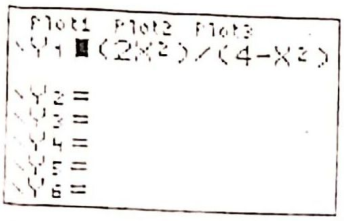
A. x = 2
B. y = 0
C. y = 2
D. no horizontal asymptotes

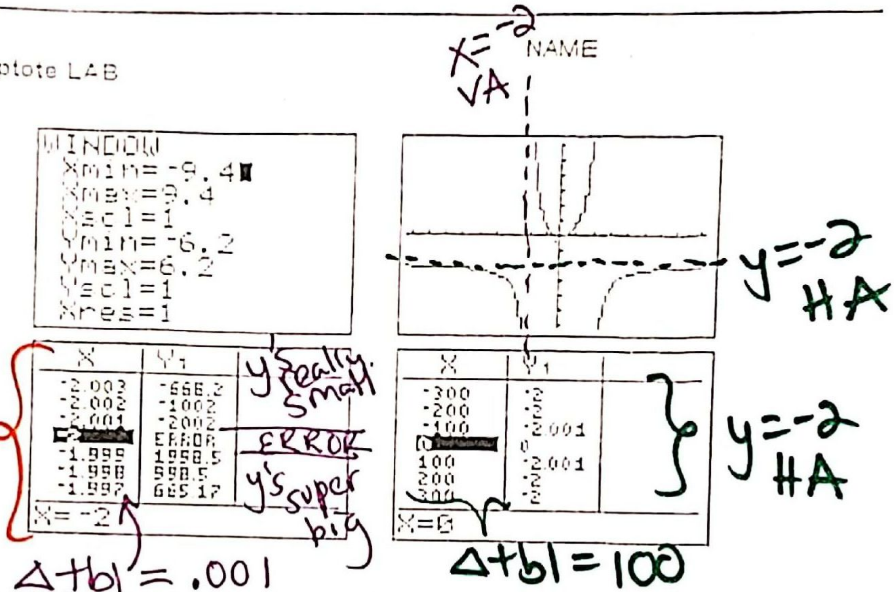
Asymptote Lab Classwork Day 3

PRE – CALCULUS
Graphing Calculator Asymptote LAB

NAME

Enter equations in Y_1.
Set window as indicated.





Examine behavior of horizontal and vertical asymptotes using features of the table.

x=-2 is VA Δtbl = .001

Examine and write equations for the horizontal asymptotes, vertical asymptotes or holes in each of the functions below. If none exist, write none. Look for patterns in the types of asymptotes that occur so you can answer the questions on the next page.

1. f(X)= $\frac{3X^2-1}{2X^2+1}$	2. f(x)= $\frac{3X}{X^2+1}$	3. f(x)= $\frac{x^2-4}{x+2}$	4. f(x)= $\frac{x+2}{x^2-4}$
v.a	v.a	v.a	v.a
hole:	hole:	hole:	hole:
h.a $y=3/2$	h.a $y=0$	h.a None	h.a $y=0$
5. f(x)= $\frac{x}{x^2-9}$	6. f(x)= $\frac{5x^2-9}{3x^2-3}$	7. f(x)= $\frac{3x^4}{x^2-16}$	8. f(x)= $\frac{x^3+1}{x+3}$
v.a	v.a	v.a	v.a
hole:	hole:	hole:	hole:
h.a $y=0$	h.a $y=5/3$	h.a None	h.a None
9. f(x)= $\frac{4x^3-3x^2+2x}{x^3-8}$	10. f(x)= $\frac{4x^2-12x+9}{(2x+3)^2}$	11. f(x)= $\frac{x^2-6x+9}{x-3}$	12. f(x)= $\frac{4}{x^3+8}$
v.a	v.a	v.a	v.a
hole:	hole:	hole:	hole:
h.a $y=4$	h.a $y=1$	h.a None	h.a $y=0$

Graphing Rational Functions

❶ Always FACTOR and cancel first	
❷ x value at hole	x from <u>cancelled</u> factor(s)
❸ y value at hole	Plug hole's x value into "REDUCED FUNCTION"
❹ vertical asymptote (VA)	x from <u>remaining</u> factor(s) in **denominator**
❺ x-intercept	x from <u>remaining</u> factor(s) in **numerator**
❻ y-intercept	<u>Plug 0</u> into x in "REDUCED FUNCTION"
❼ end behavior asymptote (EBA)	Is x "bigger" in numerator or denominator? (see examples at bottom)

Example:

❶ factor and cancel

$$f(x) = \frac{x^2 - x - 20}{x^2 + 5x + 4} \rightarrow \frac{(x \cancel{+ 4})(x - 5)}{(x \cancel{+ 4})(x + 1)} \rightarrow$$

"REDUCED FUNCTION"

$$\boxed{\frac{(x - 5)}{(x + 1)}}$$

❷&❸ hole at (-4, 3)

$$\boxed{\frac{(-4 - 5)}{(-4 + 1)} = 3}$$

❹ VA : x = -1

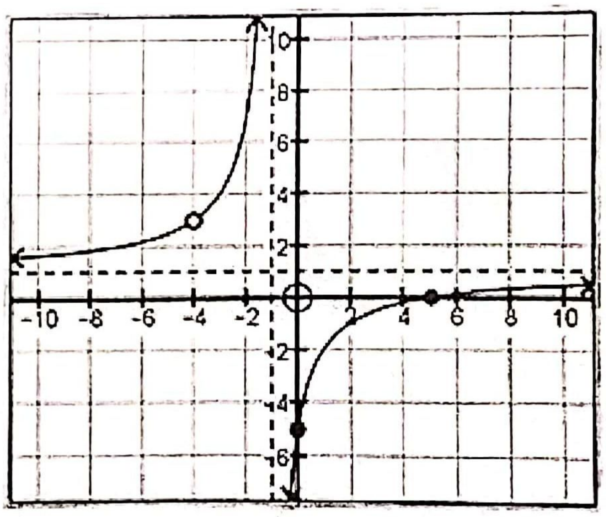
❺ x-intercept: (5, 0)

❻ y-intercept: (0, -5)

$$\boxed{\frac{(0 - 5)}{(0 + 1)} = -5}$$

❼ EBA: y = 1

$$\boxed{\frac{(x - 5)}{(x + 1)} = 1}$$



← END BEHAVIOR ASYMPTOTE (EBA) EXAMPLES →

$\frac{x^2 + 1}{x^2 + 7}$	$\frac{2x + 1}{x + 7}$	$\frac{x + 1}{2x + 7}$	$\frac{x + 1}{x^2 + 7}$
EBA: Y = 1	EBA: Y = 2	EBA: Y = 1/2	EBA: Y = 0

End Behavior can help you to determine HA.

Day 2/3