KEY

UNIT 5 ~ DAY 8

Applications

WARM UP

❖**Find the derivative of each function.**

1. $f(x) = \sqrt[3]{(3x^2 + 4x)^5}$

2. $g(x) = x^4\sqrt{2x-3}$

WARM UP

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1. $f(x) = \sqrt[3]{(3x^2 + 4x)^5}$

2. $g(x) = x^4\sqrt{2x-3}$

WARM UP ANSWERS

❖**Find the derivative of each function.**

1. $f(x) = \sqrt[3]{(3x^2 + 4x)^5}$ $\quad f'(x) = \frac{5}{3}(3x^2 + 4x)^{\frac{2}{3}}(6x+4)$

$$= (10x + \frac{20}{3})(3x^2 + 4x)^{\frac{2}{3}}$$

2. $g(x) = x^4\sqrt{2x-3}$

$$g'(x) = x^4(\frac{1}{2}(2x-3)^{-\frac{1}{2}} \cdot 2) + (2x-3)^{\frac{1}{2}}(4x^3)$$

$$= x^4(2x-3)^{-\frac{1}{2}} + (2x-3)^{\frac{1}{2}}(4x^3)$$

HW QUESTIONS?

NOTES: APPLICATIONS OF DERIVATIVES

Definitions

Velocity: the rate of motion in a specific **direction**

Acceleration: the rate at which velocity (speed) is changing.

→ If an object is moving with a constant velocity, then its acceleration is zero, since the velocity never changes.

RECALL

- Position vs Time graph (original function)
- Velocity vs Time graph (first derivative)
- Acceleration vs Time graph (second derivative)

Position y

Time t

$$v = \frac{dy}{dt}$$

Velocity is the derivative of position with respect to time.

$$a = \frac{dv}{dt} = \frac{d^2y}{dt^2}$$

Acceleration is the derivative of velocity with respect to time.

$$y = \int v\,dt$$

$$= \int (v_0 + at)\,dt$$

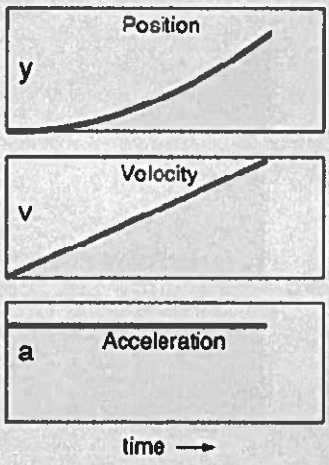
$$y = y_0 + v_0 t + \frac{1}{2}at^2$$

Integrate velocity to get position ⇧

$$v = \int a\,dt = v_0 + at$$

Integrate acceleration to get velocity ⇧

a = constant



Motion relationships in one dimension.

$$y = y_0 + v_0 t + \frac{1}{2}at^2$$

⇩ Derivative of position is velocity

$$v = \frac{dy}{dt}$$

$$v = v_0 + at$$

⇩ Derivitive of velocity is acceleration

$$a = \frac{dv}{dt} = a$$

APPLICATIONS

- Maximums and Minimums – find in calculator
- **Instantaneous Velocity** – Use the velocity function (**first derivative**) and find the value at the specific x.
- **Acceleration** – Use the **second derivative** …the derivative of the velocity function.

FORMULA...

❖The position of a free falling object (neglecting air resistance) under the influence of gravity can be given by the equation:

☆ $s(t) = \frac{1}{2}gt^2 + v_0 t + s_0$

s_0=initial height

v_0=initial velocity (or speed for our purposes)

g = acceleration due to gravity (-32ft/sec^2 or -9.8m/sec^2) ☆

□Fix typo

□Fix units

EXAMPLE 1

❖If a billiard ball is dropped from a height of 100ft, its height **s** at time **t** is given by the position function **$s = -16t^2 + 100$**, where **s** is measured in feet and **t** is measured in seconds.

❖Find the velocity at t = 1 and t = 2.

$s' = v(t) = -32t$

$s(1) = -32(1) = -32$ ft/sec

$s(2) = -32(2) = -64$ ft/sec

EXAMPLE 1 ANSWERS

❖If a billiard ball is dropped from a height of 100ft, its height **s** at time **t** is given by the position function **$s = -16t^2 + 100$**, where **s** is measured in feet and **t** is measured in seconds.

❖Find the velocity at t = 1 and t = 2.

$s'(t) = -32t$

$s'(1) = -32$ *ft* / sec

$s'(2) = -32(2) = -64$ *ft* / sec

EXAMPLE 2

❖At time t = 0, a diver jumps down from a platform diving board that is 50 feet above the water. The position of the diver is given by $\mathbf{s(t) = -16t^2 + 18t + 50}$, where **s** is the measured in feet and **t** is measured in seconds.

1) When does the diver hit the water?

at $s(t)$ = position = 0 ft $\rightarrow$ $0 = -16t^2 + 18t + 50$

↳ use calc to get zero

$t = 2.418$ sec

2) What is the velocity at impact?

$v(t) = s'(t) = -32t + 18$

$s'(2.418) = -32(2.418) + 18 = -59.376$ ft/sec

EXAMPLE 2 ANSWERS

❖At time t = 0, a diver jumps down from a platform diving board that is 50 feet above the water. The position of the diver is given by $\mathbf{s(t) = -16t^2 + 18t + 50}$, where **s** is the measured in feet and **t** is measured in seconds.

1) When does the diver hit the water?

After 2.418 seconds

2) What is the velocity at impact?

$s'(2.418) = -32(2.418) + 18$

$= -59.36$ *ft* / sec

EXAMPLE 3

❖An arrow is shot straight up in the air with a speed of 900m/s.

The arrow is launched with an initial height of 2.4m.

↳ $v_0 = 900\ m/s$

↳ $s_0 = 2.4\ m$

$$s(t) = \frac{1}{2}gt^2 + v_0t + s_0$$

*Remember g = acceleration from gravity $= -9.8\ m/sec^2$

1. What is the equation relating its height as a function of time?

$$s(t) = \frac{1}{2}\left(-9.8\ \frac{m}{sec^2}\right)t^2 + 900t + 2.4$$

$$s(t) = -4.9t^2 + 900t + 2.4$$

2. Find the instantaneous velocity at t=1.5s

$$v = s(t) = -9.8t + 900$$

$$s'(1.5) = -9.8(1.5) + 900$$

$$885.3\ m/sec$$

EXAMPLE 3 ANSWERS

❖An arrow is shot straight up in the air with a speed of 900m/s.

The arrow is launched with an initial height of 2.4m.

$$s(t) = \frac{1}{2}gt^2 + v_0t + s_0$$

1. What is the equation relating its height as a function of time?

$$h(t) = -4.9t^2 + 900t + 2.4$$

2. Find the instantaneous velocity at t=1.5s

$$h'(t) = -9.8t + 900 \qquad h'(1.5) = -9.8(1.5) + 900$$

$$= 885.3\ m/\sec$$

EXAMPLE 4

❖The position of a particle is given by the equation

$s(t) = \frac{2}{3}t^3 - 12t^2 + 54t - 8$, where t is measured in seconds and **s** in meters.

1. What is the particle's velocity function?

$v(t) = s'(t) = 2t^2 - 24t + 54$

2. What is the particle's acceleration function?

$a(t) = v'(t) = 4t - 24 = s''(t)$

s'' means 2nd Deriv of s

3. When is the particle <u>at rest</u> **(velocity=0)? ← write down!** *

"at rest" means not moving

Set $0 = 2t^2 - 24t + 54$

and find zero in calc OR by factoring

$t = 3$ sec and 9 sec

EXAMPLE 4 ANSWERS

❖The position of a particle is given by the equation

$s(t) = \frac{2}{3}t^3 - 12t^2 + 54t - 8$, where t is measured in seconds and **s** in meters.

1. What is the particle's velocity function?

$s'(t) = 2t^2 - 24t + 54$

2. What is the particle's acceleration function?

$s''(t) = 4t - 24$

3. When is the particle <u>at rest</u> **(velocity=0)? ← write down!**

$0 = 2t^2 - 24t + 54$, *so* $t = 3$ and 9 sec

HW if you do not finish!

GET IN GROUPS TO WORK ON THE APPLICATION QUESTIONS

OMIT #1 AND #9

#3, 7, AND 11 HAVE <u>NEGATIVE</u> VELOCITY

<u>Additional Hint</u>: If an object is **DROPPED**, the initial velocity is 0 (#5 and #10)
(because at start, ball is still and in the person's hand)