Key
Fall '18

Warm Up ~ Day 3

1. Find the domain, then convert to fractional/rational exponent.

$$f(x) = \sqrt{7-x}$$

2. Simplify completely: $3(2x + 5)^2$.

3. Find the domain, x & y intercepts, and label any discontinuities:

$$h(x) = \frac{\sqrt{16-x^2}}{x-3}$$

Warm Up ~ Day 3 ANSWERS

1. Find the domain, then convert to fractional/rational exponent.

$$f(x) = \sqrt{7-x} = (7-x)^{\frac{1}{2}}$$

$\boxed{\text{Domain: } (-\infty, 7]}$

1)
$7 - x = 0$
$7 = x$
$\boxed{\text{x-int } (7,0)}$

7

test x = 0: $= \sqrt{7-0} = \sqrt{7}$ ✓

test x = 10: $y = \sqrt{7-10}$, $y = \sqrt{-3}$

2. Simplify completely: $3(2x + 5)^2$.

$3(2x+5)(2x+5)$

$\boxed{12x^2 + 60x + 75} = 3(4x^2 + 20x + 25)$

3. Find the domain, x & y intercepts, and label any discontinuities:

$$h(x) = \frac{\sqrt{16-x^2}}{x-3}$$

$\boxed{\text{Domain: } [-4,3) \cup (3,4]}$

$\boxed{\text{x-int: } (-4, 0) \,\&\, (4, 0)}$

$\boxed{\text{y-int: } (0, -\tfrac{4}{3})}$

Nonremovable Discontinuity (Vertical Asymptote at x = 3)

y-int
$y = \frac{\sqrt{16-0^2}}{0-3}$
$y = \frac{4}{-3}$ → $\boxed{\text{y-int } (0, -4/3)}$

x-int
$0 = \frac{\sqrt{16-x^2}}{x-3}$
$0 = \sqrt{16-x^2}$
$0 = 16 - x^2$
$0 = (4-x)(4+x)$
$x = 4, -4$

$\boxed{\text{x-int } (-4,0) \; (4,0)}$

-4 3 4

$\boxed{\text{VA at } x=3}$ from denom.

test x = -10: $\sqrt{16-(-10)^2}$

test x = 0: $y = \frac{\sqrt{16-0^2}}{0-3}$

test x = 3.5: $\sqrt{16-(3.5)^2} = 4$ ✓

test x = 10: $y = \frac{\sqrt{16-(10)^2}}{10-3} = \sqrt{-84}$ X

Summary

Domain:

Consider the **vertical asymptotes** and the x-value of the **hole**

Make sure values under the radical are positive

x-intercept:

Set **y = 0** and solve for **x**.

y-intercept:

Set **x = 0** and solve for **y**.

Definition of Degree

- **Degree of a polynomial in one variable:** the value of the greatest exponent

  Ex: $f(x) = 4x^2 + 9x + 8$

  Degree: 2

  Ex: $g(x) = -5x^3 + 6x^2 + 4x$

  Degree: 3

- **Degree can help us with determining the horizontal asymptote of rational functions...**

☐ Add

* Be careful... polynomial may not be in order

$h(x) = 7x^4 - 6x^5 + x^2$

Degree: 5

Asymptote Lab
Packet p. 4-5

Let's do one or two together!

Horizontal Asymptotes

For horizontal asymptotes, think BOSTON for ***polynomials***! Looking at the degree of top & bottom…

Degree Bigger
Bottom > Top
y=O

$f(x)=\dfrac{2x}{x^2+3x}$ D1 / D2 $\boxed{H.A.: y=0}$

Same = ratio of coeff.

$g(x)=\dfrac{2x^3}{5x^3+4x^2}$ D3 / D3 $\boxed{H.A.: y=\frac{2}{5}}$

Top > Bottom
O No HA.
N

$h(x)=\dfrac{5x^2}{7x+3}$ D2 / D1 $\boxed{No\ H.A.}$

What is End Behavior?

A Graphical Approach

End Behavior of Graphs

- Polynomials
- Rational Functions

End Behavior: what the graph does as $x \to \infty$ or $x \to -\infty$ (What is the graph doing at huge x values and itty bitty x values?)

* End Behavior is similar to finding Horizontal Asymptotes (if there is a HA)!

End Behavior: Rational Functions

- Rational functions: Ratio of two polynomials
- As you may have guessed, degrees of these two polynomials play a key role in determining the end behavior.

Consider the following scenarios:

1) The degree of the numerator is bigger than the degree of the denominator.
2) The degree of the numerator is the same as the degree of the denominator.
3) The degree of the numerator is smaller than the degree of the denominator.

Determine a way to predict end behavior without graphing.

Definition of a Limit

- If f(x) becomes arbitrarily close to a unique number L as x approaches c from either side, the **limit of f(x)** as x approaches c is L.

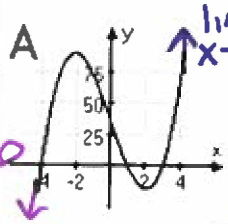
- L is a y-value! c is an x-value!

$$\lim_{x \to c} f(x) = L$$

As x gets closer to some value c) then the y-value (f(x)) gets closer to L

Before an investigation on End Behavior, let's evaluate limits at ∞ and $-\infty$ for these graphs.

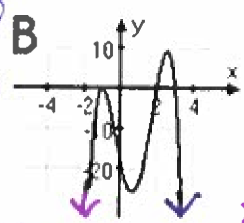
A



$\lim_{x\to\infty} f(x) = \infty$

$\lim_{x\to -\infty} f(x) = -\infty$

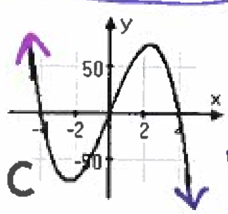
B



$\lim_{x\to\infty} f(x) = -\infty$

$\lim_{x\to -\infty} f(x) = -\infty$

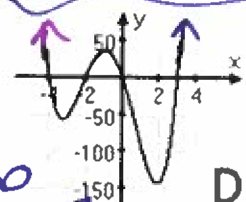
C



$\lim_{x\to\infty} f(x) = -\infty$

$\lim_{x\to -\infty} f(x) = \infty$

D

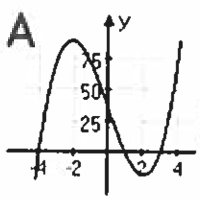


$\lim_{x\to\infty} f(x) = \infty$

$\lim_{x\to -\infty} f(x) = \infty$

Before an investigation on End Behavior, let's evaluate limits at ∞ and $-\infty$ for these graphs.

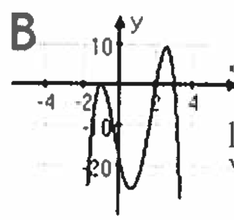
A



$$\lim_{x\to\infty} f(x) = \infty$$

$$\lim_{x\to -\infty} f(x) = -\infty$$

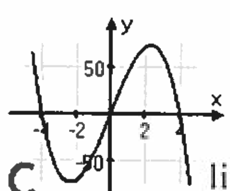
B



$$\lim_{x\to\infty} f(x) = -\infty$$

$$\lim_{x\to -\infty} f(x) = -\infty$$

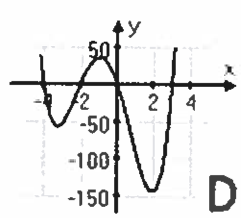
C



$$\lim_{x\to\infty} f(x) = -\infty$$

$$\lim_{x\to -\infty} f(x) = \infty$$

D



$$\lim_{x\to\infty} f(x) = \infty$$

$$\lim_{x\to -\infty} f(x) = \infty$$

skip if low on time

End Behavior - Polynomials

Graph the following in your calculator and take note of the end behavior. Find the limits at ∞ and -∞ of each. Express them using proper Limit notation. Then, determine a way to predict end behavior without graphing and **using limits.**

$y = 4x^2 + 9x + 8$

$y = -5x^3 + 6x^2 + 4x$

$y = 2x^5 + 3$

$y = -x^4 + 9x$

$y = -x^2 + 3x + 7$

$y = 9x^5 + 7x^4 + 3x + 2$

Hint: look at Degree and Leading Coefficient

End Behavior - Polynomials

Graph the following in your calculator and take note of the end behavior. Determine a way to predict end behavior without graphing and **using limits.** (Hint: Degree)

$y = 4x^2 + 9x + 8$ $\quad \lim_{x\to\infty} f(x) = \infty \quad \lim_{x\to-\infty} f(x) = \infty$

$y = -5x^3 + 6x^2 + 4x$ $\quad \lim_{x\to\infty} f(x) = -\infty \quad \lim_{x\to-\infty} f(x) = \infty$

$y = 2x^5 + 3$ $\quad \lim_{x\to\infty} f(x) = \infty \quad \lim_{x\to-\infty} f(x) = -\infty$

$y = -x^4 + 9x$ $\quad \lim_{x\to\infty} f(x) = -\infty \quad \lim_{x\to-\infty} f(x) = -\infty$

$y = -x^2 + 3x + 7$ $\quad \lim_{x\to\infty} f(x) = -\infty \quad \lim_{x\to-\infty} f(x) = -\infty$

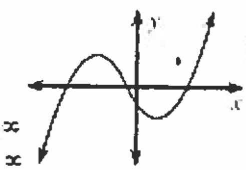
$y = 9x^5 + 7x^4 + 3x + 2$ $\quad \lim_{x\to\infty} f(x) = \infty \quad \lim_{x\to-\infty} f(x) = -\infty$

Skip if low on time

End Behavior Summary

Degree: odd
Leading coefficient: positive

$f(x) \to +\infty$ as $x \to +\infty$



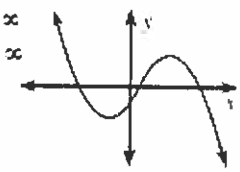
$f(x) \to -\infty$ as $x \to -\infty$

Odd, +
→ starts down, ends up

(Like + Slope)

Degree: odd
Leading coefficient: negative

$f(x) \to +\infty$ as $x \to -\infty$



$f(x) \to -\infty$ as $x \to +\infty$

Odd, —
→ starts up, ends down

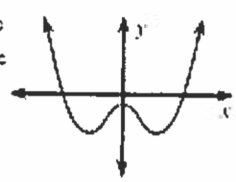
(Like – Slope)

<u>O</u>dd → Ends go <u>O</u>pposite (of each other)

End Behavior Summary

Degree: even
Leading coefficient: positive

$f(x) \to +\infty$ as $x \to -\infty$

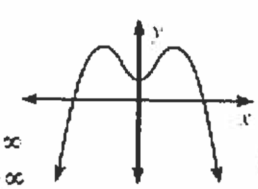


$f(x) \to +\infty$ as $x \to +\infty$

Even, +
→ both ends point up

(Like + Parabola)

Degree: even
Leading coefficient: negative



$f(x) \to -\infty$ as $x \to -\infty$

$f(x) \to -\infty$ as $x \to +\infty$

Even, —
→ both ends point down

(Like – Parabola)

<u>E</u>ven → Ends go <u>E</u>xactly the same

Together!

You Try! What is the EQUATION of the horizontal asymptote for the following functions? Then write the end behavior using limits.

HA $y = 3/4$

$$f(x) = \frac{3x^2+9}{7x+4x^2+11}$$ $\frac{D2}{D2}$ $\boxed{HA: y = 3/4}$

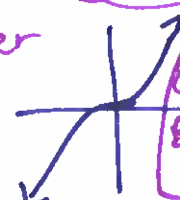
Same degree → use ratio of coeff.

End Beh:
$\lim_{x\to\infty} f(x) = 3/4$
$\lim_{x\to-\infty} f(x) = 3/4$

Bottom > Top
y=0
Same = ratio
Top > Bottom
↑ O No HA.
N

You Try!!

$$g(x) = \frac{4x^3}{5x^2+9}$$ $\frac{D3}{D2}$ Top degree bigger $\boxed{\text{No HA}}$



End Beh
$\lim_{x\to\infty} g(x) = \infty$
$\lim_{x\to-\infty} g(x) = -\infty$

$$h(x) = \frac{7x+15}{2x^2}$$ $\frac{D1}{D2}$ Bottom degree bigger $\boxed{HA: y = 0}$

End Behavior:
$\lim_{x\to\infty} h(x) = 0$
$\lim_{x\to-\infty} h(x) = 0$

You Try! What is the EQUATION of the horizontal asymptote for the following functions? Then write the end behavior using limits.

Bottom > Top
y=0
Same = ratio
Top > Bottom
↑ O No HA.
N

$$f(x) = \frac{3x^2+9}{7x+4x^2+11}$$
$H.A.: y = 3/4$

$\lim_{x\to\infty} f(x) = \frac{3}{4}$

$\lim_{x\to-\infty} f(x) = \frac{3}{4}$

$$g(x) = \frac{4x^3}{5x^2+9}$$

$\lim_{x\to\infty} g(x) = \infty$

$\lim_{x\to-\infty} g(x) = -\infty$

$H.A.: none$

$$h(x) = \frac{7x+15}{2x^2}$$

$\lim_{x\to\infty} h(x) = 0$

$\lim_{x\to-\infty} h(x) = 0$

$H.A.: y = 0$

Day 4 started here Fall '18

Remember: least to greatest so go bottom to top

x-int: $0=\sqrt{2x+7}$
$0=2x+7$
$x=-7/2$
$\boxed{\text{xint: } (-7/2, 0)}$

y-int: $y=\sqrt{2(0)+7}$
$y=\sqrt{7}$
$\boxed{\text{y-int } (0, \sqrt{7})}$ above x-int

Finding the Range of a Function

- Use numeric, algebraic and graphical approaches simultaneously.
- Keep in mind we are finding ALL y-coordinates of points on the graph. * Radicals typically have a vertex * Rationals skip HA & y of hole
- Write the range of the following functions in interval notation.

You Try!

$f(x)=\sqrt{2x+7}$ $\boxed{R: [0, \infty)}$

$g(x)=\dfrac{8}{2x+12}$ $\boxed{R: (-\infty, 0) \cup (0, \infty)}$

$\boxed{VA: x=-6}$
$2x+12=0$
$\frac{D0}{D1} \to \boxed{HA: y=0}$
$\boxed{\text{x-int: none}}$ $0=\dfrac{8}{2x+12}$
$0=8$
y-int $y=\dfrac{8}{2(0)+12}=\dfrac{8}{12}$
$\boxed{\text{y-int } (0, 2/3)}$
$\boxed{\text{Hole: none}}$

Horizontal Asymptotes? ???

$m(x)=\dfrac{\sqrt{x}}{x-9}$

x-int $0=\dfrac{\sqrt{x}}{x-9}$
$0=\sqrt{x}$
$0=x$
$\boxed{\text{x-int } (0,0)}$
y-int $y=\dfrac{\sqrt{0}}{0-9}=0$
$\boxed{\text{y-int } (0,0)}$

$\frac{D\,1/2}{D\,1} \to \boxed{HA: y=0}$ BUT hit this HA at $\sqrt{}$ vertex

So... $\boxed{R: (-\infty, \infty)}$

Finding the Range of a Function

- Use numeric, algebraic and graphical approaches simultaneously.
- Keep in mind we are finding ALL y-coordinates of points on the graph.
- Write the range of the following functions in interval notation.

$f(x)=\sqrt{2x+7}$

Range: $[0, \infty)$

$g(x)=\dfrac{8}{2x+12}$

Range: $(-\infty, 0) \cup (0, \infty)$

Horizontal Asymptotes? ???

$m(x)=\dfrac{\sqrt{x}}{x-9}$

Range: $(-\infty, \infty)$

Summary

Domain:

Consider the $\underbrace{\text{vertical asymptotes}}_{}$ and the x-value of the hole (,)

x = #

Make sure values under the radical are positive (find x-int for √)

Range:

Consider the horizontal asymptotes and the y-value of the hole

y = #

*for √

x-intercept:

Set **y = 0** and solve for **x.** (,)

y-intercept:

Set **x = 0** and solve for **y.** (,)

Practice!

Find the...

- -Domain
- -x & y intercepts
- -End Behavior using limits
- -Range

Remember, get hole & VA to help with domain

*Remember, get hole & HA to help with range

24.) $g(x)=\dfrac{x}{x-2}$

* Do #11 together *

& #23 together

11.) $f(x)=\dfrac{1\cancel{(x-1)}}{(x+3)\cancel{(x-1)}}$

Hole at $x=1$

$y=\frac{1}{x+3}=\frac{1}{1+3}=\frac{1}{4}$ $\boxed{\text{Hole at } (1, 1/4)}$

x-int $0=\frac{1}{x+3}$ $0=1 \rightarrow$ $\boxed{\text{No x-int}}$

y-int $y=\frac{1(0-1)}{(0+3)(0-1)}=\frac{1}{3}$ $\boxed{\text{y-int } (0, 1/3)}$

VA: $\boxed{x=-3}$ from extra denom.

21.) $g(x)=\dfrac{3}{x}+1$

23.) $f(x)=\dfrac{|x-1|}{x}$

End Beh

$\boxed{\lim_{x\to\infty} f(x)=0\ ;\ \lim_{x\to-\infty} f(x)=0}$

$\boxed{\text{HA: } y=0}$

D1 / D2

Textbook: p.98

$\boxed{R: (-\infty,0)\cup(0,1/4)\cup(1/4,\infty)}$

Skip HA   Skip y of hole

-3   1

$\boxed{D: (-\infty,-3)\cup(-3,1)\cup(1,\infty)}$

Skip VA   Skip x of Hole

1/4 0

x-int
$0 = \frac{x}{x-2}$
$0 = x$
$\boxed{\text{x-int } (0,0)}$
$\boxed{\text{y-int } (0,0)}$
$y = \frac{0}{0-2} = 0$

Textbook: p.98

Find the…
- -Domain
- -x & y intercepts
- -End Behavior using limits
- -Range

24.) $g(x) = \dfrac{x}{x-2}$ $\quad \frac{D1}{D1} \rightarrow$ ratio $y = \frac{1}{1}$

$\boxed{\text{VA: } x = 2}$ $\boxed{\text{HA: } y = 1}$

$\boxed{\text{Hole: none}}$
no common factor to "slash"

End Beh
$\lim_{x\to\infty} g(x) = 1$; $\lim_{x\to-\infty} g(x) = 1$

$\boxed{D: (-\infty, 2) \cup (2, \infty)}$
skip VA

$\boxed{R: (-\infty, 1) \cup (1, \infty)}$
skip HA

11.) $f(x) = \dfrac{1(\cancel{x-1})}{(x+3)(\cancel{x-1})}$

$\boxed{\text{Hole at } (1, 1/4)}$ $\quad \frac{1}{1+3}$

$\boxed{\text{VA: } x = -3}$

$\boxed{\text{HA: } y = 0}$ $\quad \frac{D1}{D2}$

x-int

---

x-int:
$0 = \frac{3}{x} + 1$
$-1 = \frac{3}{x}$ $\quad -x = 3$
$x = -3$
$\boxed{\text{x-int } (-3, 0)}$

y-int
$y = \frac{3}{0} + 1 \rightarrow$ $\boxed{\text{y-int none}}$

Textbook: p.98

Find the…
- -Domain
- -x & y intercepts
- -End Behavior using limits
- -Range

21.) $g(x) = \dfrac{3}{x} + 1$

End Beh
$\lim_{x\to\infty} g(x) = 1$
$\lim_{x\to-\infty} g(x) = 1$

$\boxed{\text{VA: } x = 0}$

$\boxed{\text{HA: } y = 1}$ look in calc with Δtbl = 100

$\boxed{\text{Holes: none}}$

$\boxed{D: (-\infty, 0) \cup (0, \infty)}$

$\boxed{R: (-\infty, 1) \cup (1, \infty)}$

23.) $f(x) = \dfrac{|x-1|}{x}$

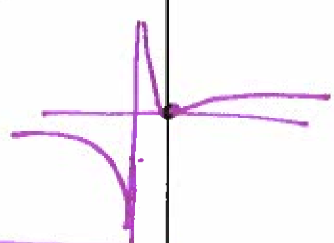
x-int
$0 = \frac{|x-1|}{x}$
$0 = |x-1|$
$x = 1$
$\boxed{\text{x-int } (1, 0)}$

y-int
$y = \frac{|0-1|}{0}$
$\boxed{\text{y-int: none}}$

$\boxed{\text{VA: } x = 0}$

$\boxed{\text{Holes: none}}$

$\boxed{D: (-\infty, 0) \cup (0, \infty)}$
skip VA



$\boxed{\text{End Beh } \lim_{x\to\infty} f(x) = 1 \; ; \; \lim_{x\to-\infty} f(x) = -1}$

$\boxed{R: (-\infty, -1) \cup [0, \infty)}$

Textbook: p.98

Find the...
- -Domain
- -x & y intercepts
- -End Behavior using limits
- -Range

$$24.)\ g(x)=\frac{x}{x-2}$$

$\lim\limits_{x\to\infty} g(x)=1$

$\lim\limits_{x\to-\infty} g(x)=1$

$Domain:\ (-\infty,2)\cup(2,\infty)$

$Range:(-\infty,1)\cup(1,\infty)$

$x-\text{int}:(0,0)$

$y-\text{int}:(0,0)$

$$11.)\ f(x)=\frac{x-1}{(x+3)(x-1)}$$

$\lim\limits_{x\to\infty} f(x)=0$

$\lim\limits_{x\to-\infty} f(x)=0$

$Domain:\ (-\infty,-3)\cup(-3,1)\cup(1,\infty)$

$Range:(-\infty,0)\cup(0,\frac{1}{4})\cup(\frac{1}{4},\infty)$

$x-\text{int}:none$

$y-\text{int}:(0,\frac{1}{3})$

$Hole:(1,\frac{1}{4})$

Textbook: p.98

Find the...
- -Domain
- -x & y intercepts
- -End Behavior using limits
- -Range

$$21.)\ g(x)=\frac{3}{x}+1$$

$\lim\limits_{x\to\infty} g(x)=1$

$\lim\limits_{x\to-\infty} g(x)=1$

$Domain:\ (-\infty,0)\cup(0,\infty)$

$Range:(-\infty,1)\cup(1,\infty)$

$x-\text{int}:(-3,0)$

$y-\text{int}:none$

$$23.)\ f(x)=\frac{|x-1|}{x}$$

$\lim\limits_{x\to\infty} f(x)=1$

$\lim\limits_{x\to-\infty} f(x)=-1$

$Domain:\ (-\infty,0)\cup(0,\infty)$

$Range:\ (-\infty,-1)\cup[0\infty)$

$x-\text{int}:(1,0)$

$y-\text{int}:none$

Fall '18

Asymptote Lab

Packet p. 4-5

HW

Packet p. ~~4~~ 4-5

&

Finish PSAT Handouts