need same degree so need x or another factor on top

In-Class Review: Unit 4 Functions and Limits

$f(x) = \frac{3(x-7)x}{4(x-7)(x+2)}$

Part 1:

Hole at (7, #)

1. Write an equation of a rational function, f(x) with Removable Discontinuity at 7, Non-Removable Discontinuity at -2, and Horizontal Asymptote of y = 3/4.

VA: x = -2 → (x+2) extra denom.

Same degree → coef 3/4

2. State the following and graph $g(x) = \frac{2x^2 - 10x + 8}{4x^2 - 4x}$

$g(x) = \frac{2(x^2-5x+4)}{4x(x-1)}$

$g(x) = \frac{2(x-4)\cancel{(x-1)}}{4x\cancel{(x-1)}}$

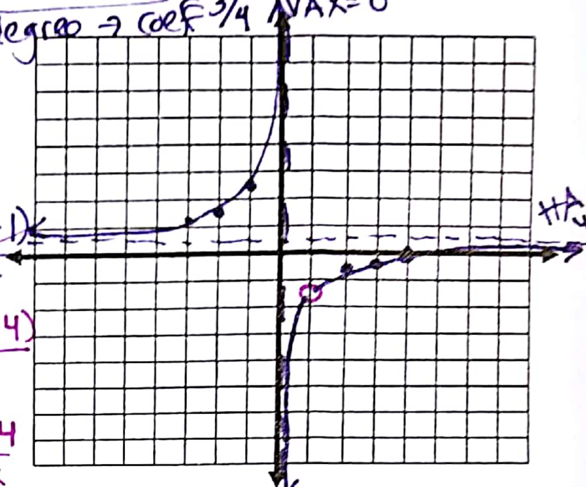
$g(x) = \frac{2(x-4)}{4x}$

or $g(x) = \frac{x-4}{2x}$ for $x \neq 1$

- Domain: $(-\infty, 0) \cup (0, 1) \cup (1, \infty)$
- Range: $(-\infty, -3/2) \cup (-3/2, 1/2) \cup (1/2, \infty)$
- x & y intercepts: (4, 0) x-int, no y-int
- Removable Discontinuity: $(1, -3/2)$
- Non-Removable Discontinuity: $x = 0$
- Horizontal Asymptote: $y = 1/2$
- Limits at discontinuities: $\lim_{x\to 1} g(x) = -3/2$
- End Behavior using limits:

$\lim_{x\to\infty} g(x) = 1/2$ $\quad \lim_{x\to -\infty} g(x) = 1/2$

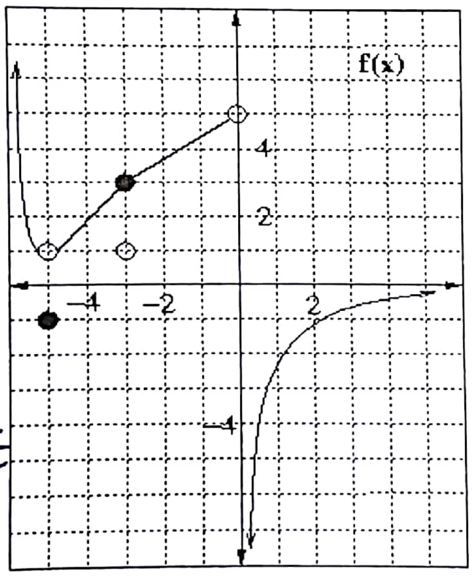
VA x = 0



Part 2:

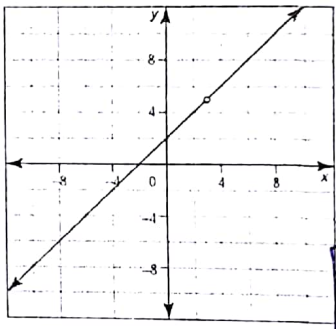
Using the graph of *f(x)* below, find the following limits.

1. $\lim_{x\to -5} f(x) = 1$
2. $\lim_{x\to -3} f(x) = 3$
3. $\lim_{x\to -\infty} f(x) = \infty$
4. $\lim_{x\to 0^-} f(x) = 5$
5. $\lim_{x\to \infty} f(x) = 0$
6. $\lim_{x\to 0} f(x)$ DNE
7. $f(-5) = -1$



Write an equation for the graphed rational function.

8.



y = 1x + 2 but has

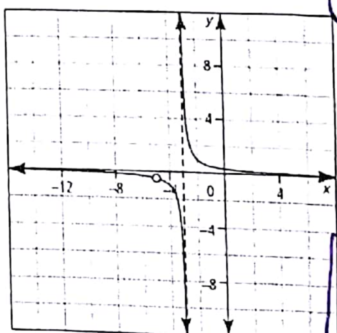
Hole (3, 5)

No VA → no extra denom

No HA → top degree larger

$\boxed{y = \frac{(x-3)(x+2)}{(x-3)}}$

9.



Hole (-5, -1/2)

VA x = -3

HA y = 0 ↳ bottom degree bigger

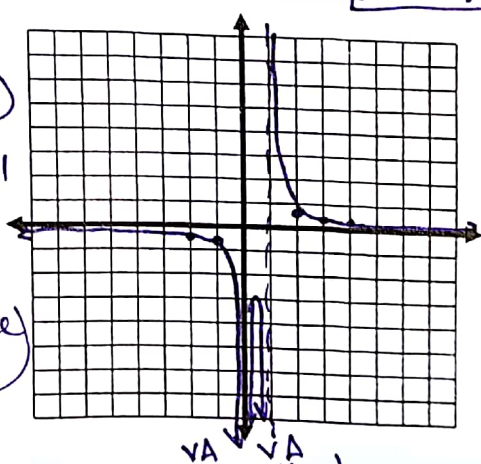
$\boxed{y = \frac{(x+5)}{(x+3)(x+5)}}$

Part 3: State the following and make a graph $g(x) = \frac{\sqrt[3]{x}}{x^2 - x} = \frac{\sqrt[3]{x}}{x(x-1)}$

- Domain: $(-\infty, 0) \cup (0, 1)(1, \infty)$
- Range: $(-\infty, 0) \cup (0, \infty)$
- x & y intercepts: none
- Max and Min: -3.07 max at x = 0.4 local, no local min
- Increasing: $(0, 0.4]$
- Decreasing: $(-\infty, 0) \cup [0.4, 1) \cup (1, \infty)$
- Limits at discontinuities:
- End Behavior using limits:

VA at x = 0, x = 1

HA at y = 0 (bottom degree bigger)



$\lim_{x\to\infty} g(x) = 0$ $\quad \lim_{x\to -\infty} g(x) = 0$

$\lim_{x\to 0^-} g(x) = -\infty$ $\quad \lim_{x\to 0^+} g(x) = -\infty$

$\lim_{x\to 1^-} g(x) = -\infty$ $\quad \lim_{x\to 1^+} g(x) = \infty$

Extra Practice Unit 4 ICM **Functions and Limits** Name: Key

1. When finding the **domain**, some key items to consider are x-intercept, x-value of hole(s), and vertical asymptotes
2. When finding the **range**, some key items to consider are x-intercept (esp. if HA $y=0$), y-value of hole(s), and horizontal asymptotes

Graph each function, showing the key features and plotting at least 3 points per curve. Also find the requested values. (Hint, see #1 and 2 for help with domain and range.

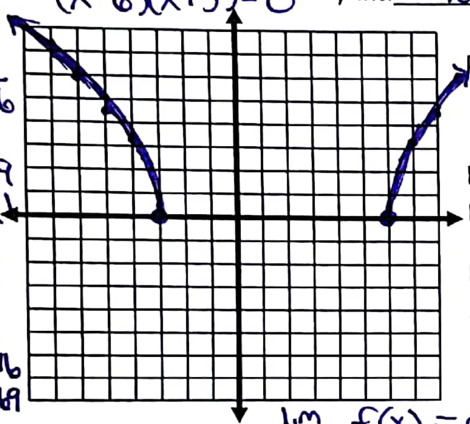
3. $f(x)=\sqrt{x^2-3x-18}$

$(x-6)(x+3)=0$

x-int: $(-3,0)\ (6,0)$

y-int: none $\sqrt{-18}$

x	y
-4	3.16
-6	6
-5	4.69
-7	7.21
-3	0
6	0
7	3.16
8	4.69
9	6



Removable Disc.: none

Nonremovable Disc.: none

Horizontal Asymp: none

End Behavior, written as Limits: $\lim_{x\to-\infty} f(x)=\infty$ $\lim_{x\to\infty} f(x)=\infty$

Domain: $(-\infty,-3]\cup[6,\infty)$

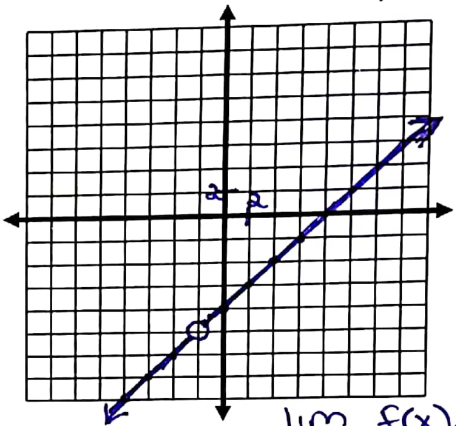
Range: $[0,\infty)$

4. $f(x)=\dfrac{x^2-6x-16}{x+2}$

$\dfrac{(x-8)(x+2)}{x+2} \rightsquigarrow y=x-8$

x-int: $(8,0)$

y-int: $(0,-8)$



Removable Disc.: $(-2,-10)$

Nonremovable Disc.: none

Horizontal Asymp: none (top degree is bigger)

End Behavior, written as Limits: $\lim_{x\to-\infty} f(x)=-\infty$ $\lim_{x\to\infty} f(x)=\infty$

Domain: $(-\infty,-2)\cup(-2,\infty)$

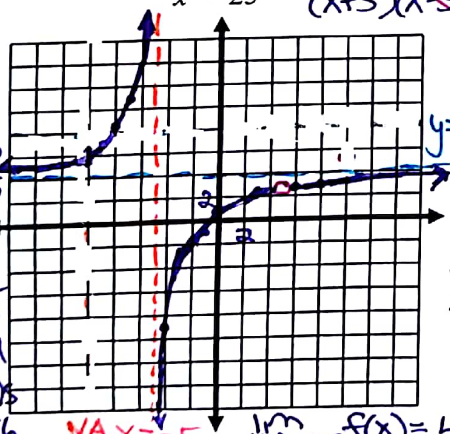
Range: $(-\infty,-10)\cup(-10,\infty)$

5. $f(x)=\dfrac{4x^2-13x-35}{x^2-25}=\dfrac{(4x+7)\cancel{(x-5)}}{(x+5)\cancel{(x-5)}}$

$-20\cdot7=-140$
$-20+7=-13$

$4x^2-13x-35$
$\dfrac{4x^2-20x}{4x}\ \dfrac{+7x-35}{+7}$
$4x(x-5)+7(x-5)$
$(4x+7)(x-5)$

x	y
-9	7.25
-8	8.33
-7	10.5
-6	17
-5	
-4	-9
-3	-2.5
-1	.75
0	1.4
2	
3	2.375
4	2.56
5	—
6	2.82
8	3
10	3.13



VA $x=-5$

Removable Disc.: $(5,\frac{27}{10})$ $y=\dfrac{4\cdot5+7}{5+5}=\dfrac{27}{10}$

Nonremovable Disc.: $x=-5$ VA

Horizontal Asymp: $y=4$

Same degree $\rightsquigarrow$ Ratio of leading Coeff.

End Behavior, written as Limits: $\lim_{x\to-\infty} f(x)=4$ $\lim_{x\to\infty} f(x)=4$

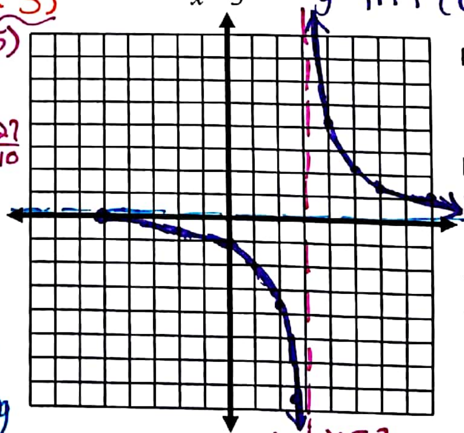
Domain: $(-\infty,-5)\cup(-5,5)\cup(5,\infty)$

Range: $(-\infty,\frac{27}{10})\cup(\frac{27}{10},4)\cup(4,\infty)$

6. $f(x)=\dfrac{\sqrt{2x+10}}{x-3}$

x-int $(-5,0)$

y-int $(0,\frac{-\sqrt{10}}{3})$



VA $x=3$

Removable Disc.: none

Nonremovable Disc.: $x=3$ VA

Horizontal Asymp: $y=0$

End Behavior, written as Limits: $\lim_{x\to\infty}=0$ $\lim_{x\to-\infty}=0$

x	y
-5	0
-2	-.49
0	-1.05
2	-3.74
3	—
4	4.24
5	2.24
6	1.56
8	1.01

Domain: $[-5,3)\cup(3,\infty)$

Range: $(-\infty,\infty)$

we don't skip HA $y=0$ here because x-int $(-5,\underline{0})$