KEY

ICM ~Unit 4 ~ Day 1

Section 1.2—Domain, x & y intercepts

Warm Up ~ What do you remember???

For #1 and #2, for each question:

a. Factor, if possible
b. Solve

1. $25 - x^2 = 0$
2. $x^2 - 17 = 0$

Simplify Completely

3. $(9x^2y^3)^{\frac{1}{2}}$

4. $f(x-5)$ *given* $f(x) = 3x^2 - 2x$

$\boxed{(5-x)(5+x)}$ Factor

1) $(5-x)(5+x) = 0$

$\boxed{x = -5, 5}$ solve

2) Not factorable

$x^2 - 17 = 0 \quad x^2 = 17 \quad \boxed{x = \pm\sqrt{17}}$

3) $9^{1/2}(x^2)^{1/2}(y^3)^{1/2}$

$\boxed{3xy^{3/2}}$

4) $f(x-5) = 3(x-5)^2 - 2(x-5)$

$= 3(x-5)(x-5) - 2x + 10$

$= 3(x^2 - 10x + 25) - 2x$

$= 3x^2 - 30x + 75 -$

$\boxed{f(x-5) = 3x^2 - 32}$

Warm Up ~ What do you remember???

For #1 and #2, for each question:

a. Factor, if possible
b. Solve

1. $25 - x^2 = 0$ $\boxed{x = -5, 5}$
2. $x^2 - 17 = 0$ $\boxed{x = \pm\sqrt{17}}$

Simplify Completely

3. $(9x^2y^3)^{\frac{1}{2}}$ $\boxed{3xy^{\frac{3}{2}}}$

4. $f(x-5)$ *given* $f(x) = 3x^2 - 2x$

$\boxed{f(x-5) = 3x^2 - 32x + 85}$

Discuss HW

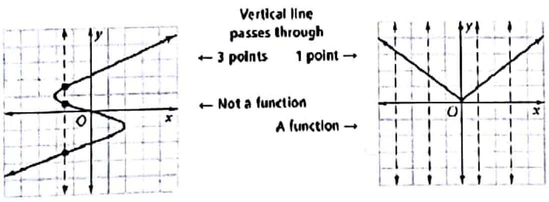
- Factoring & Function Review

What is a Function?

- A rule that associates every value in a set of numbers (***domain***) with a SINGLE value in another set of numbers (***range***).
- Notation: *f(x)* is the value of the function at a given value of "*x*"

Function vs Not Function

- ***Vertical line test***—used to graphically verify that the domain values are associated with only ONE range value



Definition of Domain:

- Set of real numbers that result in a "defined" or mathematically possible statement.
- Use interval notation!
  ( ) Not included   [ ] included
- Examples:
  (0, 10] $(-\infty,\infty)$ $(-\infty,0]\cup[1,\infty)$
- Never write ones like $(0,-\infty)$
  → Smallest value must always be on the left!

Real Life Examples

- **Domain for ages of students at GHHS**
  D: [13, 21] ages
  R: [1, 500] number of students per grade
- **What is the domain for a cash payment of $6.47?**
  (what are all possible ways to pay this amount?)

  D: pennies, nickels, dimes, quarters, $1, $5, dollar coin, $2
  R: amount of each coin, [0, 647]

X and Y intercepts – Special Points!

- X-intercepts (X, 0)
  - Where the function crosses the x-axis
  - Zero or root of the function
    ***Set y = 0 and solve for x**

Ex 1. $y=\frac{x^2-5}{2}$

$0=\frac{x^2-5}{2}$

$0=x^2-5$

$5=x^2$

$\pm\sqrt{5}$

$\boxed{(\sqrt{5},0)(-\sqrt{5},0)}$

Ex 2. $y=4x^2-18x+20$

$0=4x^2-18x+20$

$0=2(2x^2-9x+10)$

$0=2(2x^2-5x-4x+10$

$0=2(x(2x-5)-2(2x-5))$

$0=2(x-2)(2x-5)$

$\boxed{(2,0)\ (5/2,0)}$

X and Y intercepts – Special Points!

- X-intercepts (X, 0)
  - Where the function crosses the x-axis
  - Zero or root of the function
    *Set y = 0 and solve for x

Ex 1. $y = \frac{x^2 - 5}{2}$ $\quad 0 = \frac{x^2 - 5}{2}, \quad x^2 = 5, \quad x = \pm\sqrt{5}$

$(\sqrt{5}, 0)$ *and* $(-\sqrt{5}, 0)$

Ex 2. $y = 4x^2 - 18x + 20$

$(2, 0)$ *and* $(\frac{5}{2}, 0)$

X and Y intercepts – Special Points!

- Y-intercepts (0, Y)
  - Where the function crosses the y-axis
  - Where x equals 0
    *Set x = 0 and solve for y

Ex 1. $y = \frac{x^2 - 5}{2}$ $\quad y = \frac{0^2 - 5}{2} = \frac{-5}{2}$

$(0, -5/2)$

Ex. 2 $y = 4x^2 - 18x + 20$ $\quad y = 4(0)^2 - 18(0) + 20$

$(0, 20)$ $\quad y = 20$

(

X and Y intercepts – Special Points!

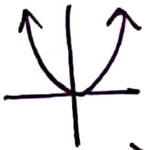
- Y-intercepts (0, Y)
  - Where the function crosses the y-axis
  - Where x equals 0
    *Set x = 0 and solve for y

Ex 1. $y = \frac{x^2 - 5}{2}$ $\quad y = \frac{0^2 - 5}{2}, \quad y = -\frac{5}{2} \quad (0, -\frac{5}{2})$

Ex. 2 $y = 4x^2 - 18x + 20$ $\quad (0, 20)$

State the Domain and x & y int:

$f(x) = x^2$

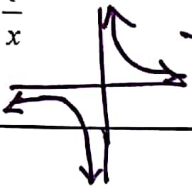


D: $(-\infty, \infty)$
x-int: $(0,0)$
y-int: $(0,0)$

$f(x) = \sqrt{x}$

D: $[0, \infty)$
x-int $(0,0)$
y-int $(0,0)$

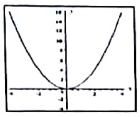
$f(x) = \frac{1}{x}$



D: $(-\infty, 0) \cup (0, \infty)$
x-int: none
y-int: none

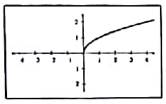
State the Domain and x & y int:

$f(x) = x^2$



Domain: $(-\infty, \infty)$
x-int: $(0,0)$
y-int: $(0,0)$

$f(x) = \sqrt{x}$



Domain: $[0, \infty)$
x-int: $(0,0)$
y-int: $(0,0)$

$f(x) = \frac{1}{x}$

Domain: $(-\infty, 0) \cup (0, \infty)$
x-int: none
y-int: none

Radical Examples: Find the Domain

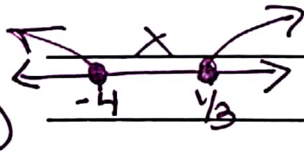
$f(x) = \sqrt{3x-5}$

$[5/3, \infty)$

$g(x) = \sqrt{3x^2 + 11x - 4}$

$\sqrt{(3x-1)(x+4)}$

$(-\infty, -4] \cup [1/3, \infty)$



Rational Examples: Find the Domain

$f(x) = \frac{x-3}{5x+4}$

$(-\infty, -4/5) \cup (-4/5, \infty)$

$g(x) = \frac{2x-3}{2x^2 + 11x - 21}$

$14 \cdot -3 = -42$
$14 + -3 = 11$

$2x^2 + 14x - 3x - 21$
$2x(x+7) - 3(x+7)$
$(2x-3)(x+7)$
$x = 3/2, -7$

$(-\infty, -7) \cup (-7, 3/2) \cup (3/2, \infty)$

Radical Examples: Find the Domain

$f(x) = \sqrt{3x-5}$ $\quad$ $g(x) = \sqrt{3x^2+11x-4}$

$Domain: [\frac{5}{3}, \infty)$ $\quad$ $Domain: (-\infty, -4] \cup [\frac{1}{3}, \infty)$

Rational Examples: Find the Domain

$f(x) = \frac{x-3}{5x+4}$

$Domain: (-\infty, -\frac{4}{5}) \cup (-\frac{4}{5}, \infty)$

$g(x) = \frac{2x-3}{2x^2+11x-21}$

$Domain: (-\infty, -7) \cup (-7, \frac{3}{2}) \cup (\frac{3}{2}, \infty)$

Square Root Functions $f(x) = \sqrt{radicand}$

Expression under the square root symbol, must be positive.

Steps to find domain of Square Root Functions:

1. Set the expression inside the square root equal to zero.
2. Solve the equation found in step 1.
3. Plot that value on a number line and test points on each side. OR check in calculator
4. Write the domain using interval notation.

*Write this down!

☐ Fix #3

☐ Fix Put f(x) here

Example: $f(x) = \sqrt{9-5x}$

$9-5x=0$

$-5x=-9$

$x=9/5$

9/5

Find the domain. D: $(-\infty, 9/5]$

Find the x and y-intercepts.

x-int: $(9/5, 0)$ $\quad$ $0=\sqrt{9-5x}$ $\quad$ $0=9-5x$ $\quad$ $5x=9$ $\quad$ $x=9/5$

y-int: $(0, 3)$ $\quad$ $y=\sqrt{9-5\cdot 0}$ $\quad$ $y=\sqrt{9}$

Square Root Functions $f(x) = \sqrt{radicand}$

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3. Plot that value on a number line and test points on each side.
4. Write the domain using interval notation.

*Write this down!

Example: $f(x) = \sqrt{9-5x}$

$9-5x=0$

$9=5x$

$x=\frac{9}{5}$

Find the domain. $Domain: (-\infty, \frac{9}{5}]$

Find the x and y-intercepts. $x-int: (\frac{9}{5}, 0)$ $\quad$ $y-int: (0, 3)$

Square Root Functions $f(x) = \sqrt{radicand}$

- Write the domain, using interval notation, of the following functions. *(verify by graphing)*
- State the x & y intercepts **You Try!**

$f(x) = \sqrt{2x+7}$

$2x+7=0$

$2x=-7$

$x=-7/2$

$-7/2$

D: $[-7/2, \infty)$

x-int: $(-7/2, 0)$ y-int $(0, \sqrt{7})$

$h(x) = \sqrt{x^2-9}$

$x^2-9=0$

$(x-3)(x+3)=0$

$x=3,-3$

-3 3

D: $(-\infty,-3] \cup [3,\infty)$

x-int: $(-3,0)$ and $(3,0)$

y-int: none

$y=\sqrt{2x+7}$

- x-int $0=\sqrt{2x+7}$ $0=2x+7$

$(-7/2, 0)$ $x=-7/2$

- y-int $y=\sqrt{2\cdot 0+7}$ $y=\sqrt{7}$

$(0,\sqrt{7})$

- x-int $0=\sqrt{x^2-9}$ $0=x^2-9$

$0=(x-3)(x+3)$

- y-int $y=\sqrt{0^2-9}$ $y=\sqrt{-9}$

non real so NONE

Square Root Functions $f(x) = \sqrt{radicand}$

- Write the domain, using interval notation, of the following functions. *(verify by graphing)*
- State the x & y intercepts **You Try!**

$f(x) = \sqrt{2x+7}$

Domain: $[-\frac{7}{2}, \infty)$

x – int: $(-\frac{7}{2}, 0)$

y – int: $(0, \sqrt{7})$

$h(x) = \sqrt{x^2-9}$

Domain: $(-\infty, -3] \cup [3, \infty)$

x – int: $(-3, 0), (3, 0)$

y – int: *None*

Rational Functions $f(x) = \dfrac{numerator}{denominator}$

Expression in the denominator can NOT be equal to zero.

Steps to find domain of Rational Functions:

1. Factor the denominator (if possible).
2. Set each factor equal to zero and solve.
3. Write the domain using interval notation.

*Write this down!

Fix

Not

$f(x) = \dfrac{x+1}{x^2+2x-35}$

shading on a number line helps

Domain:

$(-\infty,-7) \cup (-7,5) \cup (5,\infty)$

$(x+7)(x-5)$

$x \neq -7, 5$

Find the x and y-intercepts.

x-int: $(-1,0)$

y-int: $(0, -1/35)$

x-int: $0=\dfrac{x+1}{x^2+2x-35}$

$0=x+1$ $x=-1$

y-int: $y=\dfrac{0+1}{0^2+2(0)-35}$

Show your work by h

Rational Functions $f(x) = \frac{numerator}{denominator}$

Expression in the denominator can NOT be equal to zero.

Steps to find domain of Rational Functions:

1. Factor the denominator (if possible).
2. Set each factor equal to zero and solve.
3. Write the domain using interval notation.

*Write this down!

$f(x) = \frac{x+1}{x^2+2x-35}$

Find the x and y-intercepts.

Domain:
$(-\infty,-7)\cup(-7,5)\cup(5,\infty)$

x – int : $(-1, 0)$
y – int : $(0, -1/35)$

Rational Functions $f(x) = \frac{numerator}{denominator}$
You Try!

- Write the domain, using interval notation, of the following functions. *(verify by graphing)*
- State the x & y intercepts

$a.\ g(x) = \frac{8}{2x+12}$ $b.\ f(x) = \sqrt{x^2-8x+15}$

$\boxed{D: (-\infty,-6)\cup(-6,\infty)}$
$\boxed{x\text{-int}: \text{none} \quad y\text{-int} (0, 2/3)}$

$c.\ k(x) = \frac{7}{x^2-16}$

$(x-4)(x+4) \quad x \neq -4, 4$
$\boxed{D: (-\infty,-4)\cup(-4,4)\cup(4,\infty)}$

x-int: $0 = \frac{7}{x^2-16}$ none
y-int: $y = \frac{7}{0-16}$

$\boxed{x\text{-int}: \text{none} \quad y\text{-int}: (0, -7/16)}$

a) $2x+12 \neq 0$ $x \neq -6$ $D: (-\infty,-6)\cup(-6,\infty)$
x-int: $0 = \frac{8}{2x+12}$ $0 = 8 \rightarrow$ none
y-int: $y = \frac{8}{2(0)+12} = \frac{8}{12} = \frac{2}{3}$

b) $x^2-8x+15 = 0$
$(x-5)(x-3) = 0$ $x = 5, 3$
$\boxed{D: (-\infty,3]\cup[5,\infty)}$ 3 5
$\boxed{x\text{-int}: (3,0)(5,0)}$ $0 = \sqrt{x^2-8x+15}$
$0 = x^2-8x+15 - (x-5)(x-3)$
$\boxed{y\text{-int}: (0,\sqrt{15})}$

Rational Functions $f(x) = \frac{numerator}{denominator}$
You Try!

- Write the domain, using interval notation, of the following functions. *(verify by graphing)*
- State the x & y intercepts

$a.\ g(x) = \frac{8}{2x+12}$

Domain: $(-\infty,-6)\cup(-6,\infty)$
x – int : *None*
y – int : $(0, \frac{2}{3})$

$b.\ f(x) = \sqrt{x^2-8x+15}$

Domain: $(-\infty,3]\cup[5,\infty)$
x – int : $(3,0)$, $(5, 0)$
y – int : $(0, \sqrt{15})$

$c.\ k(x) = \frac{7}{x^2-16}$

Domain: $(-\infty,-4)\cup(-4,4)\cup(4,\infty)$
x – int : *None*
y – int : $(0, -7/16)$

Combined Functions

To find the domain of combined functions, a number line really helps to put the pieces together!!

- Write the domain of the following function in interval notation *(verify by graphing)*
- State the x & y intercepts

$$m(x) = \frac{\sqrt{x}}{x-9}$$

D: $[0,9)\cup(9,\infty)$

x-int: $(0,0)$

y-int: $(0,0)$

D: $x-9 \neq 0$, $x \neq 9$

0 9

set under $\sqrt{\ } = 0 \to x = 0$

x-int: $0 = \frac{\sqrt{x}}{x-9}$ $\quad 0=\sqrt{x}$ $\quad 0 = x$

y-int: $y = \frac{\sqrt{0}}{0-9} = \frac{0}{-9} = 0$

Combined Functions

To find the domain of combined functions, a number line really helps to put the pieces together!!

- Write the domain of the following function in interval notation *(verify by graphing)*
- State the x & y intercepts

$$m(x) = \frac{\sqrt{x}}{x-9}$$

Domain: $[0,9)\cup(9,\infty)$

x-int: $(0,0)$

y-int: $(0,0)$

$0 = \frac{1}{\cancel{x}} \cdot \cancel{x}(x-3) + \frac{5}{\cancel{x-3}} \cdot x\cancel{(x-3)}$

$0 = x-3+5x$

$3 = 6x$

$1/2 = x$

Practice

- Find the domain and x & y intercepts of...

$$f(x) = \frac{1}{x} + \frac{5}{x-3} \qquad h(x) = \frac{\sqrt{4-x^2}}{x-3}$$

$y = \frac{1}{x} + \frac{5}{x-3}$

D: $x \neq 0, x \neq 3$

0 3

$\boxed{D: (-\infty,0)\cup(0,3)\cup(3,\infty)}$

x-int: $0 = \frac{1}{x} + \frac{5}{x-3}$

$\boxed{\text{x-int: } (1/2, 0)}$

y-int: $y = \frac{1}{0} + \frac{5}{0-3}$

$\boxed{\text{y-int: none}}$

$$g(x) = \frac{\sqrt{4-x}}{(x+1)(x^2+1)}$$

D: $x \neq -1$

-1 4

$4-x=0$

$\boxed{D: (-\infty,-1)\cup(-1,4]}$

x-int: $0 = \frac{\sqrt{4-x}}{(x+1)(x^2+1)}$ $\quad 0=\sqrt{4-x}$ $\quad 0 = 4-x$

$\boxed{\text{x-int: } (4,0)}$

y-int: $y = \frac{\sqrt{4-0}}{(0+1)(0^2+1)} = \frac{2}{1}$

$\boxed{\text{y-int: } (0,2)}$

D: $x \neq 3$

-2 2 3

$4-x^2=0$

$(2-x)(2+x)=0$

$\boxed{D: [-2,2]}$

x-int: $0 = \sqrt{4-x^2}$ $\quad 0 = 4-$

$\boxed{\text{x-int } (-2,0),(2,0)}$

$\boxed{\text{y-int: } (0,-2/3)}$

$y = \frac{\sqrt{4}}{0-3} = \frac{-2}{3}$

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Practice

- Find the domain and x & y intercepts of...

$$f(x)=\frac{1}{x}+\frac{5}{x-3}$$

Domain: $(-\infty,0)\cup(0,3)\cup(3,\infty)$

$x-\text{int}: (0.5, 0)$

$y-\text{int}: \textit{None}$

$$h(x)=\frac{\sqrt{4-x^2}}{x-3}$$

Domain: $[-2,2]$

$x-\text{int}: (2,0), (-2,0)$

$y-\text{int}: (0,-\frac{2}{3})$

$$g(x)=\frac{\sqrt{4-x}}{(x+1)(x^2+1)}$$

Domain: $(-\infty,-1)\cup(-1,4]$

$x-\text{int}: (4, 0)$

$y-\text{int}: (0, 2)$

Homework Day 1

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