ICM ~Unit 4 ~ Day 2

Section 1.2—Domain, Continuity, Discontinuities

Warm Up Day 2

Find the domain, x-intercepts and y-intercepts.

1. $\dfrac{\sqrt{x+2}}{3x-5}$

2. $\dfrac{\sqrt{x^2+1}}{x^2-9}$

3. *Factor completely.* $6x^2-4x-16$

4. *Factor completely.* $8x^3+27$

Warm Up Day 2

Find the domain, x-intercepts and y-intercepts.

1. $\dfrac{\sqrt{x+2}}{3x-5}$

2. $\dfrac{\sqrt{x^2+1}}{x^2-9}$

3. *Factor completely.* $6x^2-4x-16$

4. *Factor completely.* $8x^3+27$

Warm Up Day 2

Find the domain, x-intercepts and y-intercepts.

1. $\dfrac{\sqrt{x+2}}{3x-5}$

Domain: $[-2,\frac{5}{3})\cup(\frac{5}{3},\infty)$

$x-\text{int}: (-2,0)$

$y-\text{int}: (0,-\sqrt{2}/5)$

2. $\dfrac{\sqrt{x^2+1}}{x^2-9}$

$(x+3)(x-3)$

Domain: $(-\infty,-3)\cup(-3,3)\cup(3,\infty)$

$x-\text{int}$: *none*

$y-\text{int}: (0,-\frac{1}{9})$

#1

$x+2=0$

-2 $\quad 5/3$

test $x=-10$ $\quad \dfrac{\sqrt{-10+2}}{3(-10)-5}$

test $x=0$ $\quad \dfrac{\sqrt{0+2}}{3(0)-5}$

$3x-5\neq 0$

$x\neq \frac{5}{3}$

$0=\dfrac{\sqrt{x+2}}{3x-5}$ $\quad 0=\sqrt{x+2}$

$\boxed{x\text{-int } (-2,0)}$

$y=\dfrac{\sqrt{0+2}}{3(0)-5}=\dfrac{\sqrt{2}}{-5}$

$\boxed{y\text{-int } (0,-\frac{\sqrt{2}}{5})}$

$x^2+1=0$

$x^2=-1$

$x=\pm i$

no restrictions from $\sqrt{\ }$

$y=\dfrac{\sqrt{0^2+1}}{0^2-9}=\dfrac{1}{-9}$

$\boxed{y\text{ int}: (0,-\frac{1}{9})}$

$x^2-9\neq 0$ $\quad x\neq\pm 3$

$0=\dfrac{\sqrt{x^2+1}}{x^2-9}$ $\quad 0=\sqrt{x^2+1}$

$x=\pm i$

$\boxed{x\text{-int}: \text{None}}$

Warm Up Day 2

3. *Factor completely.* $6x^2 - 4x - 16$

$2(3x^2 - 2x - 8)$

$-6 \cdot 4 = -24$

$-6 + 4 = -2$

$2(3x^2 - 6x + 4x - 8)$

$2(3x(x-2) + 4(x-2))$

$2(3x+4)(x-2)$

4. *Factor completely.* $8x^3 + 27$

$\sqrt[3]{8x^3}$ $\sqrt[3]{27}$

$2x$ 3

$(2x+3)((2x)^2 - 2x \cdot 3 + 3^2)$

$(2x+3)(4x^2 - 6x + 9)$

1st one, 2nd one, 1st one squared, Product of two, last squared

SOP
Same
Opp.
Positive
signs

Homework Questions?

Tonight's Homework

Packet p. 2-3

Practice

- Find the domain and x & y intercepts of...

$$f(x)=\frac{1}{x}+\frac{5}{x-3} \qquad h(x)=\frac{\sqrt{4-x^2}}{x-3}$$

$\boxed{D: (-\infty,0)\cup(0,3)\cup(3,\infty)}$

$x \neq 0, 3$

#1 f(x)

x-int: $0=\frac{1}{x}+\frac{5}{x-3}$

$0=\frac{x-3}{x(x-3)}+\frac{5x}{x(x-3)}$

$0=\frac{6x-3}{x(x-3)}$

$0=6x-3$

$\frac{1}{2}=x$

$\boxed{\text{x-int } (1/2, 0)}$

y-int: $y=\frac{1}{0}+\frac{5}{0-3}$

$\boxed{\text{y-int: none}}$

$4-x^2=0 \to 4=x^2 \to x=\pm 2$

$-2 \quad 2 \quad 3$

$x-3\neq 0 \to x\neq 3$

$\boxed{D: [-2, 2]}$

x-int: $0=\frac{\sqrt{4-x^2}}{x-3}$

$0=\sqrt{4-x^2}$

$0=4-x^2$

$x=\pm 2$

$\boxed{\text{x-int } (-2,0)\ (2,0)}$

y-int $y=\frac{\sqrt{4-0}}{0-3}$

$y=2/-3$

$\boxed{\text{y-int } (0, \tfrac{2}{-3})}$

$$g(x)=\frac{\sqrt{4-x}}{(x+1)(x^2+1)}$$

$-1 \quad 4$

test $x=-5$: $\frac{\sqrt{4-5}}{(-5+1)((-5)^2+1)}$

test $x=10$: $\frac{\sqrt{4-10}}{(10+1)(10^2+1)}$

$\boxed{D: (-\infty,-1)\cup(-1,4]}$

x-int: $0=\frac{\sqrt{4-x}}{(x+1)(x^2+1)}$

$0=\sqrt{4-x}$

$x=4$

$\boxed{\text{x-int } (4,0)}$

y-int: $y=\frac{\sqrt{4-0}}{(0+1)(0^2+1)}$

$y=2$

$\boxed{\text{y-int } (0,2)}$

Practice

- Find the domain and x & y intercepts of...

$$f(x)=\frac{1}{x}+\frac{5}{x-3} \qquad h(x)=\frac{\sqrt{4-x^2}}{x-3}$$

Domain: $(-\infty,0)\cup(0,3)\cup(3,\infty)$

x – int : (0.5, 0)

y – int : *None*

Domain: $[-2, 2]$

x – int : (2, 0), (−2, 0)

y – int : $(0, -\frac{2}{3})$

$$g(x)=\frac{\sqrt{4-x}}{(x+1)(x^2+1)}$$

Domain: $(-\infty,-1)\cup(-1,4]$

x – int : (4, 0)

y – int : (0, 2)

Notes Day 2

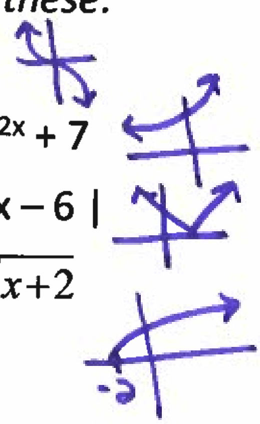
Section 1.2—Domain, Continuity, Discontinuities

Defining Continuity

A function is **<u>continuous</u>** at a point if the graph does not come apart at that point.

Try graphing these:

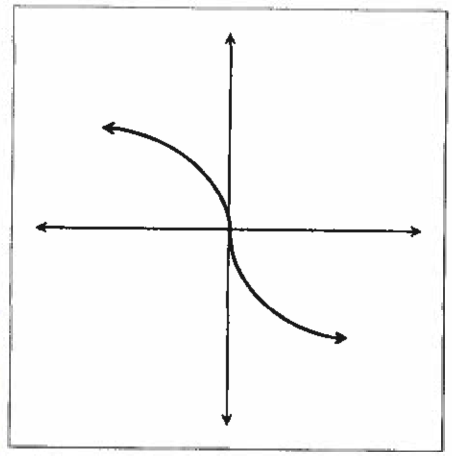
- Ex: $y = -x^3$
- Ex: $f(x) = e^{2x} + 7$
- Ex: $g(x) = |x - 6|$
- Ex: $h(x) = \sqrt{x+2}$



A function is **continuous** at all "x-values" if . . .

- There are NO breaks in the curve.

Notice: You can trace the entire graph without lifting your pencil!



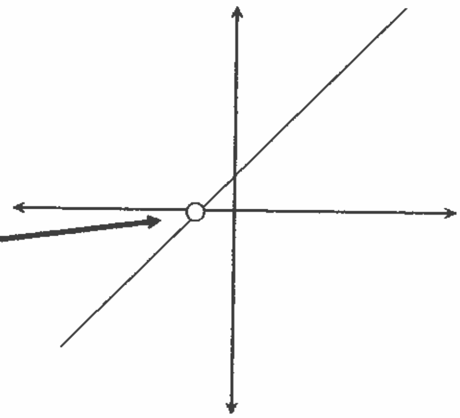
Different Types of **discontinuities:**

1. Removable – also referred to as a "*hole*"
2. Nonremovable
   a. "Jump" (occur in piecewise functions or greatest integer)
   b. Vertical Asymptote (infinite discontinuity)

Removable discontinuity — *"hole"*

- Ex: $f(x) = \dfrac{(x+2)^2}{x+2}$
  - There is a y-value for every x except at x = -2.

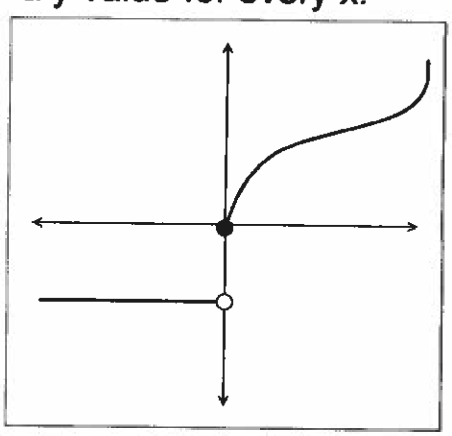
    "hole"
  - The function is "**undefined**" and "**discontinuous**" at x = -2



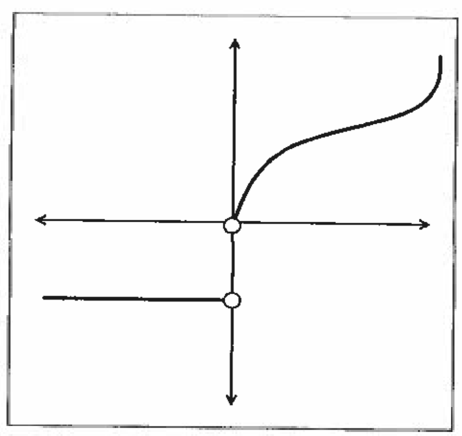
This occurs when there is a zero in the denominator that **can be "canceled out"** when algebraic steps are taken.

Jump Discontinunity – the curve "jumps" from one y-value to the next (NONREMOVABLE)

Notice: this graph still has a y-value for every x.



Notice: this one doesn't

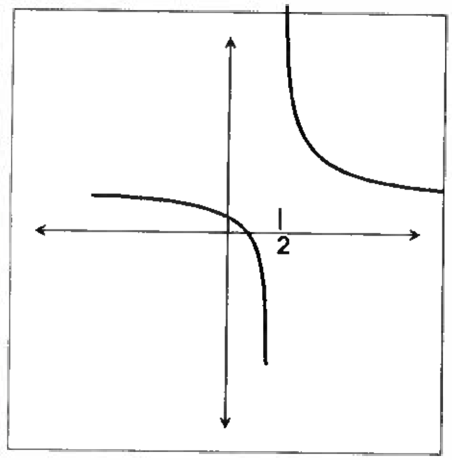


Nonremovable Discontinuity

(Vertical Asymptote)—Infinite Discontinuity

- Ex: $f(x) = \dfrac{x+3}{x-2}$

There is a zero in the denominator of a function that **cannot be "canceled out"** through algebra.



Finding Discontinuities Summary

- To find vertical asymptotes and holes, factor the problem and simplify.
  - **If the factor in the bottom cancels out, it gives us a hole.**
    - Removable $f(x) = \dfrac{\cancel{x+2}}{\cancel{(x+2)}(x-1)}$

So the hole occurs at x = -2

$(-2, -\frac{1}{3})$

How do you find the y-value of the hole?
Simplify and substitute the x-value into the leftover equation! $f(-2) = \dfrac{1}{-2-1}$

  - **If the factor does not cancel out, it gives us a vertical asymptote.**
    - Nonremovable

The vertical asymptote for f(x) is at x = 1

More Examples:

Find the discontinuities and determine whether they are removable or nonremovable. Hint: Factor first!

1) $f(x) = \dfrac{x^2 - 4}{x^2 - x - 6}$

2) $f(x) = \dfrac{x^2 - 3x}{x^3 - 7x^2 + 12x}$

More Examples ANSWERS:

Find the discontinuities and determine whether they are removable or nonremovable. Hint: Factor first!

$\dfrac{\cancel{(x+2)}(x-2)}{\cancel{(x+2)}(x-3)}$

Hole at $x+2=0$, $x=-2$ $\quad y=\dfrac{-2-2}{-2-3}$, $y=\dfrac{-4}{-5}$

$\boxed{\text{Hole at } (-2, 4/5)}$

1) $f(x) = \dfrac{x^2 - 4}{x^2 - x - 6}$

$\boxed{\begin{array}{l} Hole: (-2, \frac{4}{5}) \\ V.A. \quad x = 3 \end{array}}$

VA at $x-3=0$ (use left-over denom.)

$\boxed{\text{VA at } x=3}$

2) $f(x) = \dfrac{x^2 - 3x}{x^3 - 7x^2 + 12x}$

$\boxed{\begin{array}{l} Hole: (0, -\frac{1}{4}) \text{ and } (3, -1) \\ V.A. \quad x = 4 \end{array}}$

$\dfrac{x(x-3)}{x(x^2-7x+12)} = \dfrac{\cancel{x}\cancel{(x-3)}\ 1}{\cancel{x}\cancel{(x-3)}(x-4)}$

Hole at $x=0$ and $x=3$

$y=\dfrac{1}{0-4}=\dfrac{-1}{4}$ $\quad y=\dfrac{1}{3-4}=\dfrac{1}{-1}$

$\boxed{\text{Hole: } (0, -\tfrac{1}{4}) \text{ and } (3, -1)}$

VA: use left-over denom.

$x-4=0$

$\boxed{\text{VA: } x=4}$

You Try!

Find the discontinuities and determine whether they are removable or nonremovable. Hint: Factor first!

3) $\dfrac{x+5}{3x^2+13x-10}$

4) $\dfrac{9x^2-81}{3x+9}$

You Try! ANSWERS

Find the discontinuities and determine whether they are removable or nonremovable. Hint: Factor first!

3) $\dfrac{x+5}{3x^2+13x-10} = \dfrac{1\,\cancel{(x+5)}}{\cancel{(x+5)}(3x-2)}$

$= \dfrac{1}{(3x-2)}$

leftover equation

$\boxed{\begin{array}{l} Hole: \left(-5, -\frac{1}{17}\right) \\ V.A.\ \ x = \frac{2}{3} \end{array}}$

$\dfrac{\cancel{x+5}}{(3x-2)\cancel{(x+5)}}$

Hole at $x=-5$

$y = \dfrac{1}{3(-5)-2} = \dfrac{1}{-17}$

$\boxed{\text{Hole at } \left(-5, -\frac{1}{17}\right)}$

VA: use leftover denom.

$\boxed{\text{VA: } x = 2/3}$

4) $\dfrac{9x^2-81}{3x+9} = \dfrac{9\cancel{(x+3)}(x-3)}{3\cancel{(x+3)}}$

$= 3(x-3)$

leftover equation

$\boxed{\begin{array}{l} Hole: (-3, -18) \\ V.A.\ \ None \end{array}}$

$\dfrac{9(x^2-9)}{3(x+3)} = \dfrac{9\cancel{(x+3)}(x-3)}{3\cancel{(x+3)}}$

Hole at $x=-3$

$y = \dfrac{9(-3-3)}{3} = -18$

$\boxed{\text{Hole at } (-3, -18)}$

VA: use left-over denominator

↳ NONE because no factors left in denom.

Student Practice:

A) Classify the function as continuous or discontinuous.

B) If discontinuous, specify the type of discontinuity and where it exists.

C) State the domain and x & y intercepts.

1) $f(x) = (x+3)(x-2)$

2) $f(x) = \dfrac{x^2-4}{x-2}$

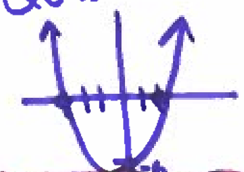
Student Practice: ANSWERS

A) Classify the function as continuous or discontinuous.

B) If discontinuous, specify the type of discontinuity and where it exists.

C) State the domain and x & y intercepts.

Quadratic



1) $f(x) = (x+3)(x-2)$

Continuous

Domain: $(-\infty, \infty)$

$x-\text{int}$: $(-3, 0), (2, 0)$

$y-\text{int}$: $(0, -6)$

2) $f(x) = \dfrac{x^2-4}{x-2}$

$\dfrac{(x+2)\cancel{(x-2)}}{\cancel{x-2}}$

Discontinuous – removable (hole)

Hole at $(2, 4)$

Domain: $(-\infty, 2) \cup (2, \infty)$

$x-\text{int}$: $(-2, 0)$

$y-\text{int}$: $(0, 2)$

Hole at $x=2$

$y = 2+2 = 4$

$\boxed{\text{Hole at } (2,4)}$

Removable Disc.

$\boxed{\text{No VA}}$

because No leftover denom.

x-int: $0 = \dfrac{x^2-4}{x-2}$

$0 = x^2-4$

$0 = (x+2)(x-2)$

$x = -2, \cancel{2}$

$\boxed{\text{x-int } (-2,0)}$

y-int:

$y = \dfrac{0^2-4}{0-2} = \dfrac{-4}{-2} = 2$

$\boxed{\text{y-int } (0,2)}$

Try this one!

$$h(x) = \frac{x^3 + x}{x} = \frac{\cancel{x}(x^2+1)}{\cancel{x}}$$

- Domain: $(-\infty, 0) \cup (0, \infty)$
- x & y intercepts:
- Continuous?
- Type of discontinuity?
- Where is the discontinuity?

• Hole at $x=0$

$y = 0^2 + 1 = 1$

Hole at $(0,1)$

• No VA because no leftover denominator

Try this one! ANSWERS

$$h(x) = \frac{x^3 + x}{x} = \frac{\cancel{x}(x^2+1)}{\cancel{x}}$$

- Domain: *Domain*: $(-\infty, 0) \cup (0, \infty)$
- x & y intercepts: $x-\text{int}: \textit{None}$ $y-\text{int}: \textit{None}$
- Continuous? No because there is a hole
- Type of discontinuity? Removable (Hole)
- Where is the discontinuity? Hole at (0, 1)

• Hole at $x=0$

$y = 0^2 + 1 = 1$

Hole at $(0,1)$

• No VA because no leftover denominator

• x-int:

$0 = \frac{x^3+x}{x}$

$0 = x^3 + x$

$0 = x(x^2+1)$

$x = \cancel{0}, \cancel{\pm i}$ not real

hole really at $x=0$

$\Rightarrow$ No x-int

y-int:

$y = \frac{0^3+0}{0}$

$\Rightarrow$ none b/c can't ÷ by 0

Let's look at this one together…

$$f(x)=\sqrt{16x^4-81x^2}$$

- Domain:
- x & y intercepts:
- Continuous?
- Type of discontinuity?

Let's look at this one… ANSWERS

$$f(x)=\sqrt{16x^4-81x^2} = \sqrt{x^2(16x^2-81)}$$

$x^2(16x^2-81)=0$

$x=0, \pm\frac{9}{4}$ $\quad 16x^2=81$ $\quad x^2=\frac{81}{16}$ $\quad x=\pm\frac{9}{4}$

- Domain: $(-\infty,-\frac{9}{4}]\cup[0]\cup[\frac{9}{4},\infty)$
- x & y intercepts: $x-\text{int}:(-\frac{9}{4},0),\ (0,0),\ (\frac{9}{4},0)$

  $y-\text{int}:(0,0)$
- Continuous? No
- Type of discontinuity? None

$-\frac{9}{4}$ $\quad \frac{9}{4}$

test $x=-10$

$\sqrt{16(-10)^4-81(-10)^2}$

$\sqrt{160{,}000-8100}$

x-int:

$0=\sqrt{16x^4-81x^2}$

$0=16x^4-81x^2$

$0=x^2(16x^2-81)$

$x=0, \pm 9/4$

x-int

$\boxed{(-\frac{9}{4},0),(0,0),(9/4,0)}$

y-int

$y=\sqrt{16(0)^4-81(0)^2}$

$y=\sqrt{0-0}=0$

$\boxed{y\text{-int}: (0,0)}$

Examine the table of values below. All of the following statements are true EXCEPT

x	y_1	y_2
-2.03	-66.67	-4.03
-2.02	-100	-4.02
-2.01	-200	-4.01
-2	ERROR	ERROR
-1.99	200	-3.99
-1.98	100	-3.98
-1.97	66.667	-3.97

$x = -2$

A. x = -2 is a vertical asymptote in y_1

B. x = -2 is an infinite discontinuity in y_1

C. x = -2 is a removable discontinuity in y_2

D. x = -2 is a vertical asymptote in y_2

Tomorrow, you'll do an Asymptote Lab to learn more about this.

Looking at y_2, the y's are in order, but y = -4 was skipped, so there is a Removable Discontinuity (Hole) at (-2, -4)

Definition of Degree

- **Degree of a polynomial in one variable:** the value of the greatest exponent

  Ex: $f(x) = 4x^2 + 9x + 8$

  Degree: 2

  Ex: $g(x) = -5x^3 + 6x^2 + 4x$

  Degree: 3

- **Degree can help us determine the horizontal asymptote of rational functions...**

Horizontal Asymptotes

For horizontal asymptotes, think BOSTON for *polynomials*! Looking at the degree of top & bottom…

Bottom > Top

y=**O**

Same = ratio

Top > Bottom

O No HA.

N

$$f(x) = \frac{2x}{x^2+3x} \qquad H.A.:\ y=0$$

$$g(x) = \frac{2x^3}{5x^3+4x^2} \qquad H.A.:\ y=\frac{2}{5}$$

$$h(x) = \frac{5x^2}{7x+3} \qquad No\ H.A.$$

You Try! What is the EQUATION of the horizontal asymptote for the following functions?

Bottom > Top

y=**O**

Same = ratio

Top > Bottom

O No HA.

N

$$f(x) = \frac{3x^2+9}{7x+4x^2+11}$$

$$g(x) = \frac{4x^3}{5x^2+9}$$

$$h(x) = \frac{7x+15}{2x^2}$$

You Try! What is the EQUATION of the horizontal asymptote for the following functions?

Bottom > Top
y=0
Same = ratio
Top > Bottom
O No HA.
N

$$f(x)=\frac{3x^2+9}{7x+4x^2+11} \qquad H.A.: y=\tfrac{3}{4}$$

$$g(x)=\frac{4x^3}{5x^2+9} \qquad H.A.: none$$

$$h(x)=\frac{7x+15}{2x^2} \qquad H.A.: y=0$$

You Try: True or False

1) The graph of function f is defined as the set of all points $(x, f(x))$ where x is in the domain of f. Justify your answer.

2) If a function is not continuous, then the domain cannot be all real numbers.

True or False ANSWERS

1) The graph of function f is defined as the set of all points $(x, f(x))$ where x is in the domain of f. Justify your answer.

True! This is the definition of a function.

2) If a function is not continuous, then the domain cannot be all real numbers.

False! It could be a piecewise function.