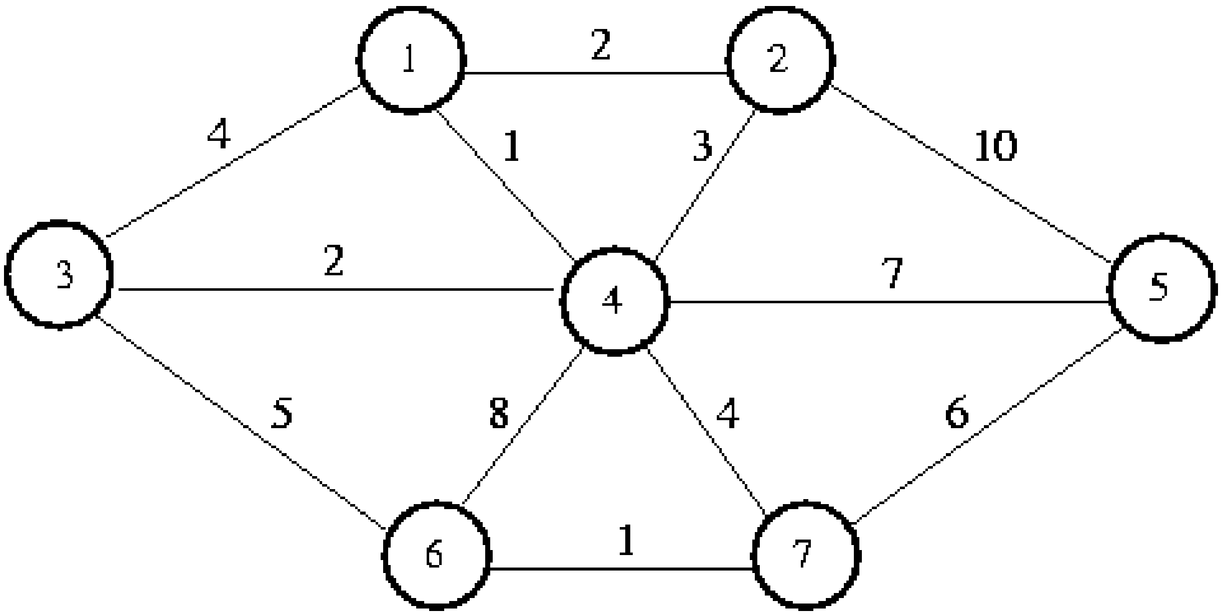
Unit 7 Day 8
Section 5.6

Binary Trees, Expression Trees, & Traversals

Warm Up Day 8

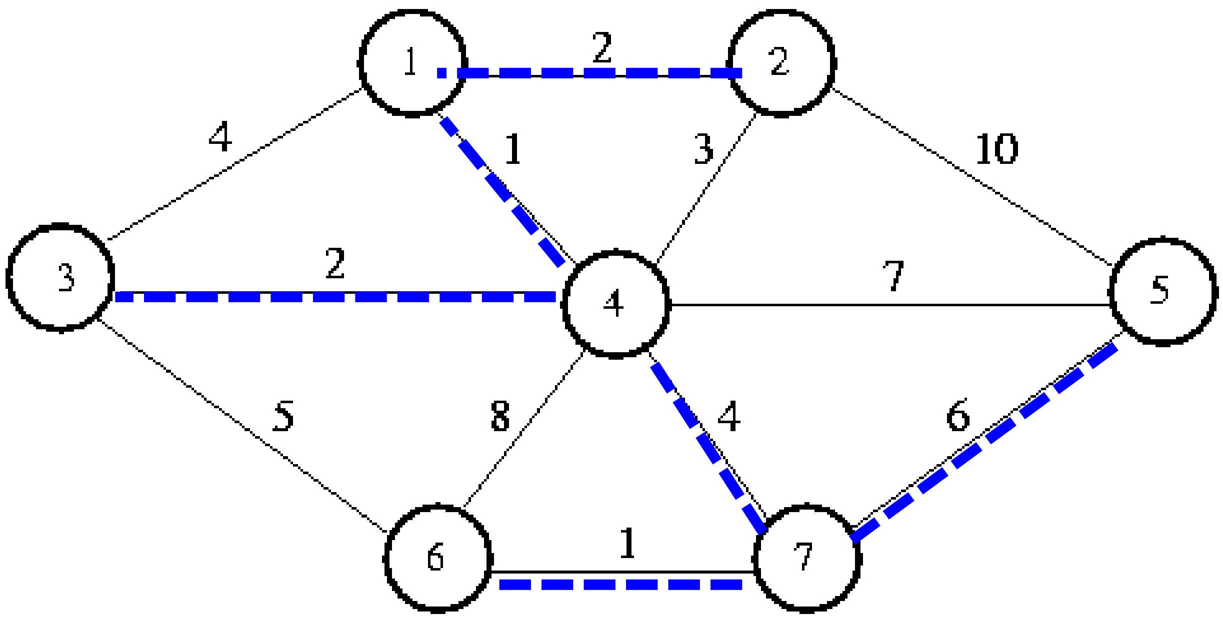
Use Kruskal's algorithm to find the minimum spanning tree and it's weight.



Edge	Weight
(2,4)	3
(1,3)	4
(3,6)	5

Warm Up Day 8 ANSWERS

Use Kruskal's algorithm to find the minimum spanning tree and it's weight.



Edge	Weight
(1,4)	**1**
(6,7)	**1**
(1,2)	**2**
(3,4)	**2**
(2,4)	3
(1,3)	4
(4,7)	**4**
(3,6)	5
(5,7)	**6**

Section 5.6

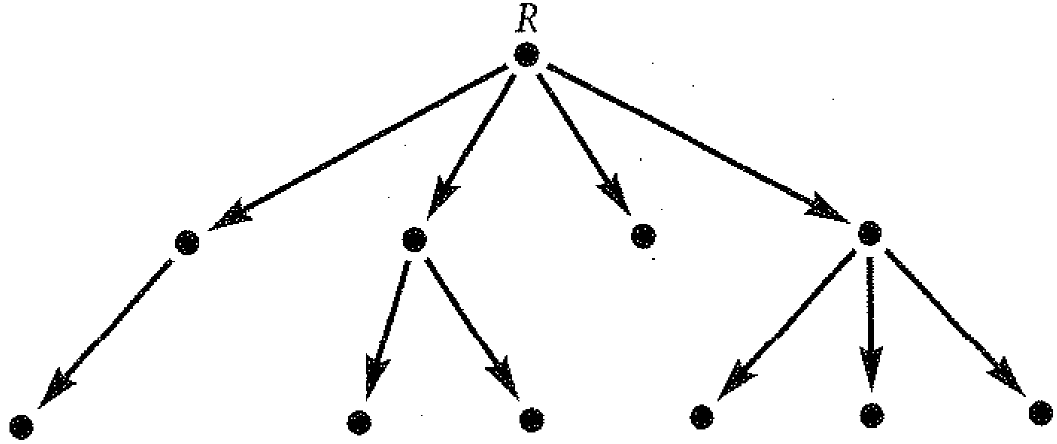
Binary Trees, Expression Trees, & Traversals

Rooted Trees

Decision trees and family trees are two examples of a kind of tree known as a <u>ROOTED TREE</u>.

A <u>Rooted Tree</u> is a directed tree in which every vertex except the root has an indegree of 1 (the root has an indegree of 0).

This is an example of a rooted tree.

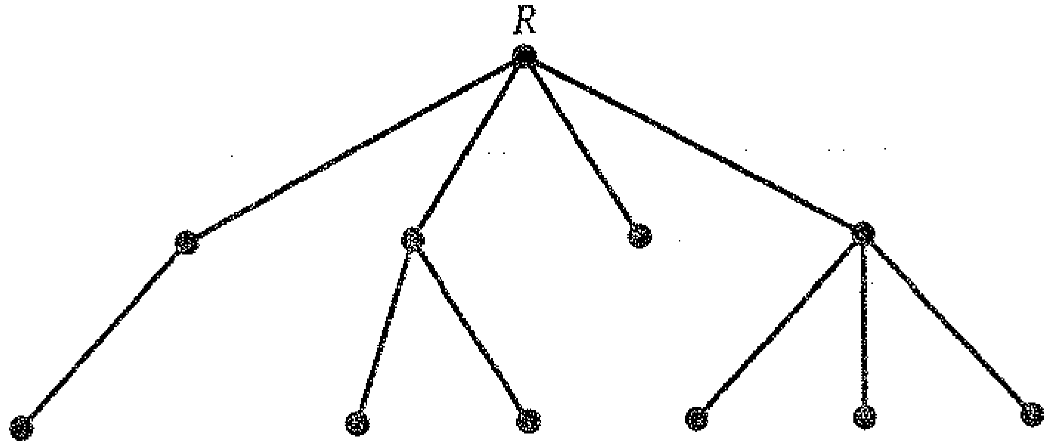


Rooted Trees

Decision trees and family trees are two examples of a kind of tree known as a <u>ROOTED TREE</u>.

A <u>Rooted Tree</u> is a directed tree in which every vertex except the root has an indegree of 1 (the root has an indegree of 0).

This is an example of a rooted tree.

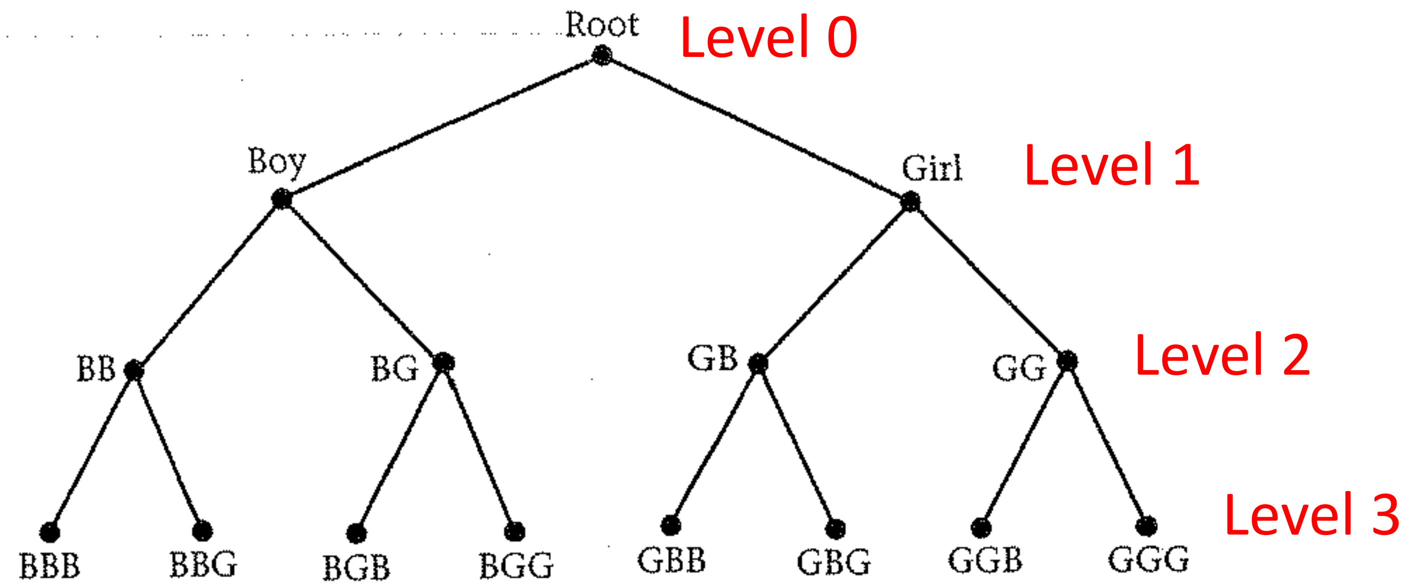


Since all the edges are directed away from the root, it is not necessary to draw the arrowheads.

Rooted trees are good for modeling situations that are multi-staged or <u>Hierarchical</u>, like staff organizations.

Rooted Trees

A couple decides to have three children. What are the possible outcomes?



<u>TREE LEVEL</u> - # of edges in direct path from root to end

<u>VERTEX PARENT</u> - _adjacent and "above" another vertex

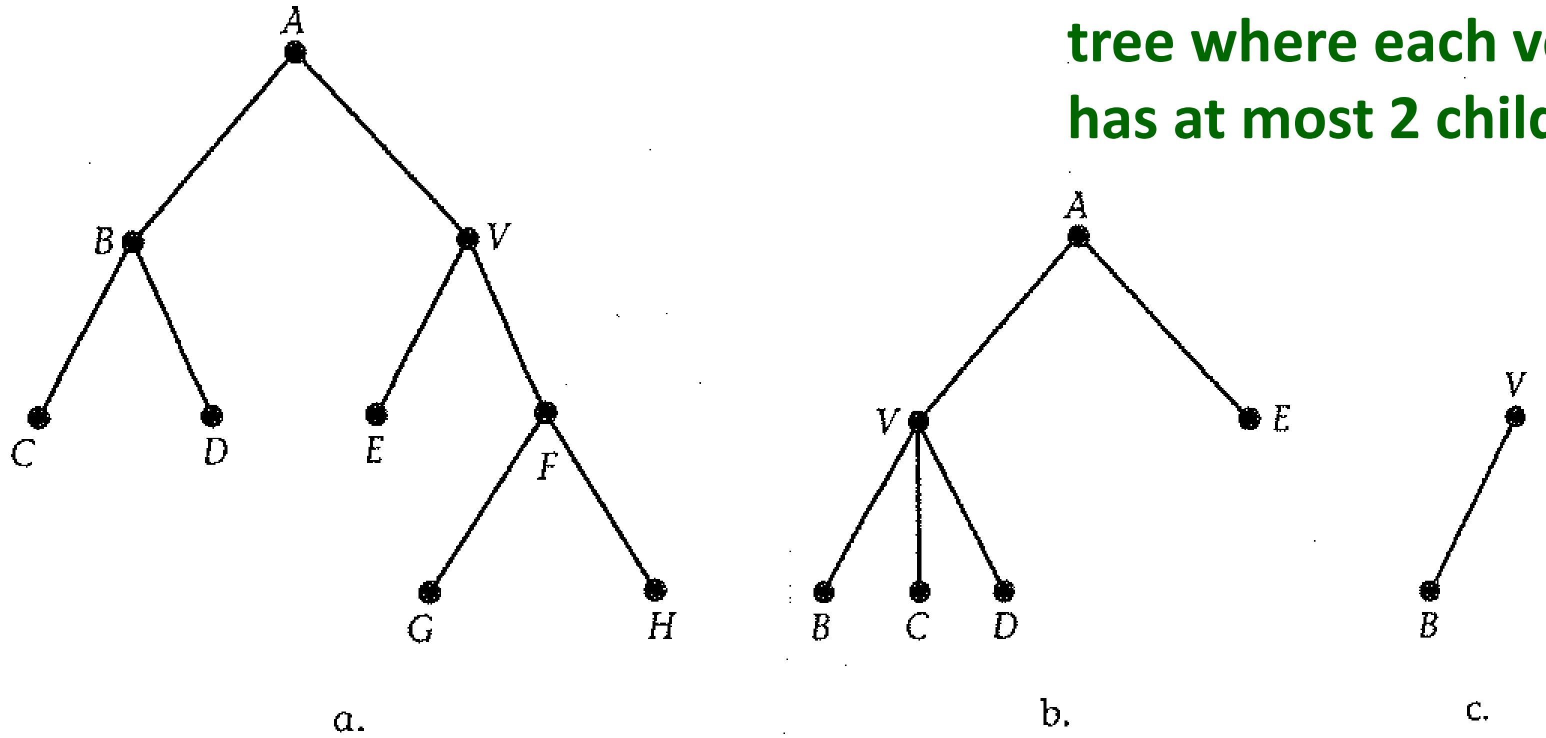
<u>VERTEX CHILD/CHILDREN</u> - adjacent and "below" parent

<u>BINARY TREE</u> – A rooted tree where each vertex has at most 2 children.
(these are used a lot in computer applications)

Binary Trees

Which trees are binary trees? For those that are binary trees, name the parent of V and the children of V.

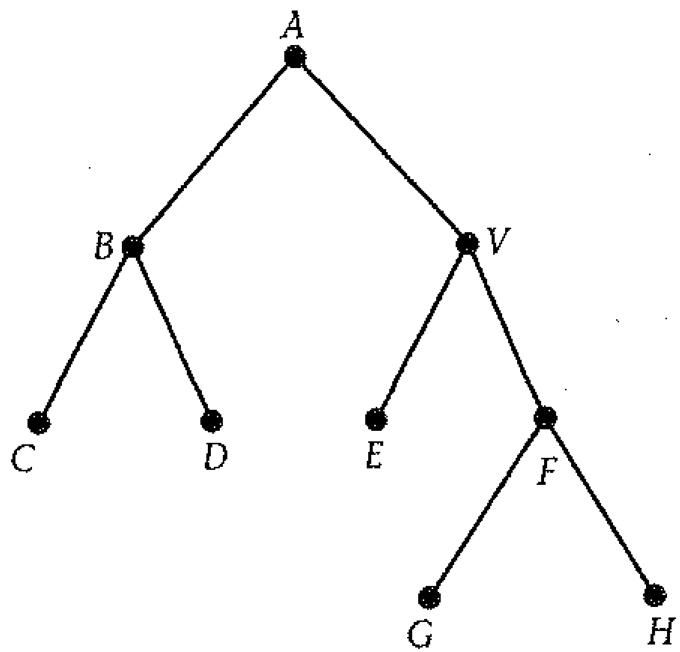
BINARY TREE – A rooted tree where each vertex has at most 2 children



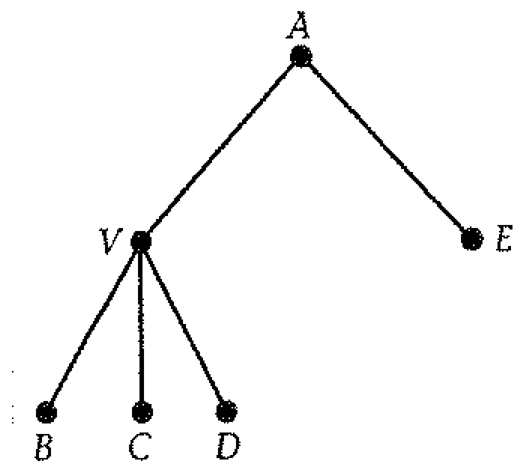
Binary Trees ANSWERS

Which trees are binary trees? For those that are binary trees, name the parent of V and the children of V.

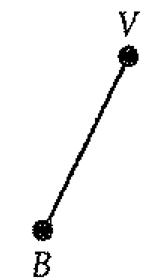
BINARY TREE – A rooted tree where each vertex has at most 2 children



a.



b.



c.

BINARY
V is at Level 1
V's parent is A
V's children are E, F

NOT BINARY
V is at Level 1
V's parent is A
V's children are B,C,D

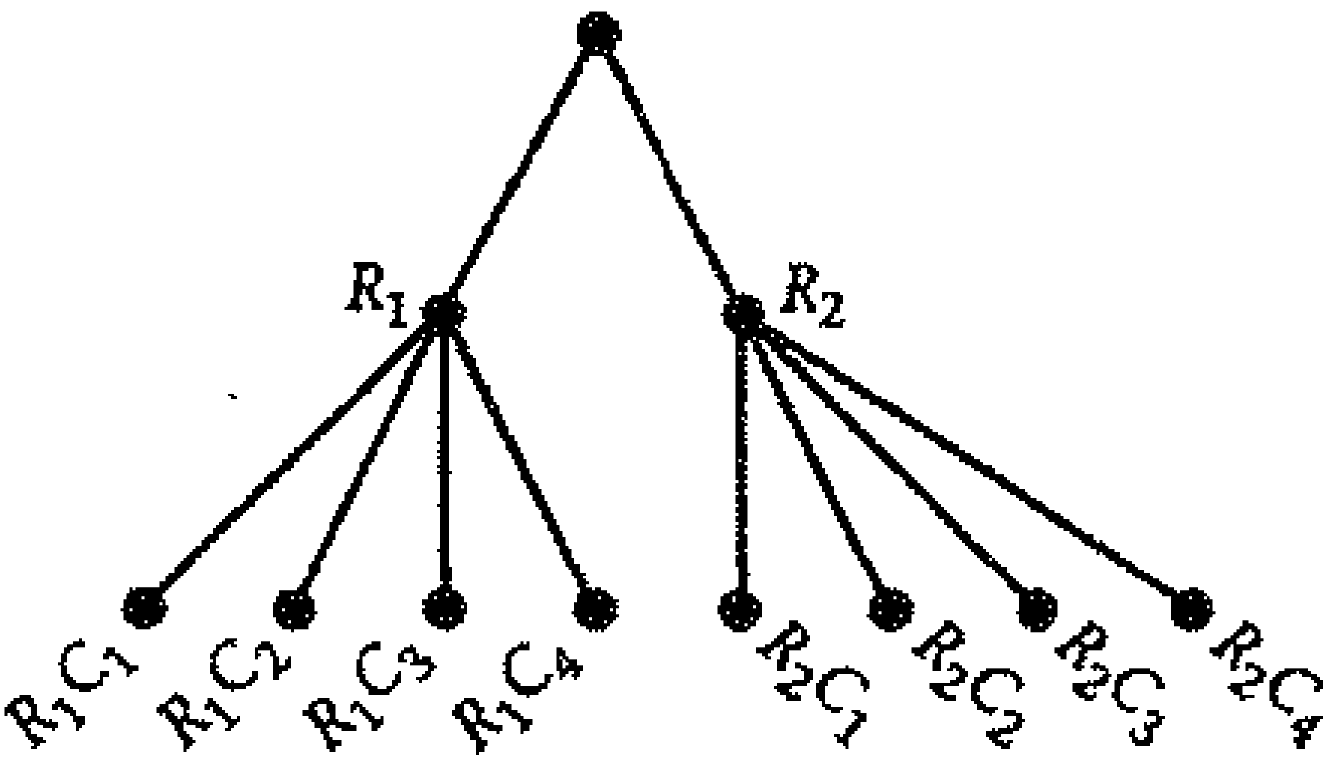
BINARY
V is at Level 0
V's parent - None
V's child is B.

Section 5.6 #1

Tony wants to buy a car. He has the options of two different brands of radios and four different exterior colors. Draw a tree diagram to show all possible outcomes choosing a radio and a color for the car.

Section 5.6 #1 ANSWER

Tony wants to buy a car. He has the options of two different brands of radios and four different exterior colors. Draw a tree diagram to show all possible outcomes choosing a radio and a color for the car.



Section 5.6 #3 – 6

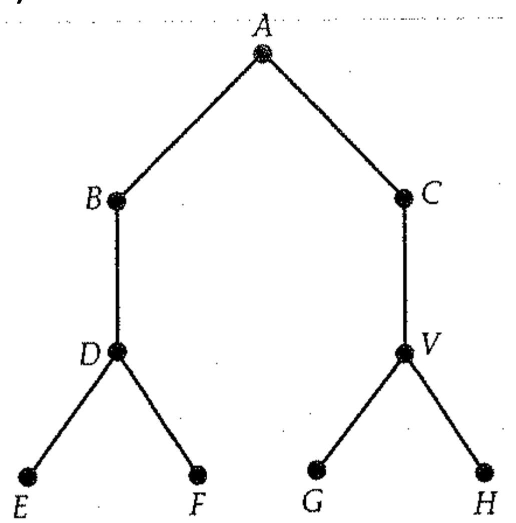
In Exercises 3-6, examine each tree. If the tree is a binary tree,

(a) Give the level of Vertex V

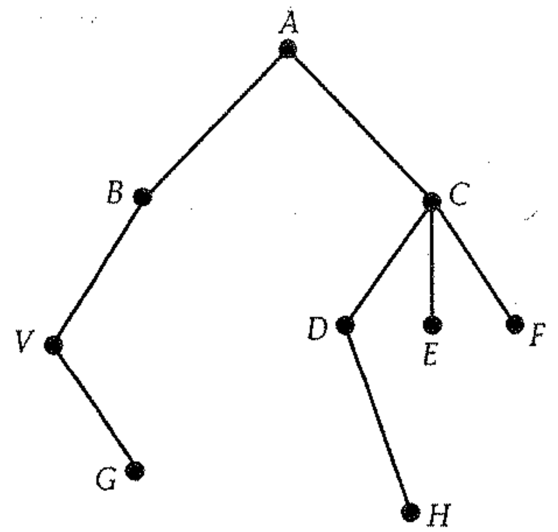
(b) Name the parent of V

(c) Name the children of V

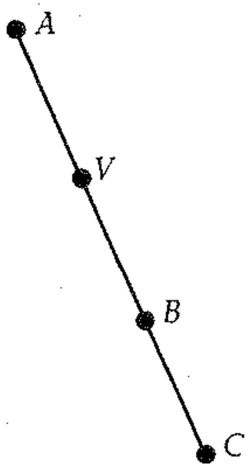
3.



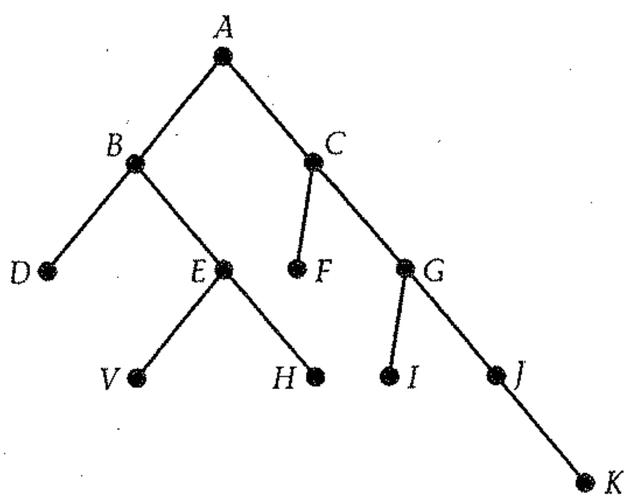
4.



5.



6.



Section 5.6 #3 – 6 ANSWERS

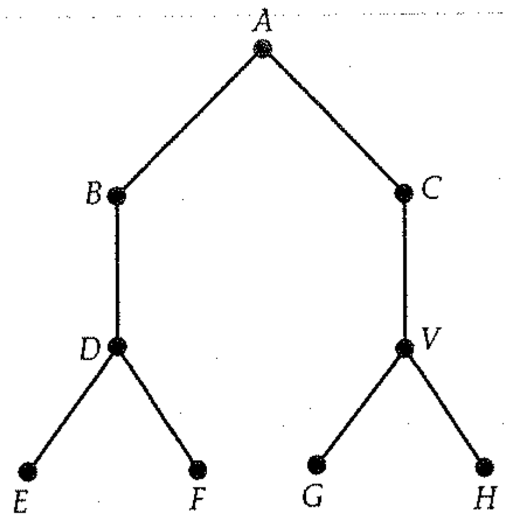
In Exercises 3-6, examine each tree. If the tree is a binary tree,

(a) Give the level of Vertex V

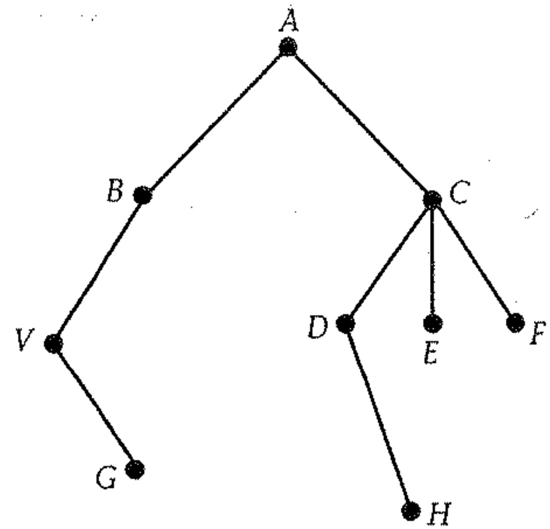
(b) Name the parent of V

(c) Name the children of V

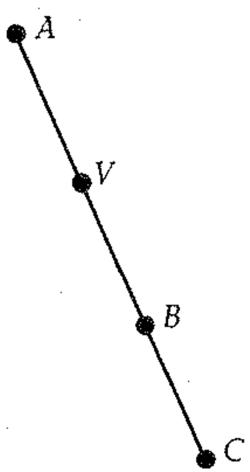
3.



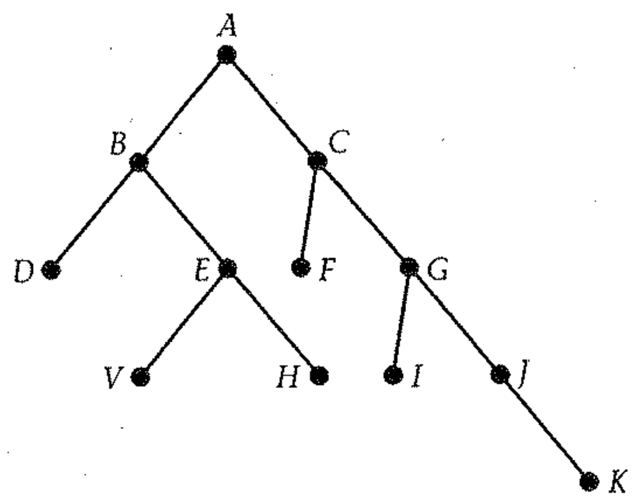
4.



5.



6.



3. Binary Tree.

a. V is level 2.

b. C is the parent.

c. G and H are children.

4. Not a binary tree.

5. Binary Tree.

a. V is level 1.

b. A is the parent.

c **B** is the child.

6. Binary Tree.

a. V is level 3.

b. E is the parent.

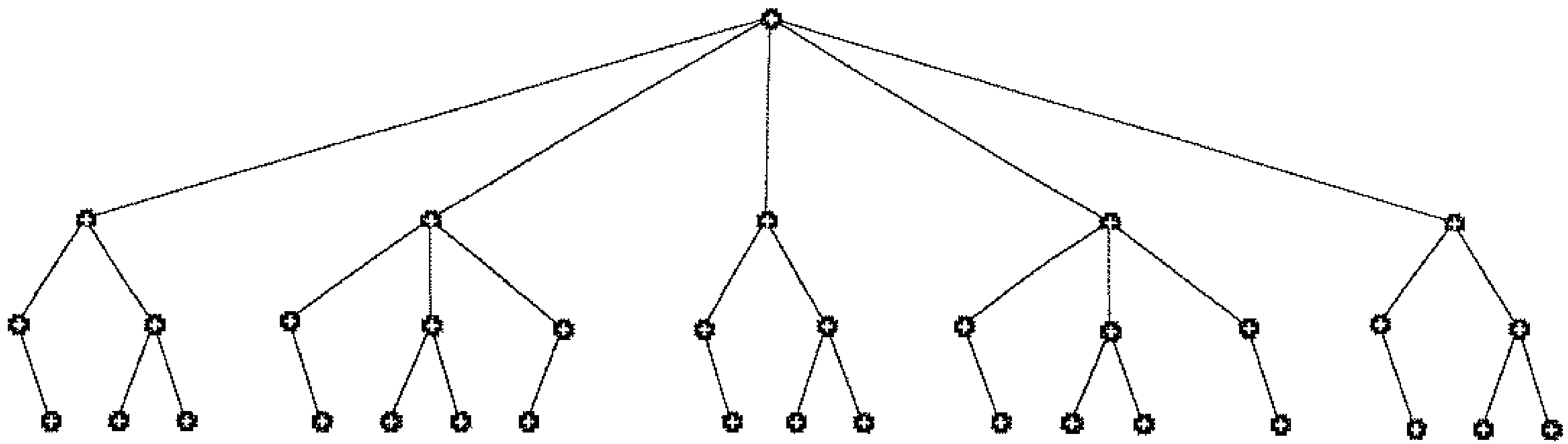
c. No children.

Section 5.6 #7

7. Jeff's brother Tom has a first-grade spelling book that contains five chapters. Each odd-numbered chapter has two lessons and each even-numbered chapter has three lessons. The second lesson of each chapter has two questions whereas all others have one. Draw a rooted tree that models Tom's book. How many questions are in the book?

Section 5.6 #7 answers

7. Jeff's brother Tom has a first-grade spelling book that contains five chapters. Each odd-numbered chapter has two lessons and each even-numbered chapter has three lessons. The second lesson of each chapter has two questions whereas all others have one. Draw a rooted tree that models Tom's book. How many questions are in the book?



There are 17 questions in the book.

Expression Trees

In computer science applications, binary trees are used to evaluate arithmetic expressions.

We know how to evaluate

$$\mathbf{(4 + 6) \bullet 8 - 4 / 2}$$

because we understand the order of operations for expressions.

A computer cannot efficiently imitate our methods, but if the expression is represented as a binary tree, a computer can evaluate it very quickly and easily.

We are going to learn how to represent an arithmetic expression as a binary tree.

But first…. Let’s do a relay race ☺

Before we start….

What is the order of operations? ☺

PEMDAS

Parenthesis

Exponents

Multiplication
Division

Addition
Subtraction

Expression Trees

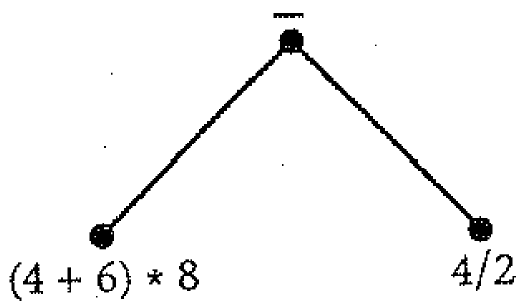
$$(4 + 6) \bullet 8 - 4 / 2$$

To represent this expression as a binary tree:

First find the operation that will be performed last. That will become the root of the tree.

PEMDAS

The right and left sides of that operation become the children of the root.



Expression Trees

$(4 + 6) \bullet 8 - 4 / 2$

Continue this process with every leaf of the tree until there are no more operations remaining on the leaves.

PEMDAS

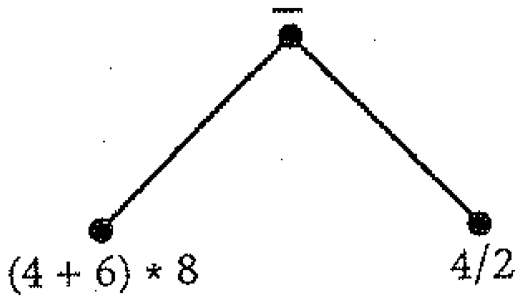
The final binary tree is called an <u>EXPRESSION TREE</u>.

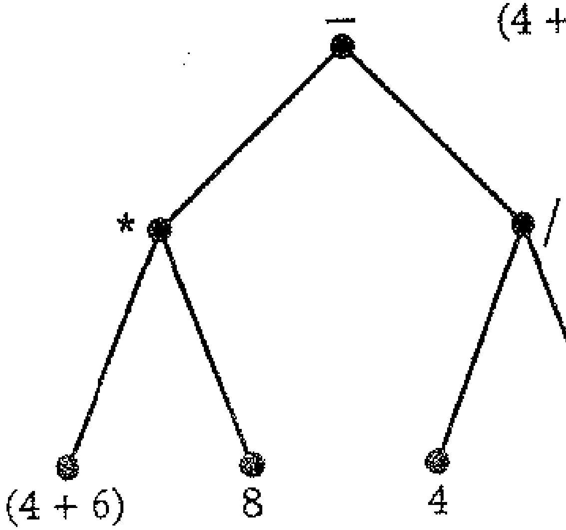
Expression Trees ANSWER

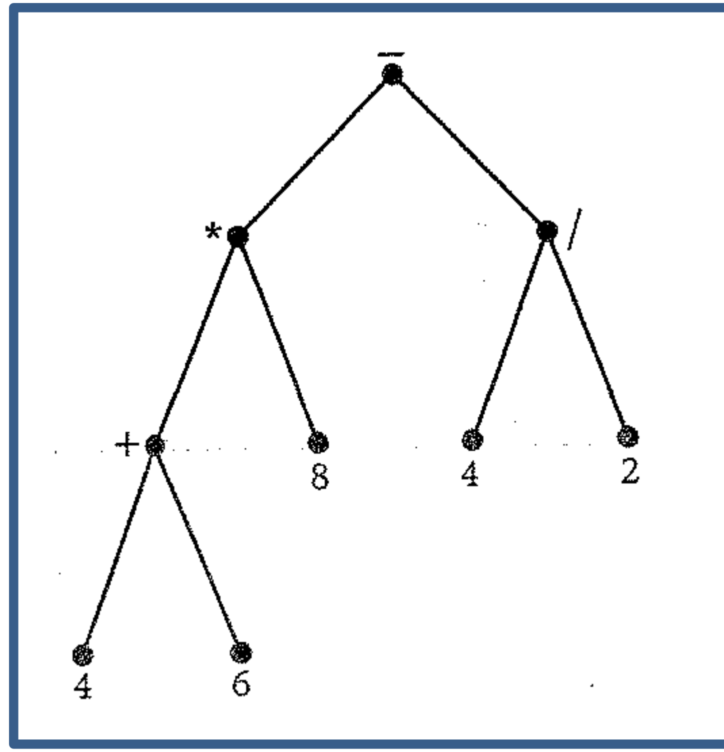
(4 + 6) • 8 – 4 / 2

Continue this process with every leaf of the tree until there are no more operations remaining on the leaves.

PEMDAS







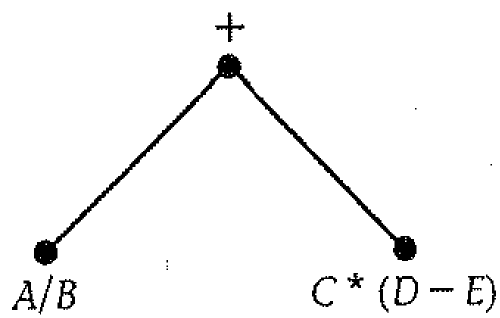
The final binary tree is called an <u>EXPRESSION TREE</u>.

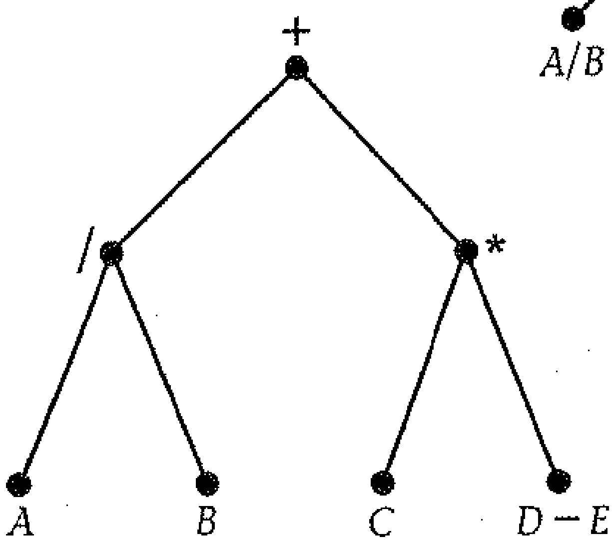
Expression Trees

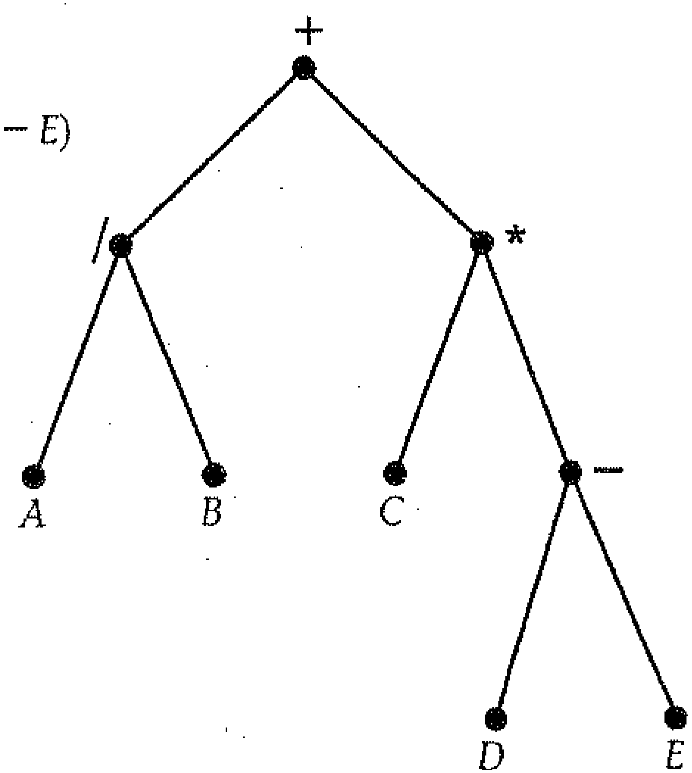
PEMDAS

A / B + C • (D – E)

Draw an expression tree.





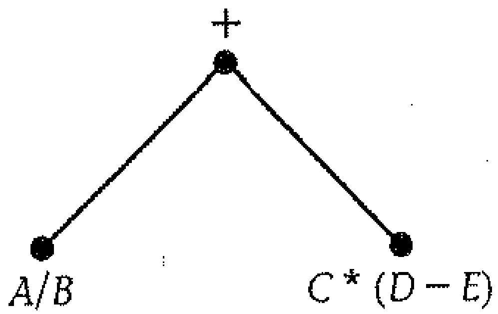


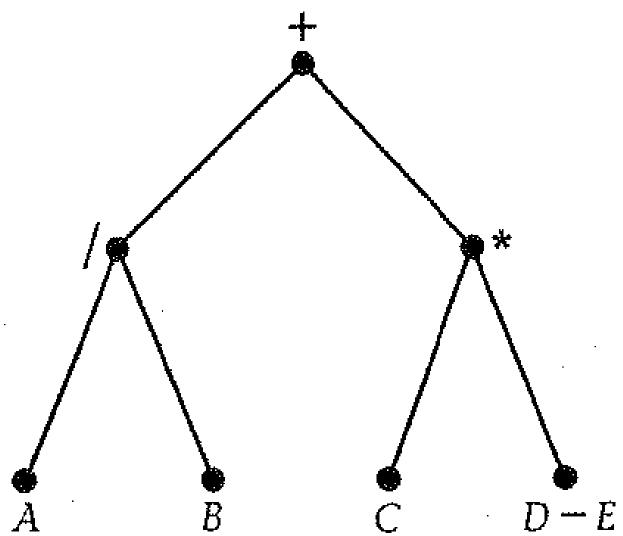
Expression Trees ANSWER

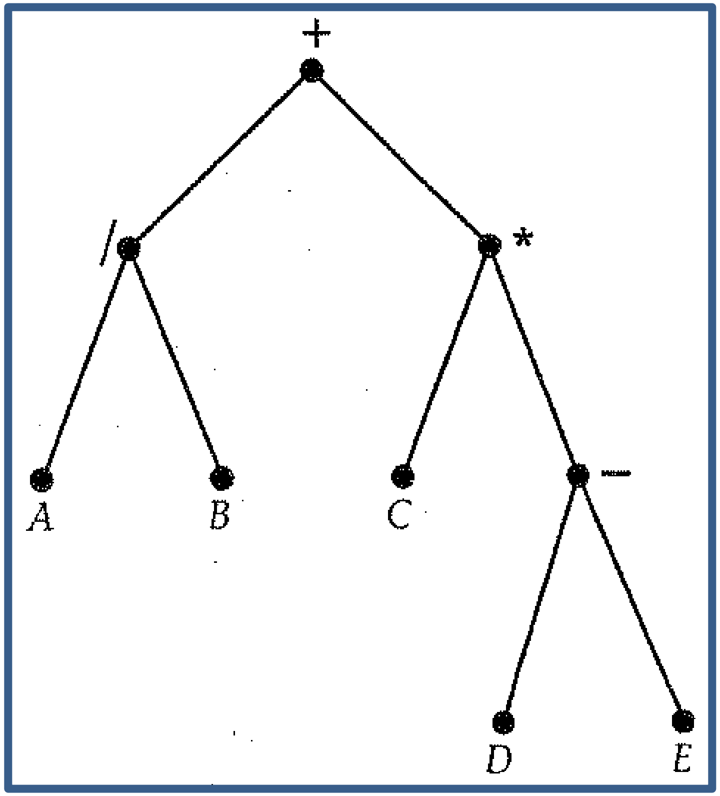
A / B + C • (D – E)

PEMDAS

Draw an expression tree.







Try this one…

PEMDAS

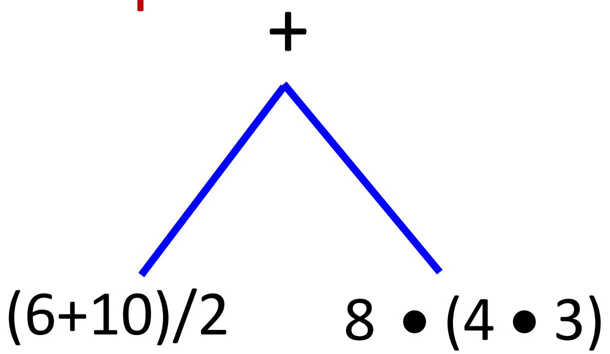
$$(6 + 10) / 2 + 8 \bullet (4 \bullet 3)$$

Try this one...

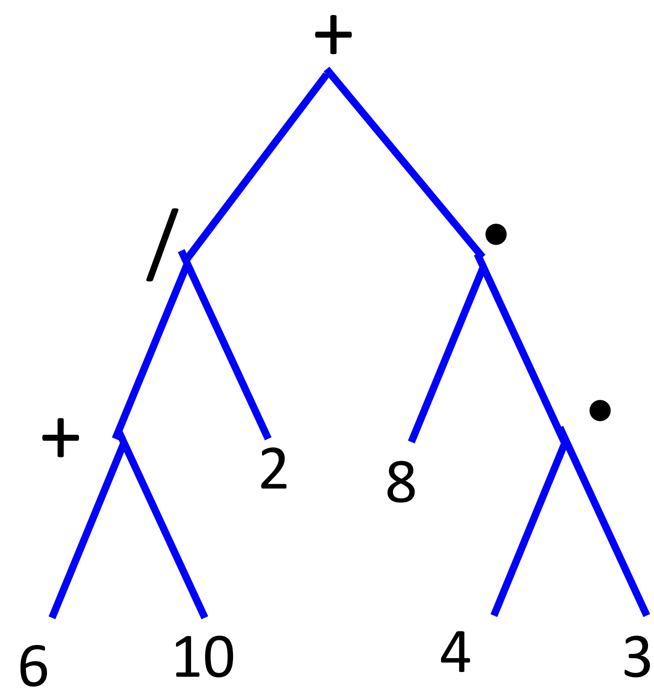
PEMDAS

$$(6 + 10) / 2 + 8 \bullet (4 \bullet 3)$$

Step 1:



Final Solution:



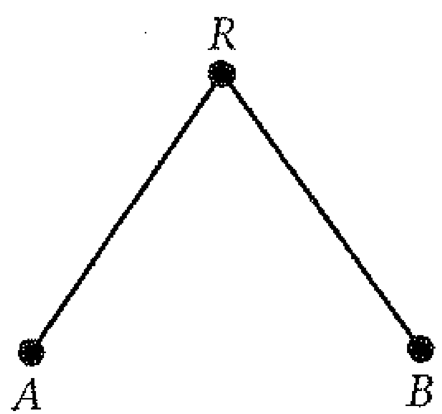
Tree Traversals

Once an expression is represented as a binary tree, the computer must have a systematic way of "looking at" the tree in order to find the value of the original expression.

The computer will obtain information by visiting each vertex of the tree exactly once which is called a TRAVERSAL of the graph.

There are many different types of traversals. The one we will learn is called the POSTORDER TRAVERSAL.

In this traversal, we will visit the left child of the tree, then the right child, then the parent or root.

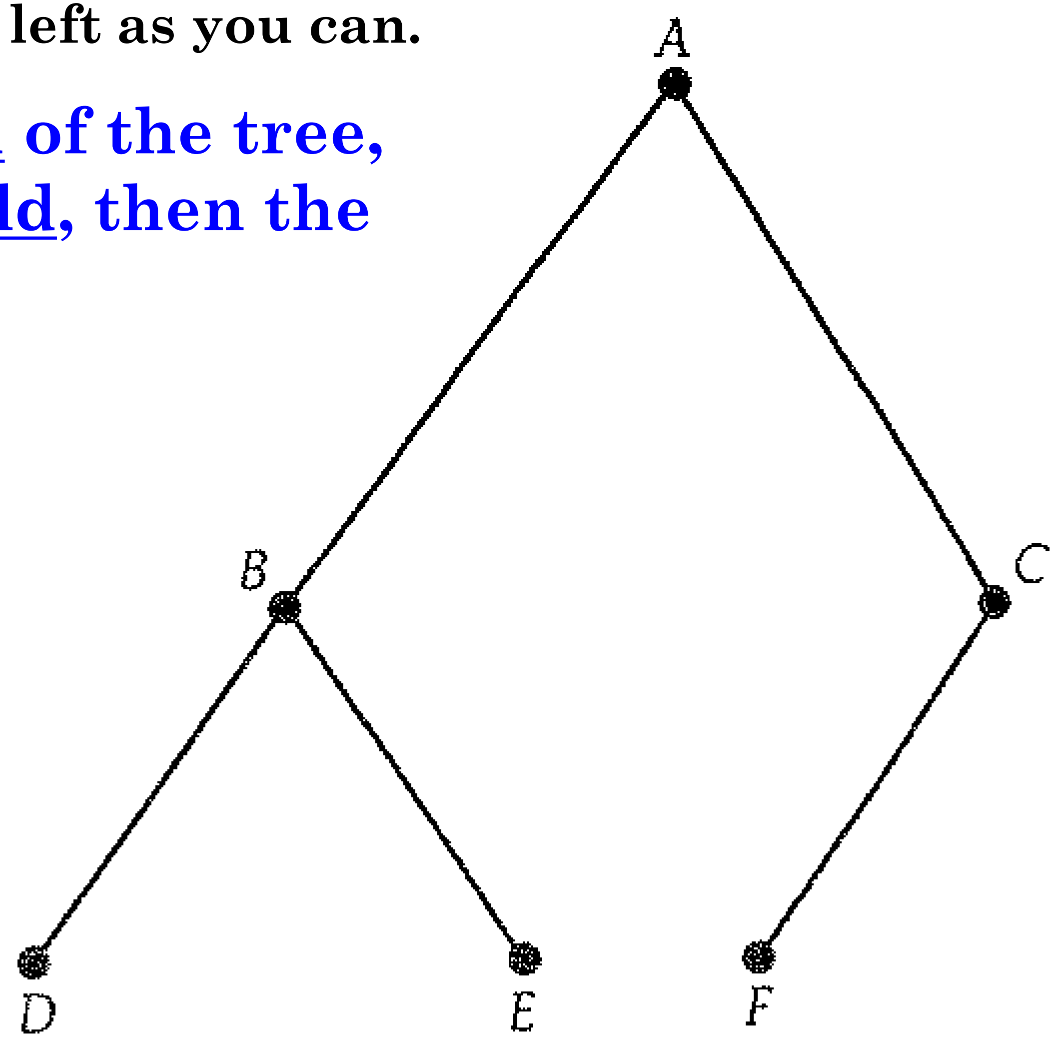


Postorder traversal A, B, R.

Tree Traversals

At each step go as far left as you can.

Visit the <u>left child</u> of the tree, then the <u>right child</u>, then the <u>parent or root</u>.



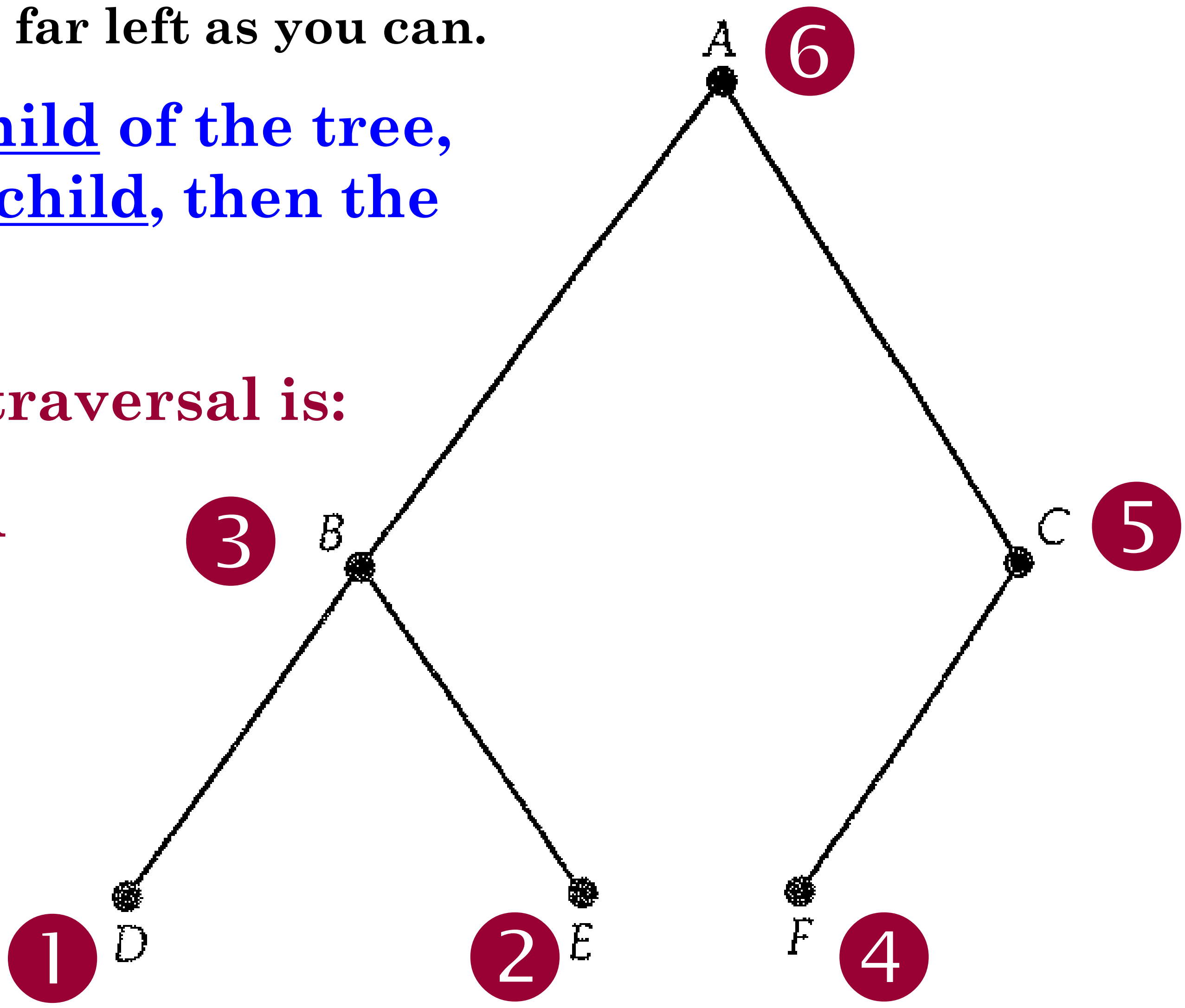
Tree Traversals ANSWER

At each step go as far left as you can.

Visit the <u>left child</u> of the tree, then the <u>right child</u>, then the <u>parent or root</u>.

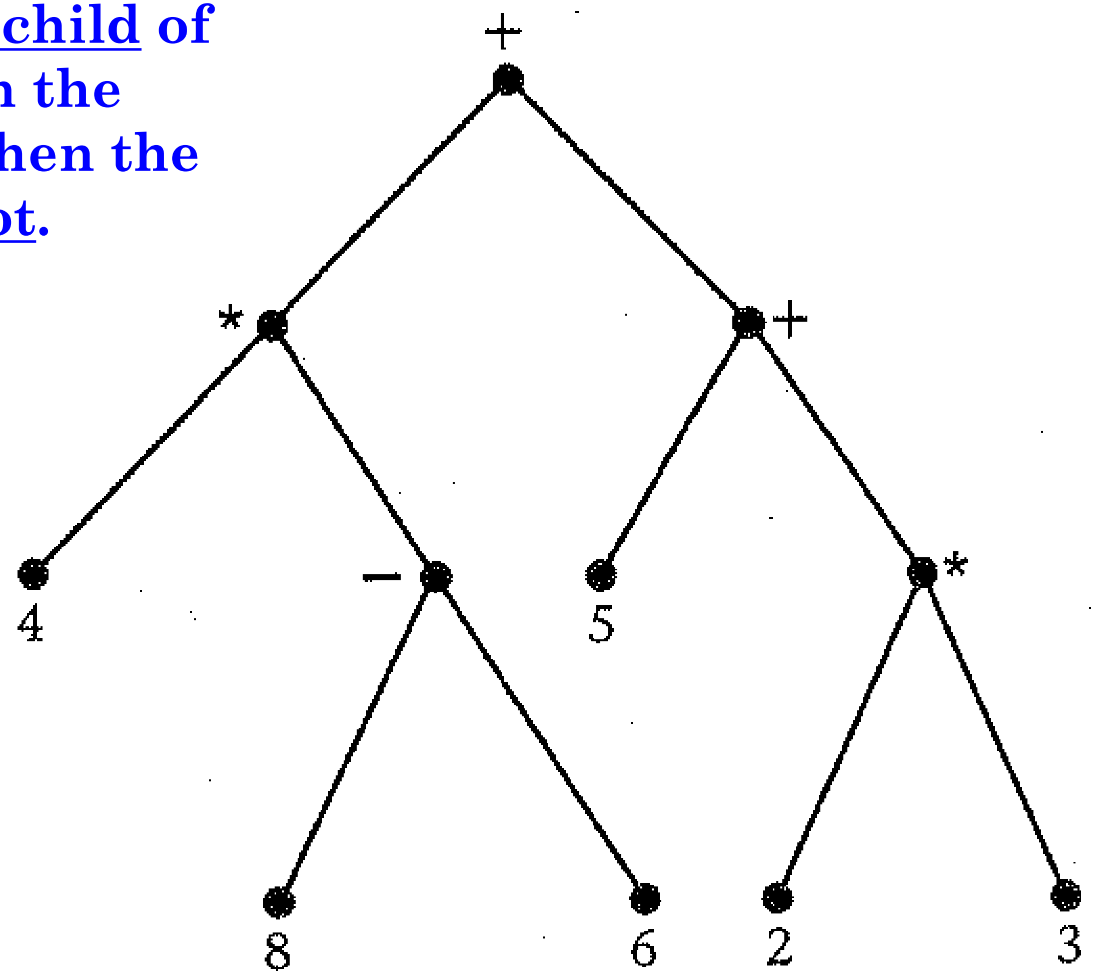
The postorder traversal is:

DEBFCA



Postorder Traversal
YOU try this one.

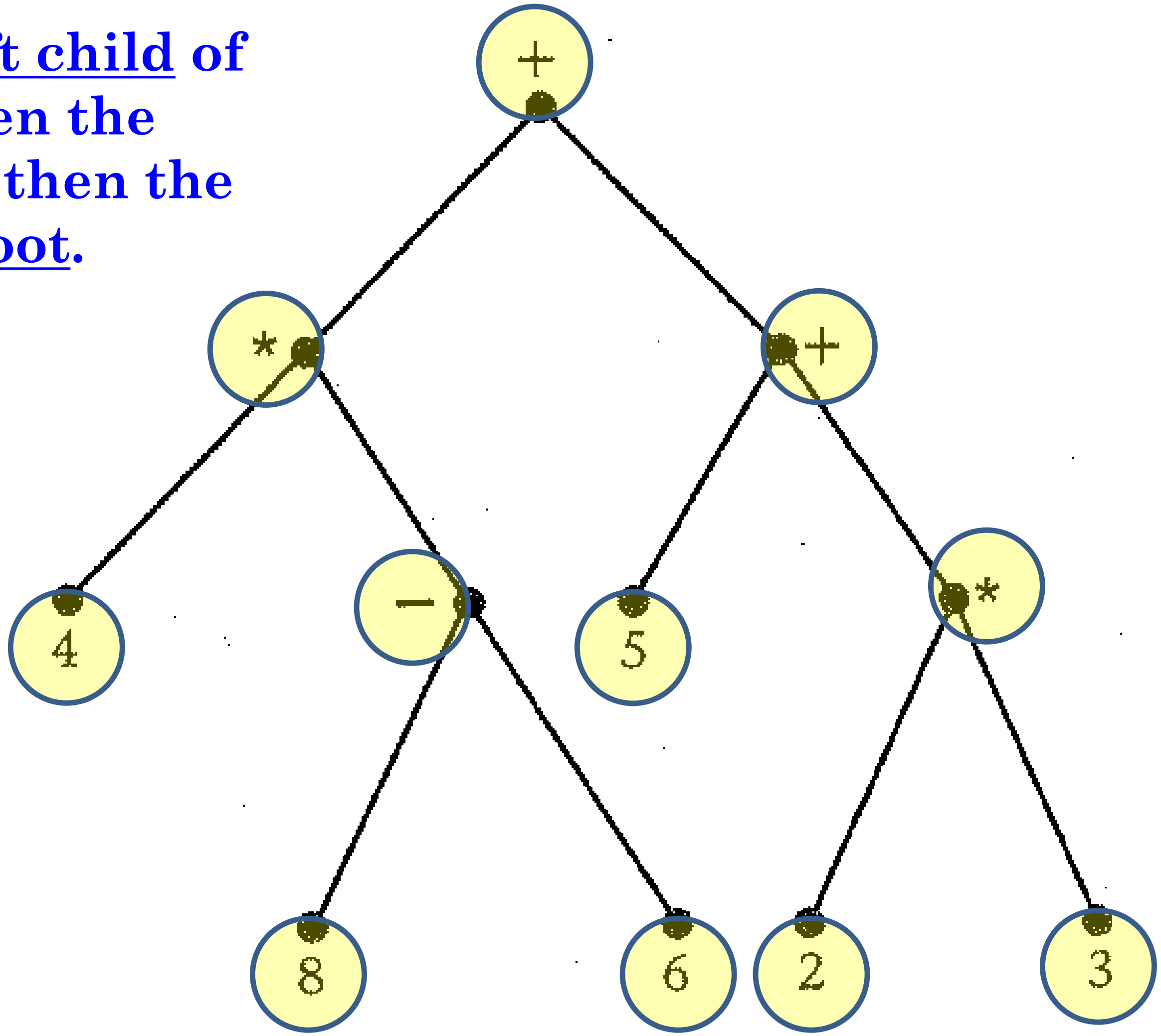
Visit the <u>left child</u> of the tree, then the <u>right child</u>, then the <u>parent or root</u>.



The postorder traversal is: ______________________

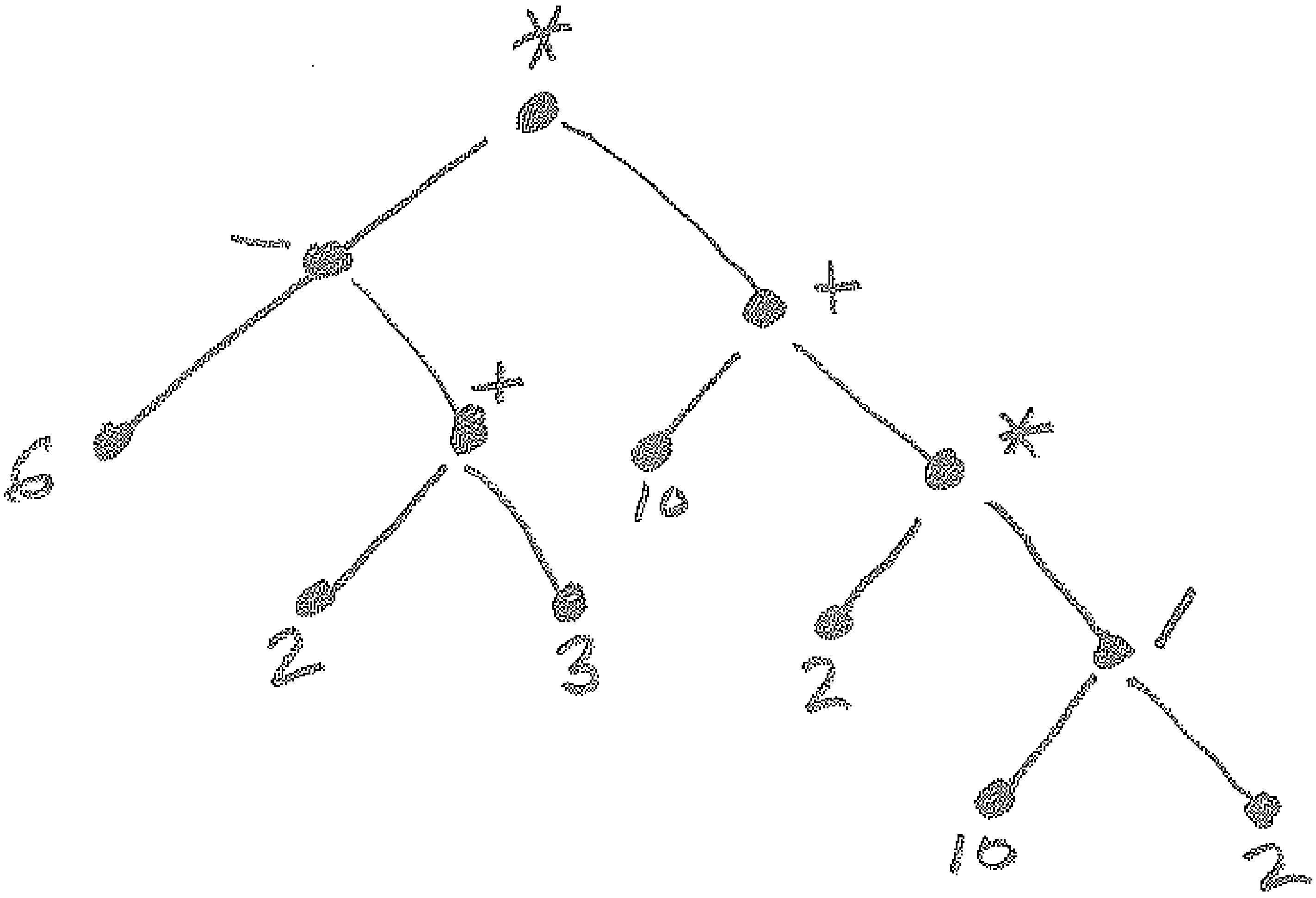
Postorder Traversal
YOU try this one. ANSWER

Visit the <u>left child</u> of the tree, then the <u>right child</u>, then the <u>parent or root</u>.



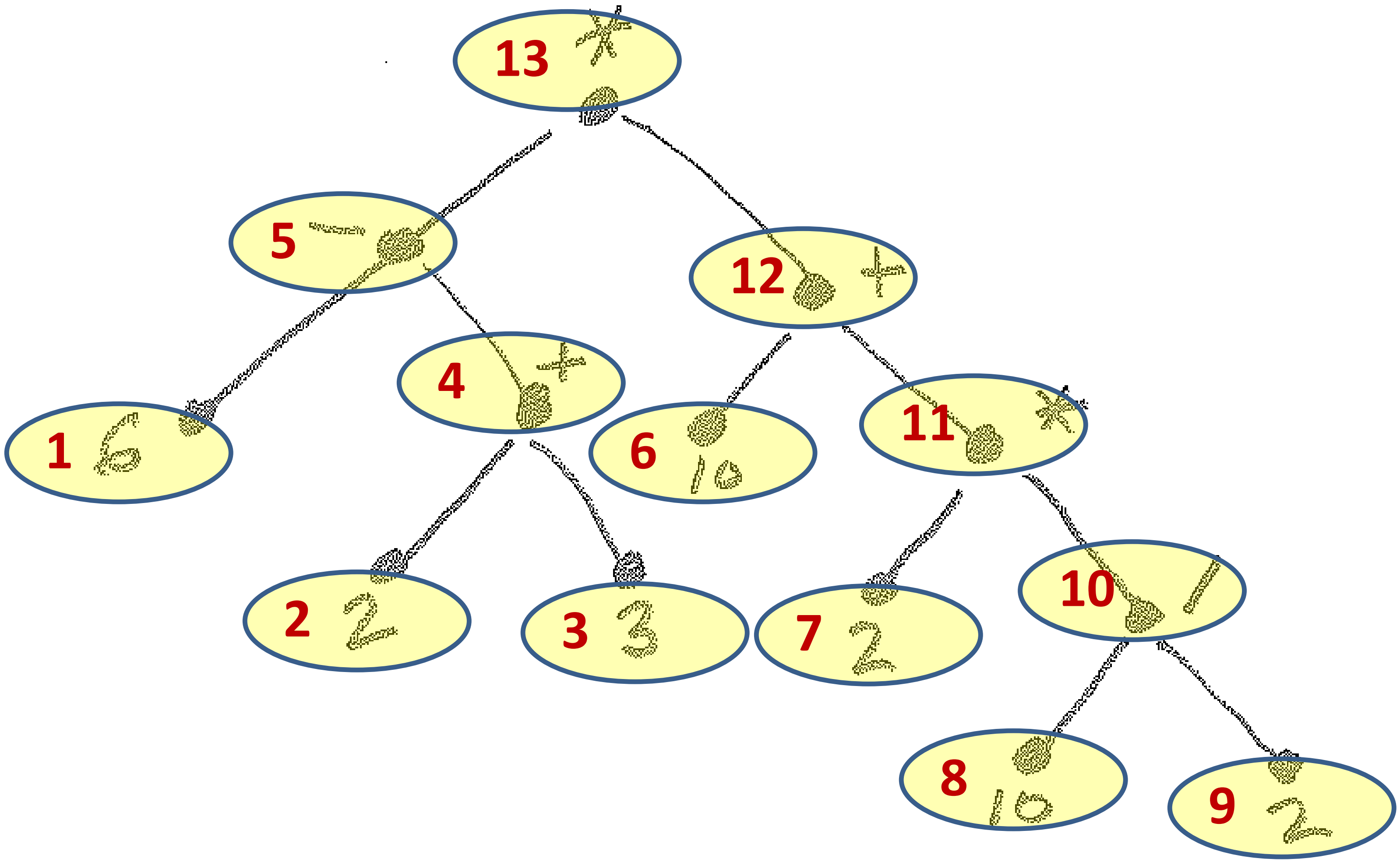
The postorder traversal is: 4 8 6 – • 5 2 3 • + +

Let's do Another…



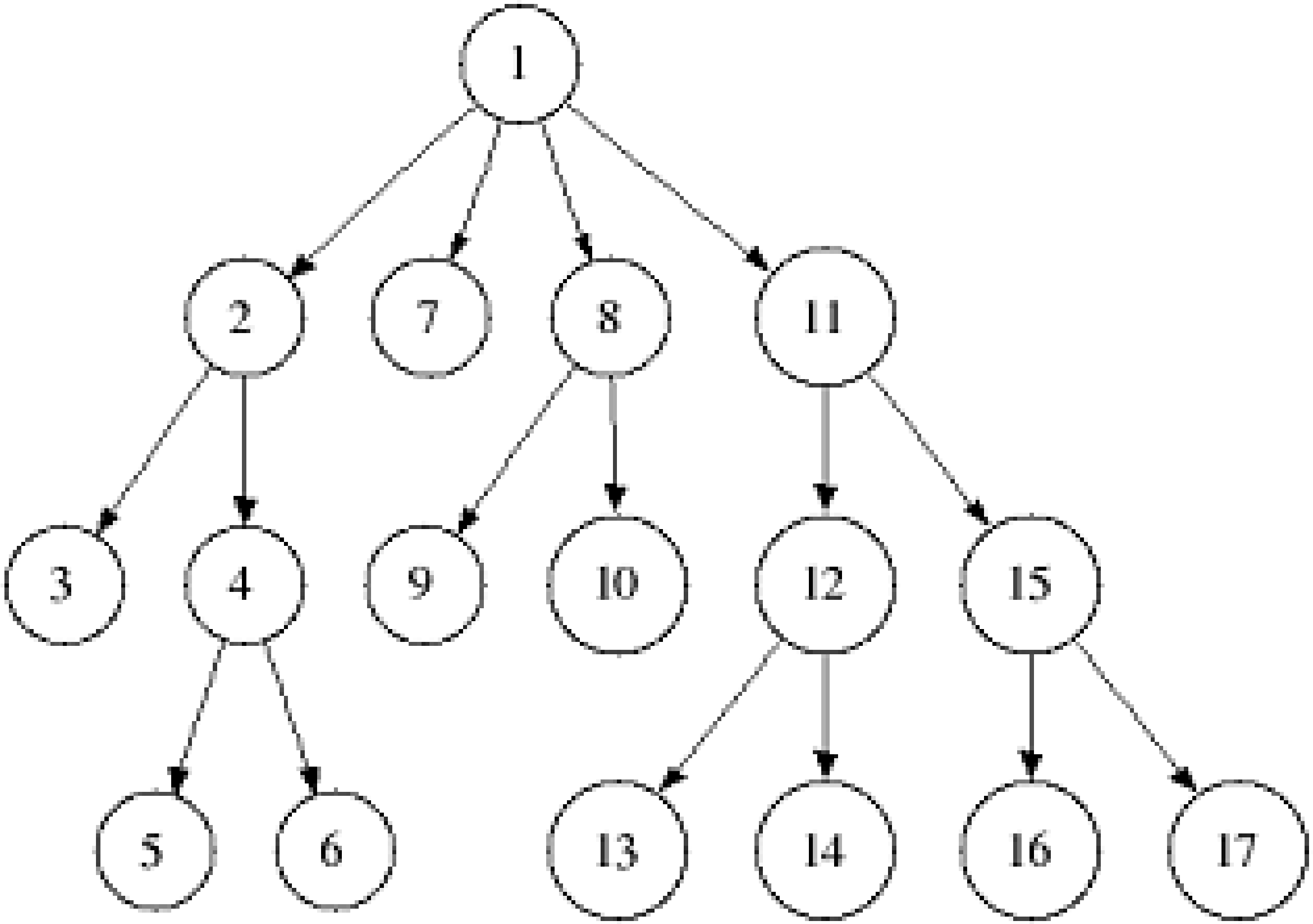
The postorder traversal is: ______________________

Let's do Another… ANSWER

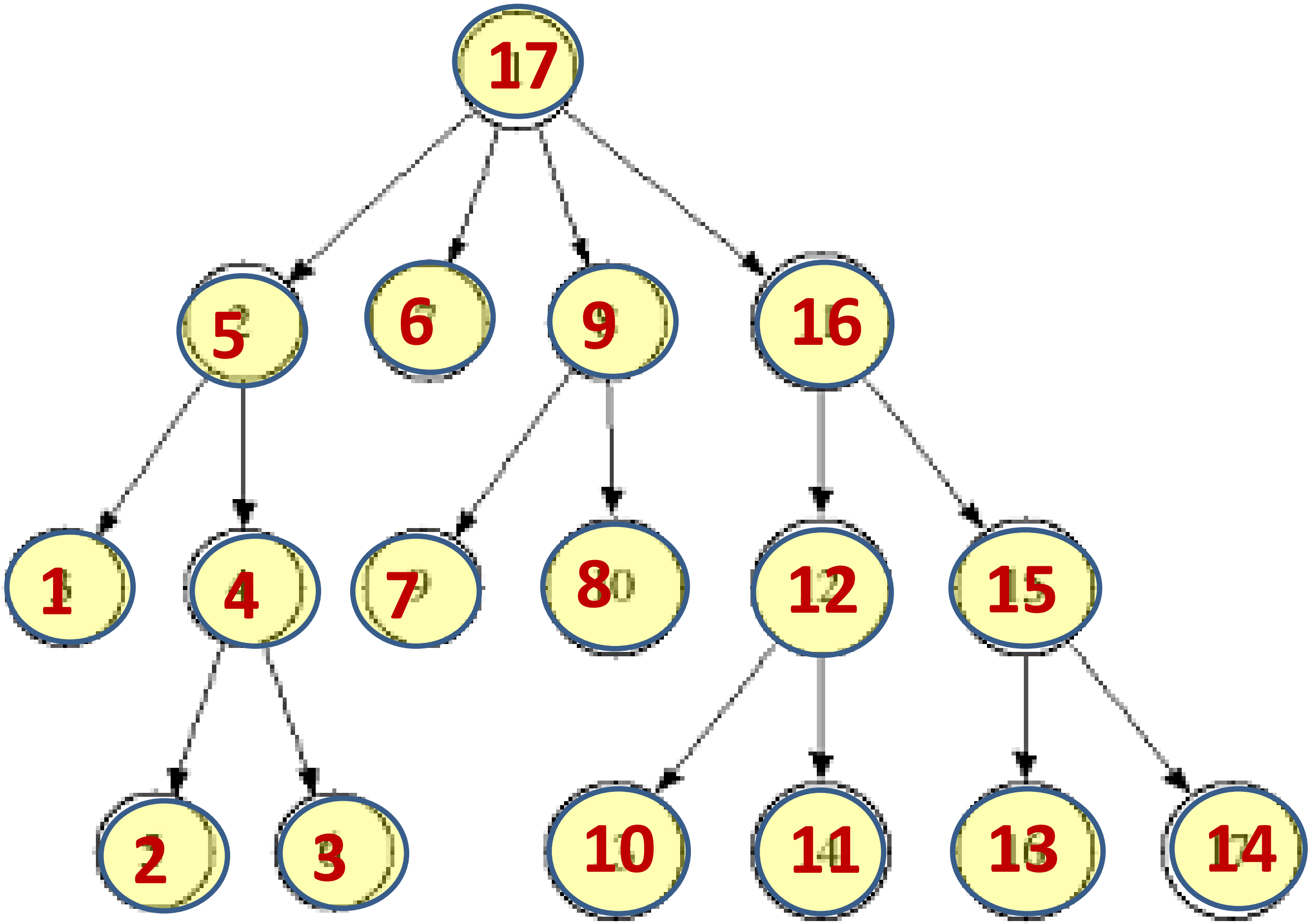


The postorder traversal is: 6 2 3 + – 10 2 10 2 / • + •

One more!



One more! ANSWER



Reverse Polish Notation

The notation obtained by doing a postorder traversal is known as <u>REVERSE POLISH NOTATION (RPN)</u>.

This notation may look strange, but computers like it. ***No parentheses are ever needed to indicate the desired order of operations.***

How do you find the value of the expression?

4 8 6 – • 5 2 3 • + +

4 8 6 – * 5 2 3 * + +

Scan the expression from <u>left to right</u> until you find two numbers followed by an operation sign.

Find the result, substitute it back in, and repeat until no more operations remain.

Reverse Polish Notation

Try it with this one. Find the value.

$$6\ 2\ 3\ +\ -\ 10\ \ 2\ \ 10\ \ 2\ \ /\ \ \bullet\ \ +\ \ \bullet$$

Reverse Polish Notation ANSWER

Try it with this one. Find the value.

$$6\ 2\ 3\ +\ -\ 10\ \ 2\ \ 10\ \ 2\ \ /\ \ \bullet\ \ +\ \ \bullet$$

$$6\ 5\ -\ 10\ \ 2\ \ 10\ \ 2\ \ /\ \ \bullet\ \ +\ \ \bullet$$

$$1\ \ 10\ \ 2\ \ 10\ \ 2\ \ /\ \ \bullet\ \ +\ \ \bullet$$

$$1\ \ 10\ \ 2\ \ 5\ \ \bullet\ \ +\ \ \bullet$$

$$1\ \ 10\ \ 10\ \ +\ \ \bullet$$

$$1\ \ 20\ \bullet \quad = 20$$

Reverse Polish Notation

Give the RPN and value for each of these expressions.

1) $4 \bullet 7 - ((3 \bullet 6) + 2) / 4$

2) $(7 - 3) + 8 - (10 / 2) \bullet 6$

3) $3 \bullet 6 + 10 / 2 - 2 \bullet 8$

Steps to finding RPN:

1. Create a binary tree
2. Find the Postorder Transversal
3. Find RPN value

Reverse Polish Notation ANSWERS

Give the RPN and value for each of these expressions.

1) $4 \bullet 7 - ((3 \bullet 6) + 2) / 4$ 23

2) $(7 - 3) + 8 - (10 / 2) \bullet 6$ -18

3) $3 \bullet 6 + 10 / 2 - 2 \bullet 8$ 7

Steps to finding RPN:

1. Create a binary tree
2. Find the Postorder Transversal
3. Find RPN value

Homework Day 8

- Packet p. 19
- Use extra piece of paper to show your work and trees!