Name: ____________________________ Per: ____

UNIT 2 Test REVIEW – Matrix Applications Leslie Model, Markov Chains, Game Theory

1. Suppose a shark population has the characteristics described in the table below.

AGE (years)	0-5	5-10	10-15	15-20	20-25	25-30	30-35
BIRTHRATE	0	0	1.2	0.8	0.7	0.2	0
SURVIVAL RATE	0.5	0.8	0.9	0.9	0.7	0.5	0
INITIAL POPULATION	10	14	25	30	20	15	8

a. What is the maximum age of this animal?

b. Construct the LESLIE MATRIX for this animal.

c. Find the population distribution after 150 years (30 cycles). Show work!

d. Find the TOTAL population after 150 years. Show work!

e. Determine the LONG TERM GROWTH RATE for this population. Show work!

f. If the maximum sustainable population for this population in its native habitat is 500, when will the maximum population be reached (answer in cycles and years)?

2. Given that a person's last soda purchase was Coke, there is a 90% chance that his next soda purchase will also be Coke and an equal chance between Pepsi and Sprite. If a person's last soda purchase was Pepsi, there is an 80% chance that his next soda purchase will also be Pepsi and he will always choose Coke over Sprite. If a person's last soda purchase was Sprite, there is a 70% chance that his next purchase will also be Sprite, 20% chance it will be Coke and the remaining choose Pepsi.

a. Create a transition matrix for this situation and label the rows and columns.

b. Given that a person is currently a Coke purchaser, what is the probability that he will purchase Pepsi **three** purchases from now? Show the matrix equation.

c. In the long run, what is the probability that a person will purchase Sprite if they are currently a Pepsi purchaser?

3. A certain species of insect lives 5 weeks. Through each of the first 4 weeks only 60% survive into the next week. Each female bug that survives into the 5th week produces, on average, 20 new female bugs. Bugs that only survive the first four weeks do not produce offspring.

a. Construct a Leslie matrix representing the life cycle of these bugs.

10 female bugs in their 5th week of life move into your garage.

b. What is the newborn population after the 6th cycle?

c. What will the total population be after 11 cycles?

Suppose the initial population looked like this $P_0 = [\,0 \quad 20 \quad 15 \quad 25 \quad 5\,]$,

d. What would the total population be after 22 cycles?

e. What is the growth rate between P_{21} and P_{22} ?

4. Each of the following matrices represents a payoff matrix for a game. Determine the best strategy for the row and column players. If the game is strictly determined, find the saddle point of the game. If the game is not strictly determined, explain why not.

a)

$$\begin{array}{c} \\ A \\ B \end{array} \begin{array}{c} \begin{array}{cc} C & D \end{array} \\ \begin{bmatrix} -2 & 5 \\ 2 & 3 \end{bmatrix} \end{array}$$

Row strategy:
Column strategy:
Strictly Determined?: yes/no
Saddle Point (if yes above):

b)

$$\begin{array}{c} \\ A \\ B \\ C \\ D \end{array} \begin{array}{c} \begin{array}{ccc} E & F & G \end{array} \\ \begin{bmatrix} 2 & -1 & 2 \\ -3 & -4 & -1 \\ 4 & 1 & 2 \\ 3 & 1 & -2 \end{bmatrix} \end{array}$$

Row strategy:
Column strategy:
Strictly Determined?: yes/no
Saddle Point(if yes above):

5. Suppose that Gary and Liz play the "penny matching game" and use the following payoff matrix.

LIZ

GARY

$$\begin{array}{c} \\ H \\ T \end{array} \begin{array}{c} \begin{array}{cc} H & T \end{array} \\ \begin{bmatrix} 3 & -2 \\ -2 & 1 \end{bmatrix} \end{array}$$

a. Use the row matrix $\begin{bmatrix} p & 1-p \end{bmatrix}$ to find Gary's best strategy for this game.

b. Use the column matrix $\begin{bmatrix} q \\ 1-q \end{bmatrix}$ to find Liz's best strategy for this game.

c. What is Gary's expectation for this game, provided that both players follow their "best strategy"? Interpret your answer in terms of how many pennies Gary can expect to win/lose over a number of games?

6. A supplier provides coffee beans to shops A, B, and C in 1 pound, 2 pound, and 3 pound bags, as shown in the table below.

	1 pound	2 pound	3 pound
Shop A	50	100	30
Shop B	60	150	40
Shop C	80	200	70

The cost of a 1 pound, 2 pound, and 3 pound bag are $10.60, $17.20, and $22.50 respectively.

a) Use matrices to calculate the total cost of the coffee bean purchases at each shop.

b) Suppose the supplier provides a 15% discount. Use matrices to determine what the total costs of the bean purchases will be at each shop with this discount.

7. A quilt maker plans to make 14 quilts this year from 113 yards of fabric. A small quilt requires 4 yards of fabric. A medium quilt requires 7 yards of fabric, and a large quilt requires 11 yards of fabric. She plans to make twice as many large quilts as small quilts. How many of each type of quilt should she make?

Define your variables:

System of linear equations:

Matrices work:

Solution:

8. The cost of admission to a popular music concert was $162 for 12 children and 3 adults. The admission was $122 for 8 children and 3 adults at another music concert. How much was the admission for each child and adult?

Space = 0 A=1 B=2 C=3 D=4 E=5 F=6 G=7 H=8 I=9 J=10 K=11 L=12 M=13
N=14 O=15 P=16 Q=17 R=18 S=19 T=20 U=21 V=22 W=23 X=24 Y=25 Z=26

9. Encode the message NEW YORK YANKEES using the encoding matrix $\begin{bmatrix} 3 & -1 & 0 \\ -2 & 0 & 1 \\ 4 & -3 & 2 \end{bmatrix}$. Show work.

10. Decode the message -2 -24 -19 17 -15 17 0 -36 -5 7 0 -38 -15 -3 using the encoding matrix $\begin{bmatrix} -1 & 3 \\ 0 & -2 \end{bmatrix}$. Show work.