Unit 1 Probability

Let U = {a, b, c, d, e, f, g, h, i, j}, A = {a, b, c, f, h, i, j}, B = {b, c, d, g} and C = {a, d, e, h, j}.

1. Find A U (B U C).
2. Find $A \cap B$.
3. Find $B \cup (A \cap C)$
4. $B^C \cap C$
5. Find $(A \cup B)^C$

6. The department of foreign languages of a liberal arts college conducted a survey of the recent graduates to determine the foreign language courses they had taken while undergraduates at the college. Of the 480 graduates: (Hint: Draw a Venn Diagram!)

   200 had at least 1 year of Spanish
   178 had at least 1 year of French
   140 had at least 1 year of German
   33 had at least 1 year of Spanish and French
   24 had at least 1 year of Spanish and German
   18 had at least 1 year of German and French
   3 had at least 1 year of all three languages

   How many of the graduates had

   a. At least 1 year of at least one of the three languages?
   b. At least 1 year of exactly one of the three languages?
   c. Less than 1 year of any of the three languages?

7. In how many ways can eight different books be arranged on a shelf?

8. In how many ways can three pictures be selected from a group of 9 different pictures?

9. Find the number of distinguishable permutations that can be formed from the letters of each word?

   a. CINCINNATI b. HONOLULU

10. Let E and F be two events of an experiment with sample space S. Suppose P(E) = 0.3 and P(F) = 0.2, and $P(E \cap F) = 0.15$

    Compute:

    $a. P(E \cup F)$ $b. P(E^C \cap F^C)$ $c. P(E^C \cap F)$

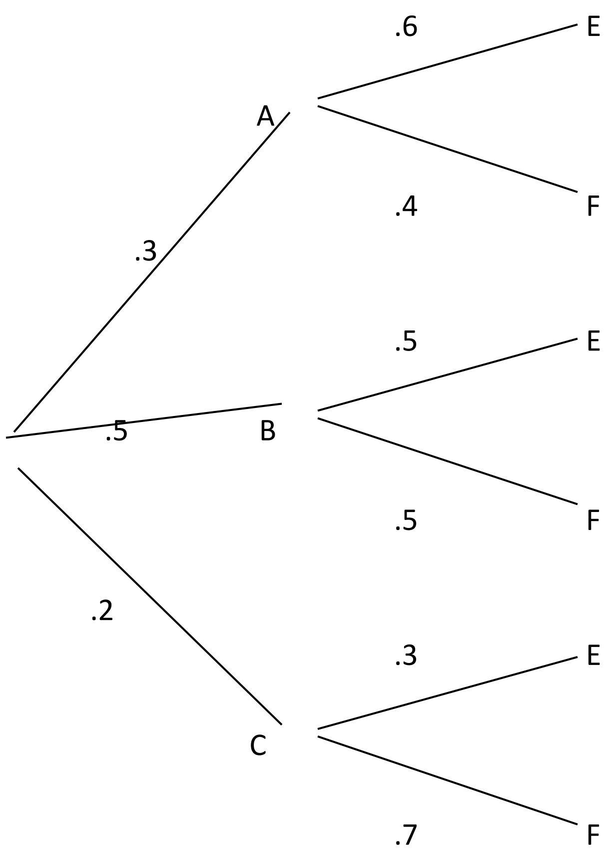
11. An urn contains six red, five black, and four green balls. If two balls are selected at random without replacement from the urn, what is the probability that a red ball and a black ball will be selected?

12. The quality control department of Starr Communications, the manufacturer of video game CD's, has determined from records that 1.5% of the CD's have video defects, 0.8% have audio defects and 0.4% have both audio and video defects. What is the probability that a video game purchased by a customer

a. Will have a video or audio defect (can be both)?
b. Will not have a video or audio defect?

13. Let E and F be two events and suppose P(E) = 0.35 and P(F) = 0.55 and P(EUF) = 0.70. Find P(E|F). (Hint: use a Venn Diagram!)

For 14-18, use the tree diagram to find the given probability.



14. $P(A \cap E)$

15. $P(B \cap E)$

16. $P(C \cap E)$

17. $P(A|E)$

18. $P(E)$

19. An experiment consists of tossing a fair coin three times and observing the outcomes. Let A be the event that at least one head is thrown and B the event that at most two tails are thrown.

   a. Find P(A).
   b. Find P(B).
   c. Are A and B independent events?

20. Five people are selected at random. What is the probability that none of the people in this group were born on the same day of the week?

21. A pair of fair dice is cast. What is the probability that the sum of the numbers falling uppermost is 8 if it known that the two numbers are different?

For 22 – 27, three cards are drawn at random from a standard deck of 52 playing cards. Find the probability of each of the given events.

22. All three cards are aces.

23. All three cards are face cards.

24. The second and third cards are red.

25. The second card is black, given that the first card was red.

26. The second card is a club, given that the first card was black.

27. Applicants who wish to be admitted to a certain professional school in a large university are required to take a screening test that was devised by an educational testing service. From past results, the testing service has estimated that 70% of all applicants are eligible for admission and that 92% of those who are eligible for admission pass the exam, whereas 12% of those who are ineligible for admission pass the exam. Using these results, what is the probability that an applicant for admission passed the exam?
(Hint: Create a tree Diagram!)

Unit 2 Game Theory and Matrices

1. A certain species of insect lives 5 weeks. Through each of the first 4 weeks only 55% survive into the next week. Each female bug that survives into the 5^{th} week produces, on average, 25 new female bugs. Bugs that only survive the first four weeks do not produce offspring.

a. Construct a Leslie matrix representing the life cycle of these bugs.

12 female bugs in their 5^{th} week of life move into your garage.

b. What is the newborn population after the 7^{th} cycle?

Suppose the initial population looked like this $P_0 = [\,0 \quad 20 \quad 15 \quad 25 \quad 5\,]$,

d. What would the total population be after 21 cycles?

e. What is the growth rate between P_{20} and P_{21} ?

2. Each of the following matrices represents a payoff matrix for a game. Determine the best strategy for the row and column players. If the game is strictly determined, find the saddle point of the game. If the game is not strictly determined, explain why not.

a)

$$\begin{array}{c} \\ A \\ B \end{array}\begin{array}{c} \begin{matrix} C & D \end{matrix} \\ \begin{bmatrix} -2 & 5 \\ 2 & 3 \end{bmatrix} \end{array}$$

Row strategy:
Column strategy:
Strictly Determined?: yes/no
Saddle Point (if yes above):

b)

$$\begin{array}{c} \\ A \\ B \\ C \\ D \end{array}\begin{array}{c} \begin{matrix} E & F & G \end{matrix} \\ \begin{bmatrix} 2 & -1 & 2 \\ -3 & -4 & -1 \\ 4 & 1 & 2 \\ 3 & 1 & -2 \end{bmatrix} \end{array}$$

Row strategy:
Column strategy:
Strictly Determined?: yes/no
Saddle Point(if yes above):

3. Two softball teams submit equipment lists for the season.
Women's Team: 12 bats, 45 balls, 15 uniforms
Men's Team: 15 bats, 38 balls 17 uniforms

Each bat costs $21, each ball costs $4, and each uniform costs $30.

(A) Set up and label two matrices.

(B) Find the total cost of equipment for each team. (Hint: Use matrix multiplication)

4. A hospital categorizes its patients as well (in which case they are discharged), good, critical, and deceased. Data show that the hospital's patients move from one category to another according to the probabilities shown in the

transition matrix: $\begin{bmatrix} 1 & 0 & 0 & 0 \\ .5 & .3 & .2 & 0 \\ 0 & .3 & .6 & .1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

a) Write an initial-state matrix for a patient who enters the hospital in critical condition.

b) If patients are reclassified daily, predict the patient's future after one week in the hospital.

c) Predict the future of any patient in the long run.

5. Decode the message: 34 -35 49 64 -17 -4 92 -25 118 70 -26 10 56 -47 53 86 -14 25 using the encoding

matrix $\begin{bmatrix} 2 & 0 & -3 \\ 4 & -1 & 5 \\ 0 & -2 & 3 \end{bmatrix}$. Show work.

6. Jose, Marianna, and Charlie went to a craft store to purchase supplies to create decorations for the upcoming prom. Jose purchased three sheets of craft paper, four boxes of markers, and five glue sticks. His bill, before tax was $24.40. Marianna spent $30.40 when she bought six sheets of craft paper, five boxes of markers, and two glue sticks. Charlie's purchases totaled $13.40 when he bought three sheets of craft paper, two boxes of markers and one glue stick. Determine the unit cost of each item, and the information requested below.

a. Define variables:

b. System of equations:

c. Matrix work:

d. Solution (as a sentence): ____

Unit 3 Limits and Functions

Use the function to answer the questions below. $y = \dfrac{x-7}{x^2 - x - 42}$

Domain:	Range:	Vertical Asymptote:	Horizontal Asymptote:	Removable discontinuity:
X-intercept:	Y-intercept:	Increasing:	Decreasing:	Nonremovable discontinuity:
$\lim_{x\to\infty} f(x) =$	$\lim_{x\to-\infty} f(x) =$	$\lim_{x\to 7} f(x) =$	$\lim_{x\to -6^-} f(x) =$	$\lim_{x\to -6} f(x) =$

2. For the following function $f(x)=x^3 + 2x^2 - 7x + 3$:
Determine all local maximums and minimums. Determine the increasing and decreasing intervals.

3. State whether the function is odd, even, or neither. Support graphically and confirm algebraically. List the type of symmetry (y-axis/origin)

a) $f(x) = \sqrt{x^3 + x - 3}$

b) $f(x) = \dfrac{x^2 + x^3}{x^3}$

4. Given $f(x) = 4x^2 - x + 3$ and $g(x) = \sqrt{x+1}$;

a) Find. $(g \circ f)(x)$ and state its domain in interval notation.

b) Find $(f(g(5))$.

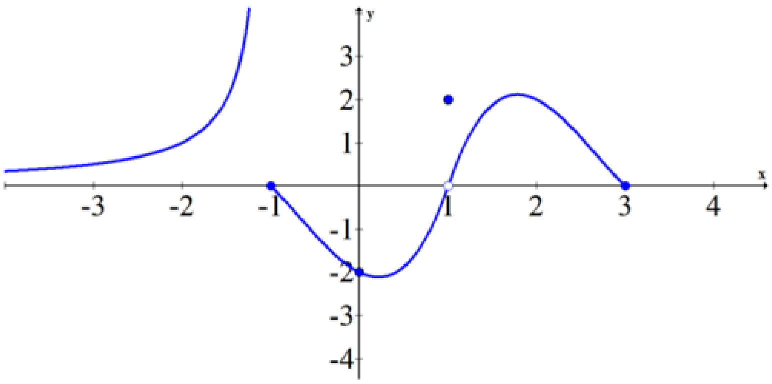
c) Find $g(x+1) - f(4)$.

5. Determine the domain, range, discontinuities (holes and asymptotes) and the horizontal asymptotes.

a) $f(x) = \dfrac{x^2 - 16}{x^3 - 64}$

b) $g(x) = \dfrac{x - 3}{2x^2 + x - 21}$

6. Use the graph below to find the following values.



$\lim\limits_{x \to \infty} f(x) =$

$f(-1) =$

$\lim\limits_{x \to -1} f(x) =$

$f(2) =$

$\lim\limits_{x \to 0} f(x) =$

$f(3) =$

$\lim\limits_{x \to 1} f(x) =$

$\lim\limits_{x \to 0} f(x) =$

Unit 4 Derivatives

Find the derivative of the function using the **limit definition** of derivative. Show all work!

1. $f(x) = \sqrt{x-3}$

2. $f(x) = 3x^2 - x + 5$

Find the **slope** of the tangent line to the graph of *f* at a given point.

3. $f(x) = (4x^3 - 5x^2)(1+2x); \quad (-1, 7)$

4. $f(x) = \dfrac{2x - 3x^2}{5x+1}; \quad x = 1$

Find the **equation** for the tangent line at the given point.

5. $f(x) = -5x^2 + 8x + 2; \quad x = 3$

6. $f(x) = \dfrac{6}{x+1}; \quad (11, 7)$

7. The position of a particle at time t sec is $s = t^3 - 6t^2 + 9t$ meters.
   (a) Find the instantaneous velocity t = 4 seconds.

   (b) Find the acceleration for each time the particle's velocity is zero.

Find the derivative of the following.

8. $g(x) = 3x^2 - \sqrt[4]{x^3}$

9. $g(x) = \dfrac{3}{x^3} - \dfrac{4}{x^2} + 5$

10. $f(x) = \dfrac{6 - x + 3x^2}{4 - 9x}$

12. $f(x) = (\sqrt{2x^2 - 4x + 1})(6x - 5)$

13. $g(x) = \dfrac{\sqrt[3]{x^2+1}}{3x-5}$

Unit 5 Election Theory and Fair Division

Aubrey, Ben, Charlie, Derek, and Emily are running for class president. Use the following preference schedules to determine the winner as indicated in questions 1-6.

# of votes	**20**	**22**	**12**	**9**
First Choice	A	B	E	D
Second Choice	C	D	C	E
Third Choice	D	C	D	C
Fourth Choice	E	A	A	A
Fifth Choice	B	E	B	B

1. Determine the winner using a 5-4-3-2-1 Borda count.

2. Determine the plurality winner.

3. Determine the runoff winner.

4. Determine the sequential runoff winner.

5. Determine the Condorcet winner, if there is one.

6. Suppose that this election is conducted by the approval method and all the voters decide to approve of the first three choices on the preference schedule. Determine the approval winner.

7. What are Arrow's 5 conditions of approval voting?

8. Anne, Beth, and Jay are heirs to an estate that includes a computer, a used car, a stereo, and $6,000. The will states that Anne is to receive 25%, Beth receives 35%, and Jay receives 40%. Each heir has submitted bids for the items in the estate as summarized in the following table.

	Anne	Beth	Jay
Computer	$1,800	$1,500	$1,650
Car	$2,600	$2,400	$2,000
Stereo	$1,000	$800	$1,200

Determine the properties that each person receives and the final amount of cash that he or she receives or pays.

9. Consider a situation in which voters A, B, and C have 1, 3, and 2 votes respectively and 4 votes are needed to pass an issue.

a. List all winning coalitions and their vote totals.

b. Find the power index of each voter.

Unit 6 Graph Theory

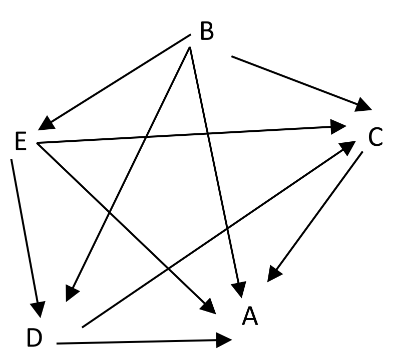
1. Ms.Collins want to remodel her house. The following task table lists the task, time needed in days, and immediate prerequisites for each task. Draw a directed graph to represent the situation, complete the table with the earliest start time and latest start time for each task, calculate the minimal project time required to complete the project, and state the critical path.

	Task	Time	Prerequisites	EST
A	Initial Clean-Up	2	None	
B	Find a Contractor	6	None	
C	Get an Inspection	4	A	
D	Framing	4	C	
E	Plumbing	3	B	
F	Electrical Work	3	D	
G	Install Windows	2	D	
H	Finish Carpentry	3	E,F	
I	Flooring	1	H,G	
J	Install appliances	2	I	
K	Reassemble house	3	I	
L	Clean Up	2	J,K	

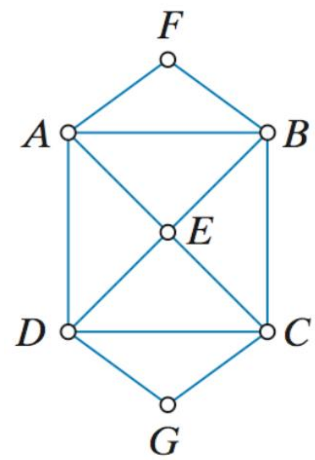
a. Minimum Project Time:

b. Critical Path:

2. Given the following tournament, find a Hamiltonian Path. Then use the path to rank the players from 1st place – 5th place



3. Consider the graph to the right:



a. Is there an Euler circuit? If so, find one. If not, explain.

b. Is there an Euler path? If so, find one. If not, explain.

c. Is there a Hamiltonian circuit? If so, find one; if not, explain why not.

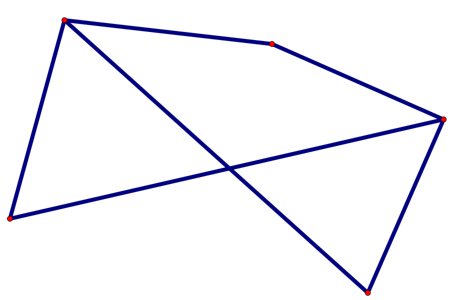
d. Is there a Hamiltonian path? If so, find one; if not, explain why not.

4. In scheduling the final exams for summer school at Green Hope High School six different exams must be scheduled. The following table shows the exams that are needed for seven different students. How many different time slots are necessary to schedule finals in a way that no student has two finals at the same time?

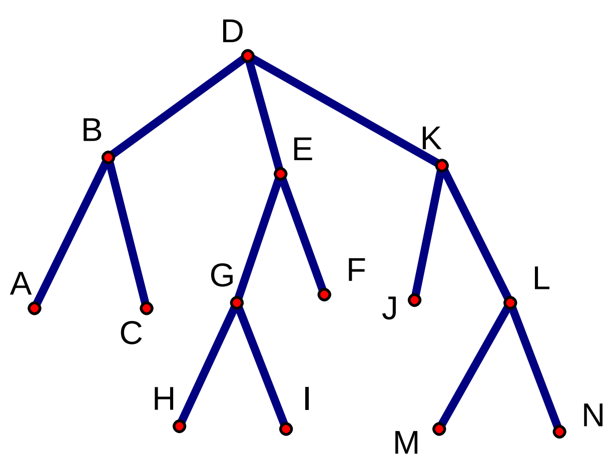
Students							
Exam	1	2	3	4	5	6	7
(C) Civics	X	-	X	-	X	-	X
(F) French	-	X	-	X	-	X	-
(S) Science	X	X	-	-	-	-	X
(H) History	-	-	X	-	-	X	-
(A) Art	-	-	-	X	X	-	-
(M) Math	X	X	-	X	X	-	X

Determine the minimum number of time slots needed to schedule the six exams.

5. Is this graph planar? How do you know?



6. Name the level, parent and children of vertex G.



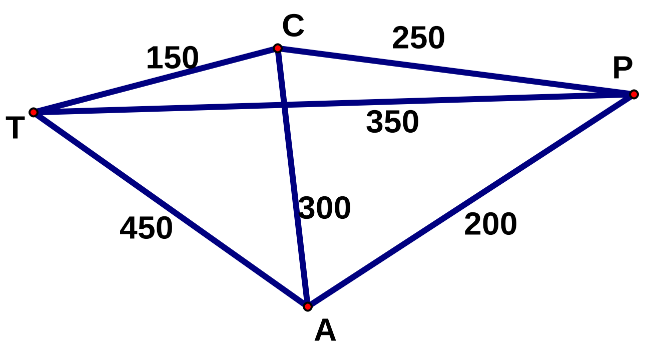
7. Draw an expression tree for each of the following and write the post-order listing for the expression.

(6 + 3) / (4 + 2) – 5 * 8 (B * C) / A + (D – E) * (F + G)

8. Solve each RPN expression:

2 5 * 4 + 3 2 * 1 + / 3 4 + 3 6 / +

9. Use our tree technique to find the shortest round trip, starting in Cleveland, visiting each city in the graph and returning to Cleveland. What is the length of the trip?



C - Cleveland

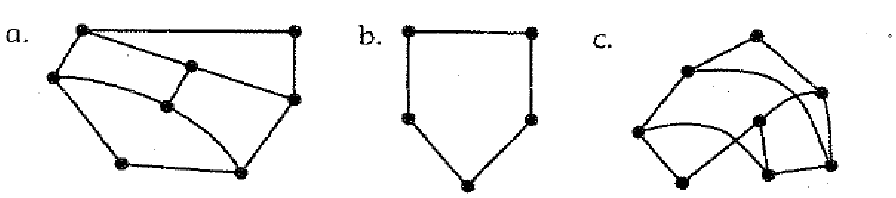
T - Toledo

A - Athens

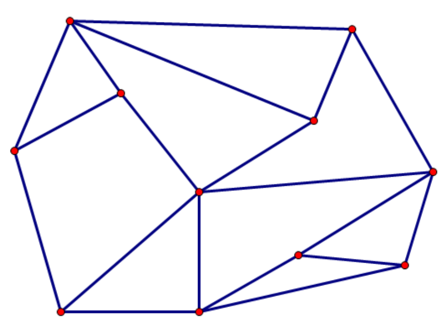
P – Pittsburgh

What route does the nearest neighbor technique produce? Does it produce same length as the tree technique?

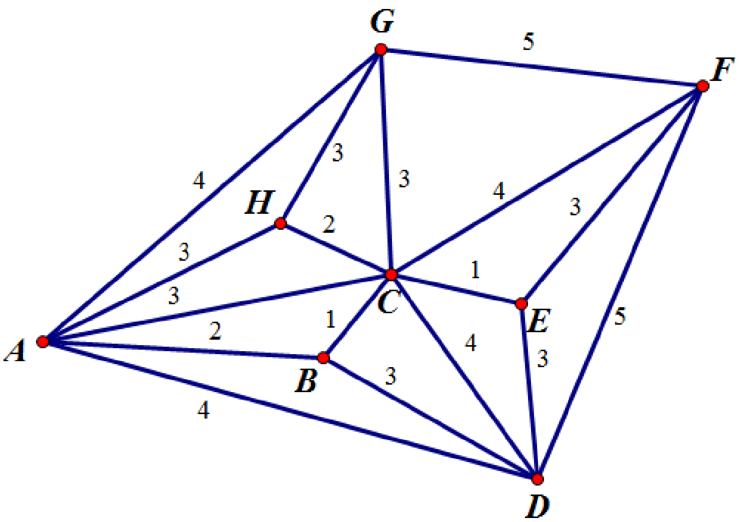
10. State whether the following graphs are bipartite. Explain why or why not.



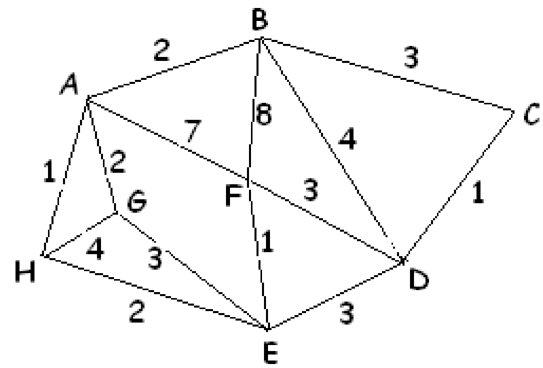
11. Use the Breadth-First Algorithm to trace a spanning tree for this graph.



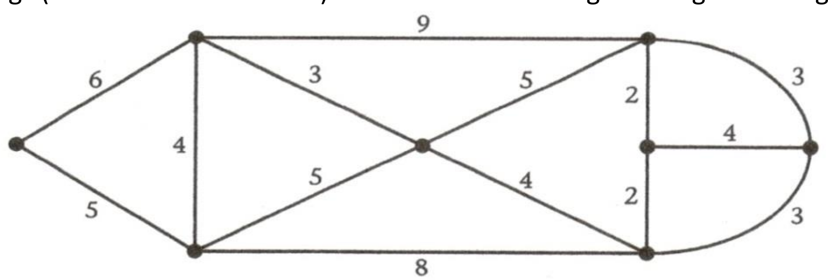
12. Use Kruskal's algorithm to find the minimum spanning tree of the graph. What is the weight of the minimum spanning tree? **Be sure to show your ordered list of edges.**



13. The graph below shows distances between various towns. Find the length of the shortest path from B to F.



14. The vertices of the following graph represent buildings on a small college campus. Administrators on the campus want to connect the buildings with fiber-optic cable and are interested in finding the least expensive way of doing so. The costs of connecting the buildings (in thousands of dollars) are shown as the weighted edges of the graph.



Find the minimum spanning tree for the graph. Darken the edges on the graph above.

What is the minimum total cost of connecting the buildings?

Unit 6 Vocabulary

1. Vertices
2. Graphs
3. Edges
4. Critical Path
5. Earliest Start Time (EST)
6. Connected Graph
7. Complete Graph
8. Adjacent
9. Degree
10. Loop

11. Multigraph
12. Euler Circuit (pronounced OILER)
13. Euler Path
14. Digraph
15. Indegree/ Outdegree
16. Hamiltonian Path
17. Hamiltonian Circuit
18. Tournament
19. Chromatic Number
20. Circuit (Cycle)

Unit 7 Finance Review the problems from the notes, HW, etc in that mini unit.