Midterm Review **Honors ICM** **Name:** Key **Per:** ___ p 1

Unit 1 Sets and Probability

* Remember to show work for credit!

1. Let **U** denote the set of all the students at Green Hope High. Let

$$D = \{x \in U \mid x \text{ has a driver's license}\} \qquad F = \{x \in U \mid x \text{ attends GHHS football games}\}$$

Write the following descriptions using symbol notation.

A. Has a driver's license and attends GHHS football games: $D \cap F$

B. Has a driver's license or attends GHHS football games: $D \cup F$

C. Has a driver's license, but does not attend GHHS football games: $D \cap F^c = F^c \cap D$

D. Does not have a driver's license nor attends GHHS football games: $(D \cup F)^c$ or $D^c \cap F^c$

2. Let **E** and **F** be two events that are mutually exclusive and suppose P(E) = 0.4 and P(F) = 0.5.

a. **Draw a Venn Diagram** for these events. Then compute the following:

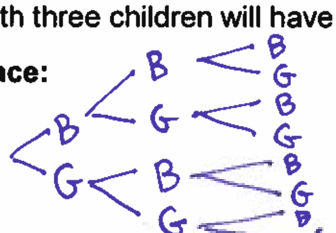
no overlap

b. $P(E \cup F) = 0.9$ c. $P(F^c) = 0.5$ not F $P(E) + P(\text{neither})$ $0.4 + 0.1$

d. $P(E \cap F) = 0$ ↳ "mutually exclusive" e. $P(E \cap F^c) = 0.4$ E and not F

3. Assume that the probability of a boy being born is the same as the probability of a girl being born. Find the probability that a family with three children will have **girls as the youngest two** children.

Write the sample space:



BBB BBG
BGB BGG
GBB GBG
GGB GGG

2/8

Probability: 1/4

4. The figure below shows the counts of earned degrees for several colleges on the East Coast. The level of degree and the gender of the degree recipient were tracked. Row & Column totals are included.

	Bachelor's B	Master's A	Professional P	Doctorate D	Total
Female F	542	128	26	18	714
Male M	438	165	38	20	661
Total	980	293	64	38	1375

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

Let F = Female, M = Male, B = Bachelor's, A = Master's, P = Professional, and D = Doctorate. Find the number of people in each of the following sets.

(a) $F \cap D =$ 18
Female + Doctrate

(b) $F \cup (B \cup A)^c =$ 772
Female or not bachelor's nor masters
714 + 64 + 38 − 26 − 18

(c) $(B \cup A) \cap M =$ 603
Bachelor's or Masters and Male
438 + 165

(d) Of those that are female, what is the probability earned a Doctorate? (fraction) 3/119
714 female total
$P(F \cap D) / P(F) = 18/714$

(e) Find $P(\text{~~Doctorate~~} \mid \text{~~Male~~})$ (fraction) 10/19

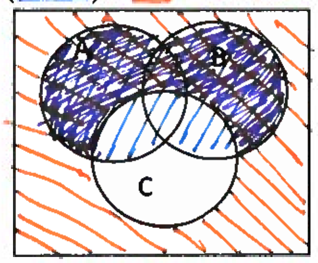
$$P(\text{Male} \mid \text{Doctorate}) = \frac{P(\text{Male} + \text{Doctorate})}{P(\text{Doctorate})} = \frac{20}{38}$$

Unit 1 continued

p 2

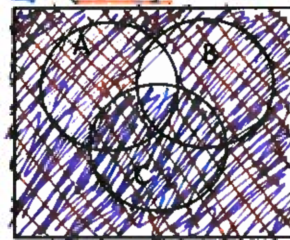
5. Given sets A, B, and C as shown, shade the following:

$(A \cup B) \cap C^c$ = A or B but not in C



$= (A \cup B) \cap C^c$

$C \cup (A \cap B)^c$ = C or not in the intersection of A and B



$= C \cup (A \cap B)^c$

6. Ms. Terrell is redecorating her office. She has a choice of 9 colors of paint, 3 kinds of curtains, 7 colors of carpet, and 2 styles of furniture. How many different ways are there to redecorate if she can choose two different colors of paint, one kind of curtain, two colors of carpet, and one style of furniture?

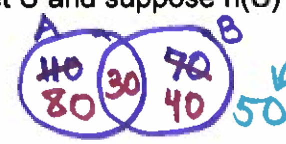
$\underbrace{{}_9C_2}_{\text{Paint}} \cdot \underbrace{{}_3C_1}_{\text{Curtain}} \cdot \underbrace{{}_7C_2}_{\text{carpet}} \cdot \underbrace{{}_2C_1}_{\text{furniture}} = 36 \cdot 3 \cdot 21 \cdot 2 = \boxed{4{,}536}$

7. Let S = $\{s_1, s_2, s_3, s_4\}$ be the sample space associated with an experiment having the probability distribution shown in the accompanying table. If A = $\{s_1, S_3\}$ and B = $\{s_1, S_4\}$ find $P(A^c)$. $= P(S_2 \cup S_4)$

Outcome	s_1	s_2	s_3	s_4
Probability	3/10	1/5	1/4	1/4
	in A	not in A	in A	not in A

$= 1/5 + 1/4 \; \boxed{\frac{9}{20}} = \boxed{.4}$

8. Let A and B be subsets of a universal set U and suppose n(U) = 200, n(A) = 110, n(B) = 70, and $n(A \cap B) = 30$. Compute $n(A^c \cap B^c)$.



200-80-30-40

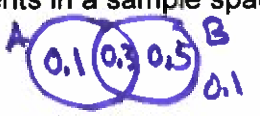
$n(A^c \cap B^c) = n(A \cup B)^c = \boxed{50}$

200-80-30-40

9. Smith & Jones, an accounting firm, employs 16 accountants, of whom 6 are CPAs. If a delegation of ~~3~~ 2 accountants is randomly selected from the firm to attend a conference, what is the probability that 2 CPAs will be selected? Round your answer to the nearest thousandth.

$\frac{6}{16} \cdot \frac{5}{15} = \frac{1}{8} = \boxed{0.125}$

10. Let A and B be events in a sample space S such that P(A) = 0.4, P(B) = 0.8, and $P(A \cap B) = 0.3$. Find $P(A|B)$.



$\boxed{0.375}$

$P(A|B) = \frac{P(A+B)}{P(B)} = \frac{P(A \cap B)}{P(B)} = \frac{0.3}{0.8} \; \boxed{\frac{3}{8}}$

11. How many different 11 letter arrangements of the letters in *Connecticut* are possible?

$11!/(3! \cdot 2! \cdot 2!) = \frac{39916800}{24} = \boxed{1{,}663{,}200}$

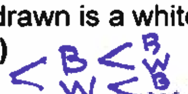
12. According to the Gallop Poll, the probability that a randomly chosen American is an Independent is ~~0.32~~. 0.42
What is the probability that in a sample of ~~12~~ 8 Americans, that at least 1 will be an Independent?

$P(\geq 1) = 1 - P(\text{none}) = 1 - (0.58)^8 = \boxed{0.9872}$

13. A pair of dice is rolled and the resulting number is odd. What is the complement of this event?

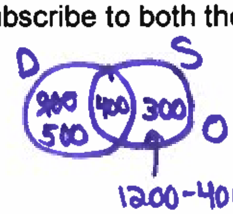
The complement is rolling an even number $P(\text{odd}^c) = \boxed{1/2}$

14. Seven black tiles and ten white tiles are placed in a bag. Two tiles are then drawn in succession. What is the probability that the second tile drawn is a white tile if the second ball is drawn without replacing the first? **(Hint: Draw a tree diagram)**



$\underbrace{7/17 \cdot 10/16}_{B,W} + \underbrace{10/17 \cdot 9/16}_{\text{or } W,W} = \frac{35}{136} + \frac{45}{136} = \frac{80}{136} \; \boxed{\frac{10}{17}}$

15. A survey of 1,200 subscribers to the *News and Observer* revealed that 900 people subscribe to the daily edition and 400 subscribe to both the daily and the Sunday editions. How many total subscribe to the Sunday edition?



$\boxed{700}$

Unit 1 continued

16. In how many ways can the president of junior class representatives select 6 of 20 representatives to be on a prom decorations committee?

$_{20}C_{6}$ $\boxed{38{,}760}$

17. Given a standard deck of 52 playing cards, what is the probability of drawing a Queen or a Heart?

$P(Q)+P(\heartsuit)-P(Q \,\&\, \heartsuit) = 4/52 + 13/52 - 1/52 = 16/52 = \boxed{\frac{4}{13}}$

18. Based on data obtained from a ~~a~~ study conducted by a national ice cream franchise, it has been determined that 41% of 2-year-olds have never had ice cream, 32% of 3-year-olds have never had ice cream, and 26% of 4-year-olds have never had ice cream. If a child is selected at random from a group of 30 toddlers comprising eight 2-year-olds, thirteen 3-year-olds, and nine 4-year-olds what is the probability that this child is 4 years old ***and*** has never had ice cream?
(Hint: Draw a tree diagram)

8/30 2yo — .41 No Ice Cr .1093 ; .59 Ice Cream .1573
13/30 3yo — .32 No Ice Cr .1387 ; .68 Ice Cr .2947
9/30 4yo — .26 No Ice Cr .0780 ; .74 Ice Cr .2220

$\left(\frac{9}{30}\right)(.26) = \boxed{.0780 = 7.8\%}$

19. Among 500 freshmen pursuing a business degree at a university, 310 are enrolled in an English course, 215 are enrolled in a Science course, and 150 are enrolled in both an English and a Science course. What is the probability that a freshman selected at random from this group is enrolled in an English **or** a Science course?

E 310: 160, 150, 65 :215 S; 125

$P(E \text{ or } S) = P(E)+P(S)-P(E \,\&\, S)$
$\frac{310}{500} + \frac{215}{500} - \frac{150}{500} = \frac{375}{500}$ $\boxed{0.75 \text{ or } 3/4}$

20. Let $U = \{1,2,3,4,5,6,7,8,9\}$ and $A = \{1,2,6,7\}$ and $B = \{1,3,6,9\}$. Find the set A^c.

$\boxed{A^c = \{3,4,5,8,9\}}$

21. How many ways are there to arrange 9 books on a shelf?

$_{9}P_{9} = \boxed{362{,}880}$

Unit 2 Matrix Applications

Suppose an animal population has the characteristics described in the table below. Round to tenths place, as needed. Fix

$P_{50} = 23615.9$
$P_{51} = 26080.8$ 10.4%

Age Groups (years)

Rates	0-5	5-10	10-15	15-20	20-25	25-30	30-35
Birth	0	0.7	1.2	0.8	0.7	0.2	0
Survival	0.5	0.8	0.9	0.9	0.7	0.4	0

Survival Rates

1. Construct the Leslie matrix for this animal.

7x7 matrix

Birth

$$L = \begin{bmatrix} 0 & 0.5 & 0 & 0 & 0 & 0 & 0 \\ 0.7 & 0 & 0.8 & 0 & 0 & 0 & 0 \\ 1.2 & 0 & 0 & 0.9 & 0 & 0 & 0 \\ 0.8 & 0 & 0 & 0 & 0.9 & 0 & 0 \\ 0.7 & 0 & 0 & 0 & 0 & 0.7 & 0 \\ 0.2 & 0 & 0 & 0 & 0 & 0 & 0.4 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$ 7x7

2. What is the maximum age of this animal?

35 (see table)

~~3.~~ 4 What is the long-term growth rate for the animal?

new − previous / previous

$P_{30} = 3242.4$
$P_{31} = 3580.9$
$\frac{3580.9 - 3242.4}{3242.4} = \boxed{10.4\%}$
$P_{50-51} \rightarrow 10.4\%$

~~4.~~ 3 For the initial female population given in the table below, find

Age (years)	0-5	5-10	10-15	15-20	20-25	25-30	30-35
Number	30	30	27	28	32	15	10

$= P_0$

$$C = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$$ 7x1

a) the female age distribution after 10 cycles $= P_0 \cdot L^{10}$

0-5	5-10	10-15	15-20	20-25	25-30	30-35
181.2	82.4	59.1	48.4	40.4	24.4	9.3

b) the total female population after 80 years. $\frac{80}{5}$ = 16 cycles

$(P_0 \cdot L^{16}) \cdot C =$ $\boxed{807.7 \text{ females}}$

Unit 2 continued

5. Use the following payoff matrix for a two-person game: $\begin{bmatrix} 0 & 4 \\ -1 & 2 \end{bmatrix}$ 0 −1 maximin } Find row minimums

Find column maximums { 0 4 minimax

What is the saddle point for this game? 0 What is the minimax? 0 What is the maximin? $\boxed{0}$

Given $A = \begin{bmatrix} 3 & 1 \\ 9 & 3 \end{bmatrix}$, $B = \begin{bmatrix} m & k & 7 \\ 4 & 9 & 0 \end{bmatrix}$, $C = \begin{bmatrix} 1 & 2 & 9 \\ 4 & 9 & 0 \end{bmatrix}$

Best Strategy is Row 1 for Row Player + Column 1 for Column Player.

6. Find B + C.

$\begin{bmatrix} m+1 & k+2 & 16 \\ 8 & 18 & 0 \end{bmatrix}$

7. Find AB. $(2\times 2)\cdot(2\times 3) = (2\times 3)$

$\begin{bmatrix} 3m+4 & 3k+9 & 21 \\ 9m+12 & 9k+27 & 63 \end{bmatrix}$

8. Find BA.

$(2\times 3)(2\times 2)$

$\boxed{\text{Undefined}}$

because # columns for 1st matrix ≠ # rows of 2nd matrix, as required to multiply

9. If $A \times \begin{bmatrix} 2 & 8 & -7 \\ 5 & 9 & 10 \\ -8 & 3 & 0 \\ 4 & 7 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 9 \\ 4 & 9 & 0 \end{bmatrix}$, what are the dimensions of matrix **A**? $\boxed{2\times 4}$

$A \times (4\times 3) = (2\times 3)$ $(2\times 4)(4\times 3) = 2\times 3$

10. A certain machine is in one of four possible states from week to week: 0 = working without a problem; 1 = working but in need of minor repair; 2 = working but in need of major repair; 3 = out-of-order. The corresponding transition probability matrix is shown at the right.

next week

$$T = \begin{array}{c} \\ 0 \\ 1 \\ 2 \\ 3 \end{array} \begin{array}{c} \begin{array}{cccc} 0 & 1 & 2 & 3 \end{array} \\ \begin{bmatrix} 0.80 & 0.14 & 0.04 & 0.02 \\ 0 & 0.6 & 0.3 & 0.10 \\ 0 & 0 & 0.65 & 0.35 \\ 0.90 & 0 & 0 & 0.10 \end{bmatrix} \end{array}$$

a) An experienced technician finds that, in her many years, she is called in 60% of the time for out-of-order machines, 20% of the time for machines needing major repairs, and 15% of the time for machines needing minor repairs. What is the typical initial state matrix, in her experience?

No Prob minor Major OoO
[0.05 0.15 0.20 0.60]

b) If this technician checks this machine after 3 weeks, what is probability distribution for states 0-3?

$\cdot T^3 = [0.56 \quad 0.17 \quad 0.17 \quad 0.09]$

c) If this technician checks this machine after 5 weeks, how likely is it that the machine is in need of major repair?

No Prob Minor Major Outoforder
$\cdot T^5 = [0.52 \quad 0.18 \quad 0.20 \quad 0.10]$ $\boxed{20\%}$

11. There are 3 popular coffee brands that are most popular in a market: Brand A, Brand B, and Brand C. Frequently, people switch from one brand to another. If they are currently using Brand A, there is 0.6 probability they will continue to use it next week, 0.3 probability they will switch to Brand B, and 0.1 probability they will switch to brand C. If they are using Brand B this week, there is 0.5 probability that they will switch to Brand A, 0.3 probability they will stay with Brand B, and 0.2 probability they will switch to Brand C. If they are using Brand C now, there is 0.4 probability they will switch to Brand A, 0.1 probability they will switch to Brand B, and 0.5 probability they will stay with Brand C.

a) Express this scenario as a transition matrix.

now

$$T = \begin{array}{c} \\ A \\ B \\ C \end{array} \begin{array}{c} \begin{array}{ccc} A & B & C \end{array} \\ \begin{bmatrix} 0.6 & 0.3 & 0.1 \\ 0.5 & 0.3 & 0.2 \\ 0.4 & 0.1 & 0.5 \end{bmatrix} \end{array}$$

% up next

b) If a family is using Brand B now, what is the probability they will be using Brand C 4 weeks later?

A B C
$I_0 = [0 \quad 1 \quad 0]$ $I_0 \cdot T^4$ (20.98%)

c) If a set of roommates is currently using Brand A, what is the long term probability that they'll stick with Brand A?

new $I_0 = [1 \quad 0 \quad 0]$ initial state matrix

$I_0 \cdot T^{30}$
$I_0 \cdot T^{40}$
$I_0 \cdot T^{50}$

$\boxed{53.23\%}$

Unit 2 continued

p 5

12. A local designer clothes manufacturer tries to keep up with her sales in Target, Old Navy, and Kohl's. The inventories of her most popular items in all three stores are recorded in the table.

	T-shirts	Jackets	Cardigans
Target	16	21	25
Old Navy	19	24	20
Kohl's	18	27	19

Label your rows and columns in your work and your answers.

a. The designer makes the T-shirts for \$10, the jackets for \$12, and the cardigans for \$15. Use **one matrix operation** to calculate the manufacturer's cost of making these popular items for each store. Call the resulting cost matrix C.

$$C = \begin{array}{c} \\ \text{Target} \\ \text{Old Navy} \\ \text{Kohl's} \end{array}\begin{array}{c} \text{Ts} \quad \text{Js} \quad \text{Cs} \\ \begin{bmatrix} 16 & 21 & 25 \\ 19 & 24 & 20 \\ 18 & 27 & 19 \end{bmatrix} \end{array} \cdot \begin{array}{c} \text{Ts} \\ \text{Js} \\ \text{Cs} \end{array}\begin{array}{c} \text{cost} \\ \begin{bmatrix} 10 \\ 12 \\ 15 \end{bmatrix} \end{array} = \begin{array}{c} \\ \text{Targ} \\ \text{ONav} \\ \text{Kohls} \end{array}\begin{array}{c} \text{cost} \\ \begin{bmatrix} 787 \\ 778 \\ 789 \end{bmatrix} \end{array} = C$$

b. The designer sells the T-shirts to the stores for \$13, the jackets for \$20, and the cardigans for \$18. Use matrices to calculate the income, I, that the designer makes from selling these popular items to the stores.

$$\begin{array}{c} \\ \text{T} \\ \text{ON} \\ \text{K} \end{array}\begin{array}{c} \text{Ts} \quad \text{Js} \quad \text{Cs} \\ \begin{bmatrix} 16 & 21 & 25 \\ 19 & 24 & 20 \\ 18 & 27 & 19 \end{bmatrix} \end{array} \cdot \begin{array}{c} \text{T} \\ \text{J} \\ \text{C} \end{array}\begin{array}{c} \text{Sell} \\ \begin{bmatrix} 13 \\ 20 \\ 18 \end{bmatrix} \end{array} = \begin{array}{c} \\ \text{Tar} \\ \text{ON} \\ \text{Koh} \end{array}\begin{array}{c} \text{Income} \\ \begin{bmatrix} 1078 \\ 1087 \\ 1116 \end{bmatrix} \end{array} = I$$

c. Use matrices to calculate the profit that the designer makes on her sales to each store.

$$\text{Profit} = \underset{I}{\text{Income}} - \underset{C}{\text{Cost}} = \begin{bmatrix} 1078 \\ 1087 \\ 1116 \end{bmatrix} - \begin{bmatrix} 787 \\ 778 \\ 789 \end{bmatrix} = \boxed{\begin{array}{c} \text{Target} \\ \text{Old Navy} \\ \text{Kohl's} \end{array}\begin{bmatrix} 291 \\ 309 \\ 327 \end{bmatrix}}$$

13. Last week, the local movie theater sold 8500 movie tickets. Their income totaled \$64,600. Tickets can be bought in one of 3 ways: a matinee admission costs \$5, student admission is \$6 all day, and regular admissions are \$8.50. How many of each type of ticket was sold if twice as many student tickets were sold as matinee tickets?

$2m = s$

$2(50) = 100$ ✓

a. Define the variables.

m = # of Matinee tix sold
s = # of student tix sold
r = # of regular tix sold

b. Create a system of equations for the problem.

$m + s + r = 8500$ total tickets
$5m + 6s + 8.50r = 64600$ income
$2m = s \rightarrow 2m - 1s + 0r = 0$ twice student vs mat.

c. Write a matrix equation to represent the scenario.

$$A \rightarrow \begin{bmatrix} 1 & 1 & 1 \\ 5 & 6 & 8.50 \\ 2 & -1 & 0 \end{bmatrix} \cdot \begin{bmatrix} m \\ s \\ r \end{bmatrix} = \begin{bmatrix} 8500 \\ 64600 \\ 0 \end{bmatrix} \leftarrow B$$

d. Use matrices to solve the problem.

Do $A^{-1} \cdot B$

$$\begin{bmatrix} m \\ s \\ r \end{bmatrix} = \begin{bmatrix} 900 \\ 1800 \\ 5800 \end{bmatrix}$$

e. Express your solution in complete sentences.

The local movie theater sold 900 matinee tickets, 1800 student tickets, and 5800 regular tickets.

Space = 0	A=1	B=2	C=3	D=4	E=5	F=6	G=7	H=8	I=9	J=10
K=11	L=12	M=13	N=14	O=15	P=16	Q=17	R=18	S=19	T=20	U=21
V=22	W=23	X=24	Y=25	Z=26						

9-0-12-15-22-5-0-7-18-5-5-14-0-8-15-16-5-0 ← Extra zero for space at end

14. Encode the message I LOVE GREEN HOPE using the encoding matrix $\begin{bmatrix} 3 & -2 & 0 \\ -1 & 0 & 2 \\ 3 & -4 & 1 \end{bmatrix}$.

$$\begin{bmatrix} 9 & 0 & 12 \\ 15 & 22 & 5 \\ 0 & 7 & 18 \\ 5 & 5 & 14 \\ 0 & 8 & 15 \\ 16 & 5 & 0 \end{bmatrix} \cdot \begin{bmatrix} 3 & -2 & 0 \\ -1 & 0 & 2 \\ 3 & -4 & 1 \end{bmatrix} = \boxed{\begin{bmatrix} 63 & -66 & 12 \\ 38 & -50 & 49 \\ 47 & -72 & 32 \\ 52 & -66 & 24 \\ 37 & -60 & 31 \\ 43 & -32 & 10 \end{bmatrix}}$$

* use Inverse to Decode

15. Using the encoding matrix $\begin{bmatrix} -2 & 3 \\ 0 & -1 \end{bmatrix}$, decode the message

-26 38 -40 52 0 -9 -38 57 -2 -20 -10 -4 -30 32 -10 15.

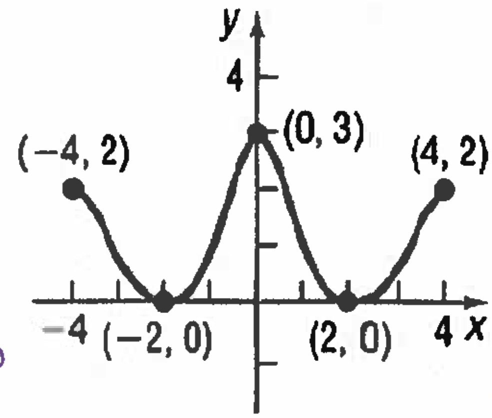
$$\begin{bmatrix} -26 & 38 \\ -40 & 52 \\ 0 & -9 \\ -38 & 57 \\ -2 & -20 \\ -10 & -4 \\ -30 & 32 \\ -10 & 15 \end{bmatrix} \cdot \begin{bmatrix} -2 & 3 \\ 0 & -1 \end{bmatrix}^{-1} = \begin{bmatrix} 13 & 1 \\ 20 & 8 \\ 0 & 9 \\ 19 & 0 \\ 1 & 23 \\ 5 & 19 \\ 15 & 13 \\ 5 & 0 \end{bmatrix}$$

$\boxed{\text{MATH\_IS\_AWESOME\_}}$

Unit 3 p6

Unit 3 Functions and Limits **Remember to show work for credit! Circle your answers!**

1. For the given graph find the following:



a) Domain [-4, 4]

b) Range [0, 3]

c) X-intercepts (-2,0) and (2,0)

d) Y-intercepts (0, 3)

e) Interval(s) that the function is increasing

[-2, 0] ∪ [2, 4] * give domain values here

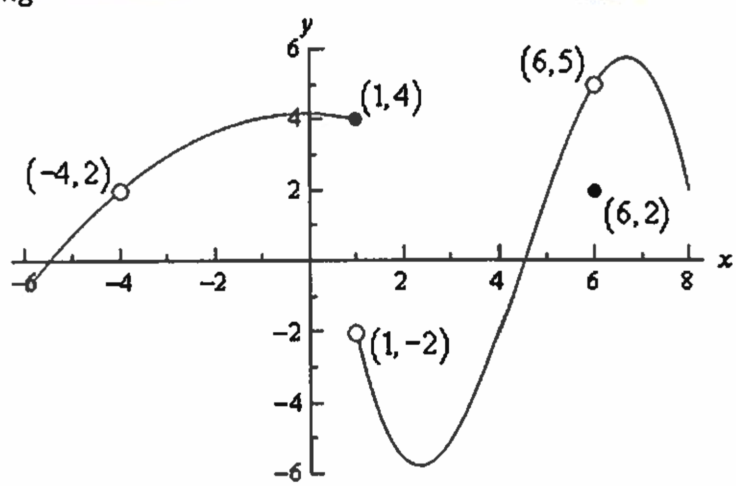
f) Interval(s) that the function is decreasing

[-4, -2] ∪ [0, 2]

g) Maximum(s): 3 at x = 0 (Absolute max + Local max) AND Local max: 2 at x = -4 and 2 at x = 4

h) Minimum(s): 0 at x = -2 ; 0 at x = 2 (Absolute mins + Local mins)

2. Given function f shown below, evaluate the following



a. $\lim_{x\to1} f(x)$ DNE the two sides 1^- and 1^+ disagree

b. $\lim_{x\to1^-} f(x) = 4$

c. $\lim_{x\to-4} f(x)$ 2

d. $\lim_{x\to-\infty} f(x)$ $-\infty$

e. $\lim_{x\to\infty} f(x)$ $-\infty$

f. $\lim_{x\to6} f(x) = 5$

g. $f(6) = 2$

f(x) find y-value when x=6 ⇒ look at dot on graph

3. Find the domain of the following functions:

a. $f(x)=\sqrt{27-3x}$

b. $f(x)=\dfrac{2x^2-10x}{x^2-4x-5}$

c. $f(x)=\dfrac{2x^2}{\sqrt{x-3}}$

d. $f(x)=\sqrt{2x^2-11x+12}$

see next page ⇒

Unit 3 continued

P7

3. Find the domain of the following functions:

a. $f(x)=\sqrt{27-3x}$

$27-3x=0$

$x=9$

$(-\infty, 9]$

b. $f(x)=\dfrac{2x^2-10x}{x^2-4x-5}$

$\dfrac{2x(x-5)}{(x-5)(x+1)}$

$x-5=0 \to x=5$ Hole at 5 so $x \neq 5$

VA at $x=-1$ so $x \neq -1$

$(-\infty,-1)\cup(-1,5)\cup(5,\infty)$

Skip VA Skip x-value of hole

c. $f(x)=\dfrac{2x^2}{\sqrt{x-3}}$

$\sqrt{x-3} \neq 0 \to x \neq 3$

$(3,\infty)$

d. $f(x)=\sqrt{2x^2-11x+12}$

$-8 \cdot -3 = 24$

$-8 + -3 = -11$

$2x^2-8x-3x+12$

$2x(x-4)-3(x-4)$

$(2x-3)(x-4)$

$x=3/2, 4$

$(-\infty, 3/2]\cup[4,\infty)$

4. Find the following for $f(x)=\dfrac{x+2}{x^2-8x-20}$

$f(x)=\dfrac{x+2}{(x-10)(x+2)}=\dfrac{1}{x-10}$

VA: $x=10$ Hole at $x=-2$

$y=\dfrac{1}{-2-10}$

$y=-1/12$ $(-2,-1/12)$

a) Domain $(-\infty,-2)\cup(-2,10)\cup(10,\infty)$

b) Range $(-\infty,-1/12)\cup(-1/12,0)\cup(0,\infty)$

c) X-intercepts NONE $0=\frac{1}{x-10} \to 0=1$ non-sense!!

d) Y-intercepts $(0,-1/10)$ $y=\frac{1}{0-10}=-1/10$

e) Removable Discontinuity a.k.a Hole at $(-2,-1/12)$

f) Non-Removable Discontinuity aka VA: $x=10$ aka Infinite Discontinuity

g) Horizontal Asymptote $y=0$ (Bottom Degree Bigger) $\frac{\text{Degree }1}{\text{Degree }2}$

h) Limits at $-\infty$ and ∞

$\lim_{x\to-\infty} f(x)=0$ + $\lim_{x\to\infty} f(x)=0$ approach H.A. $y=0$

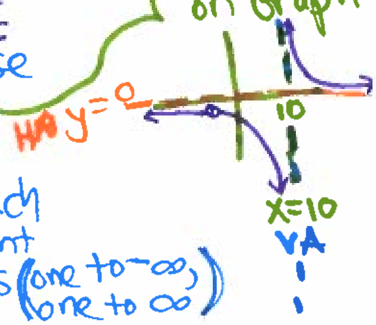
Check Limits on Graph

i) Limits at Discontinuities

$\lim_{x\to-2} f(x) = -1/12$ + $\lim_{x\to10} f(x)=$ DNE

x of hole y-value of hole VA: $x=10$

because $x\to10^-$ and $x\to10^+$ approach different values (one to $-\infty$, one to ∞)



5. Evaluate the following:

a) Given f (n) = 2n and g(n) = –n – 4. Find f(g(3)).

$f(g(3))=f(-3-4)=f(-7)=2(-7)=\boxed{-14}$

b) Given g(n) = 2n + 3 and h(n) = n – 1. Find (g ∘ h)(n) and its domain and range.

$(g\circ h)(n)=g(h(n))=g(n-1)=2(n-1)+3$

$2n-2+3$

$\boxed{2n+1}$

c) Given h(x) = x² – 2 and g(x) = 4x + 1. Find h(g(x)) and its domain and range.

$h(g(x))=h(4x+1)=(4x+1)^2-2$

$(4x+1)(4x+1)-2$

$(16x^2+8x+1)-2$ $\boxed{16x^2+8x-1}$