Unit 4 Day 8

Symmetry & Compositions

Warm Up Day 8

1. $f(x) = 4x^3$ 2. $g(x) = 2x^2 - 4x$

a. f(-1)=

b. f(3x)=

c. f(-y)=

a. -g(x)=

b. g(x+y)=

c. g(-x)=

3. Write and graph an equation that has the following:

-Nonremovable discontinuity at 5

-Removable discontinuity at -3

-Horizontal asymptote of y=5/2

Warm Up Day 8 ANSWERS

1. $f(x) = 4x^3$ 2. $g(x) = 2x^2 - 4x$

a. f(-1)= -4

b. f(3x)= $108x^3$

c. f(-y)= $-4y^3$

a. -g(x)= $-2x^2 + 4x$

b. g(x+y)=

$2x^2 + 2y^2 + 4xy - 4x - 4y$

c. g(-x)= $2x^2 + 4x$

3. Write and graph an equation that has the following:

-Nonremovable discontinuity at 5

-Removable discontinuity at -3

-Horizontal asymptote of y=5/2

$$f(x) = \frac{5(x+3)(x+1)}{2(x+3)(x-5)}$$

Homework Questions?

Packet p. 6-7

Announcements ☺

- Quiz Unit 4 #1 Corrections due Thurs 3/22
  - If you complete those corrections & do better on Unit 4 Test, the Unit 4 Test grade can replace that quiz Grade

- Tutorials for credit due Friday 3/23!
- Tutorials are
  - Most mornings at 7 AM
  - Monday & Wednesday 1st half lunch

- Make-Up work due Monday 3/26!

- Reminder: Midterm is Tuesday 3/27
  - Midterm Packet due!

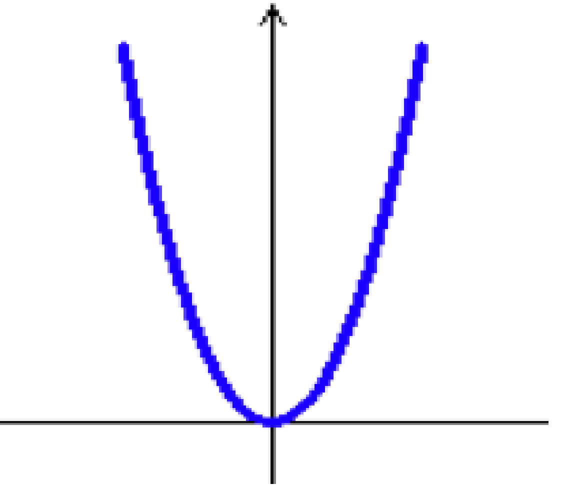
Notes:

Symmetry, Even & Odd Functions, Function Composition

Symmetry:
About the y-axis

- Example: $f(x) = x^2$

Graph:

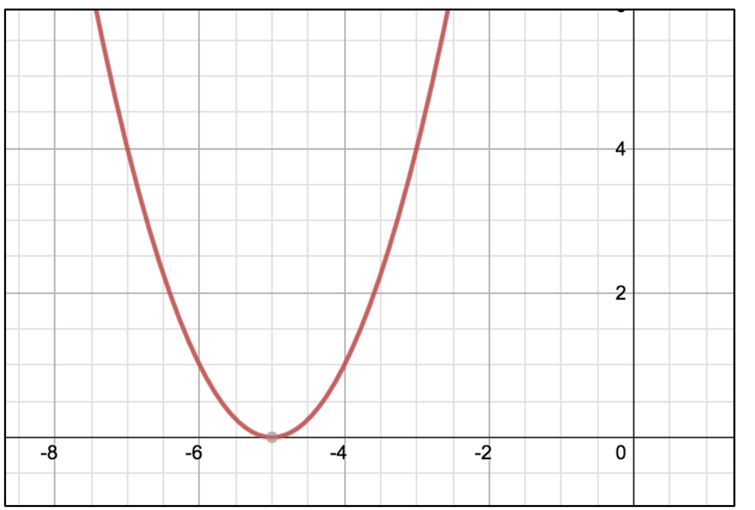


- A function is **EVEN** if it is symmetrical about the y-axis.

- **NON-Example:**

$$f(x) = (x+5)^2$$

Graph:



Symmetry with respect to the y-axis

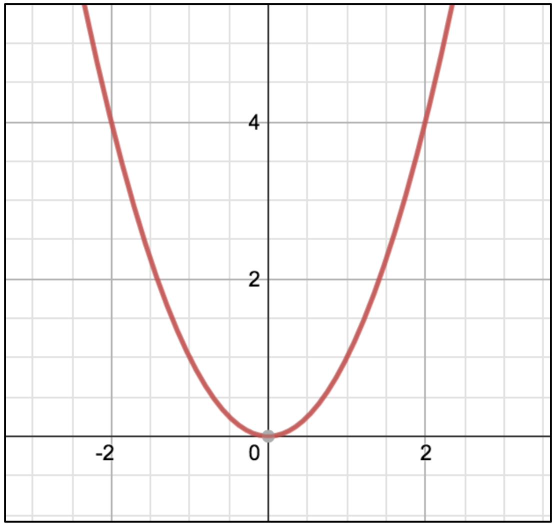
- A.k.a. Reflection about y-axis

- **To prove a function is even algebraically, show that**

$$f(-x) = f(x)$$

- Ex: $f(x) = x^n$

  where n is ***<u>even</u>***



Determining Algebraically if a function is Even, Odd, or Neither

- Prove whether $f(x) = x^4$ is even, odd, or neither.

- We'll check if $f(-x) = f(x)$

Determining Algebraically if a function is Even, Odd, or Neither

- Prove whether $f(x) = x^4$ is even, odd, or neither.
- We'll check if $f(-x) = f(x)$

$$f(-x) = (-x)^4 = x^4$$

$$f(-x) = f(x)$$

$$\therefore \boxed{f(x) \text{ is even}}$$

Symmetry— Checking in Table

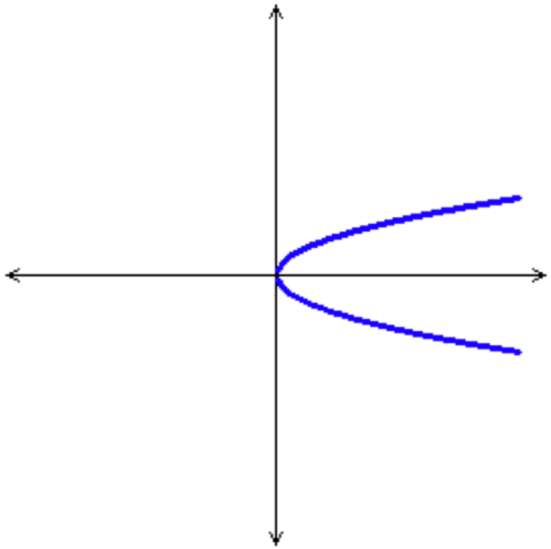
- **<u>Note:</u>** You can often double-check if a function is even in the table

x	$f(x)$
-3	9
-2	4
-1	1
1	1
2	4
3	9

- Example: $f(x) = x^2$
- BUT To prove **<u>"Even"</u>** functions you must show Algebraically that $f(-x) = f(x)$

Symmetry: x-axis

- A.k.a. Symmetric with respect to the *x*-axis.
- Graphs with this symmetry are *not usually functions*. They are mirrors.
- (*x*,-*y*) mirrors (*x*,*y*)

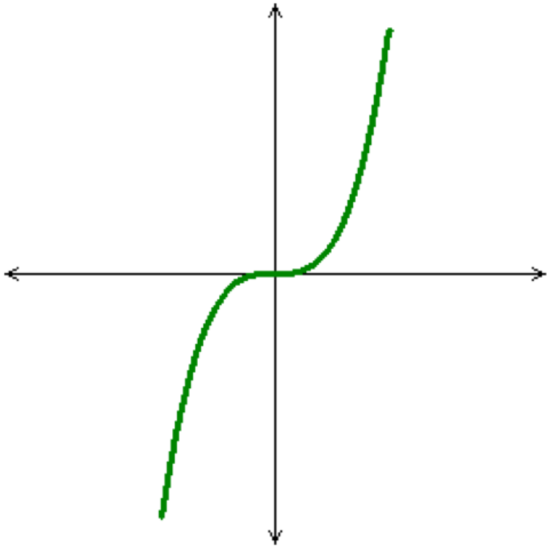


x	$f(x)$
9	-3
4	-2
1	-1
1	1
4	2
9	3

Symmetry: About the origin

- Example: $f(x) = x^3$

Graph:

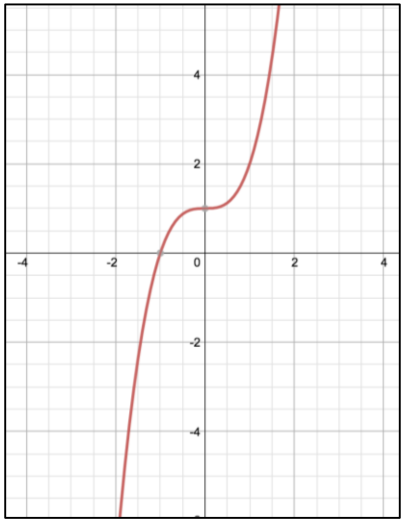


- A function is **ODD** if it is symmetrical about the origin.

- **NON-Example:**

$$f(x) = x^3 + 1$$

Graph:



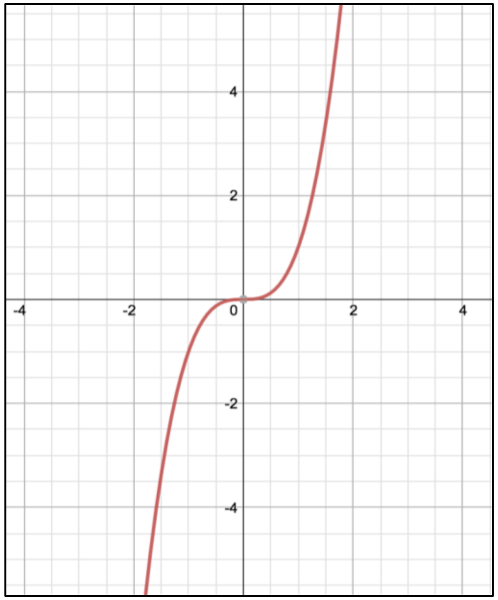
Symmetry about the Origin

- A.k.a. Symmetric with respect to the *origin.*
- *Can be seen as* Rotation about origin 180°.
- **To prove odd functions algebraically show that :**

$$f(-x) = -f(x)$$

- Ex: $f(x) = x^n$

  where *n* is ***odd***.



Determining Algebraically if a function is Even, Odd, or Neither

- Prove whether f(x) = x^5 is even, odd, or neither.
- 1st we'll check if $f(-x) = f(x)$
- 2nd we'll check if $f(-x) = -f(x)$

Determining Algebraically if a function is Even, Odd, or Neither

- Prove whether $f(x) = x^5$ is even, odd, or neither.
- 1st we'll check if $f(-x) = f(x)$

$$f(-x) = (-x)^5 = -x^5$$

$$f(-x) \neq f(x) \therefore \ f(x) \ is \ NOT \ even$$

- 2nd we'll check if $f(-x) = -f(x)$

$$-f(x) = -(x^5) = -x^5$$

$$f(-x) = -f(x) \therefore \ f(x) \ is \ odd$$

Symmetry— Checking in Table

- **<u>Note:</u>** You can often double-check if a function is odd in the table

x	$f(x)$
-3	-27
-2	-8
-1	-1
1	1
2	8
3	27

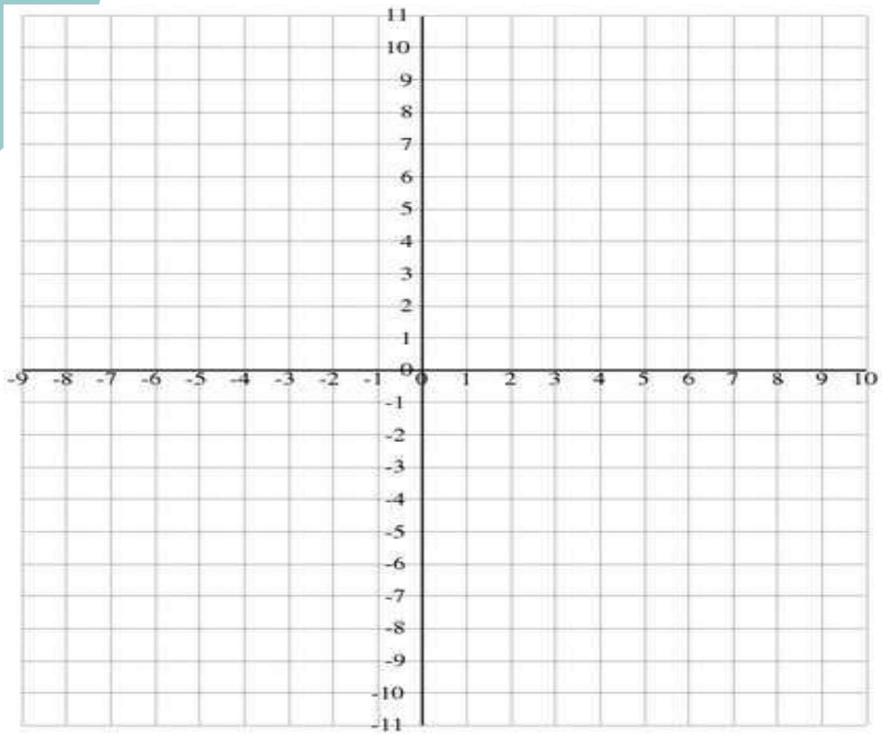
- Example: $f(x) = x^3$
- BUT To prove **<u>"Odd"</u>** functions you must show Algebraically that $f(-x) = -f(x)$

Is the function symmetric?
If so, determine the symmetry.

$$f(x) = 5 + x^2$$

Graph

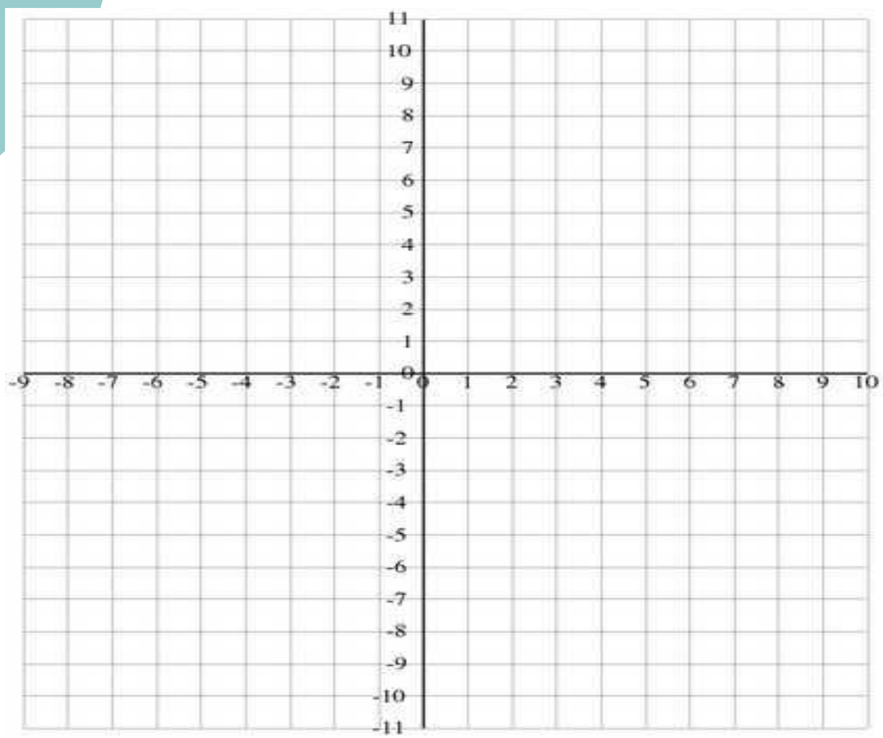
Algebraic



Is the function symmetric? ANSWER If so, determine the symmetry.

$$f\left(x\right)=5+x^2$$

Graph



Algebraic

$$f(-x)=f(x)$$

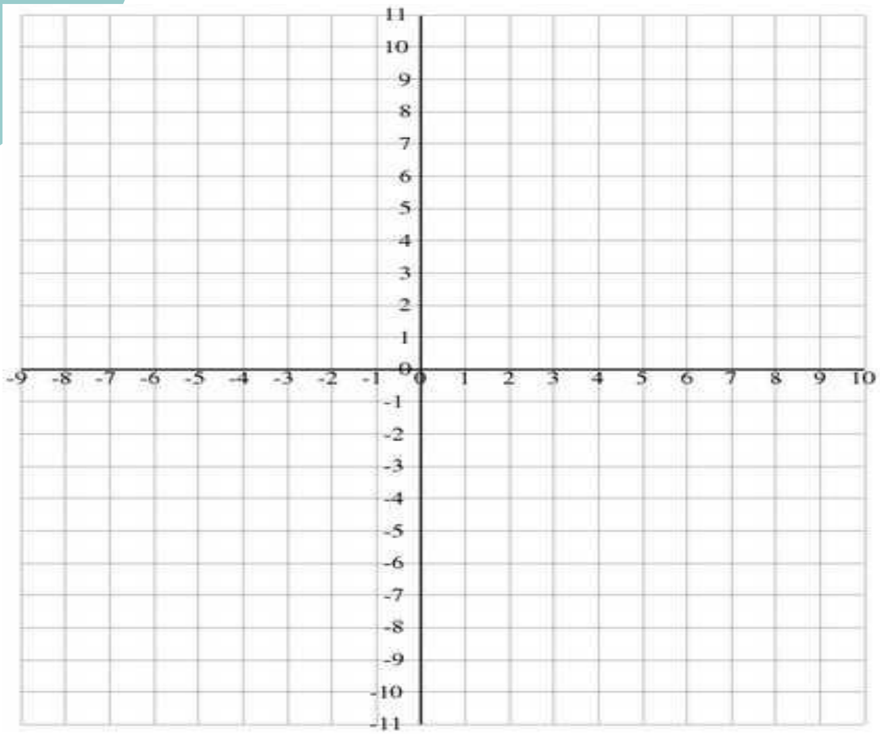
$$\therefore Even$$

$$Symmetric\ about\ the\ y-axis$$

Is the function symmetric? If so, determine the symmetry.

$$g(x) = 3x - 2x^3$$

Graph

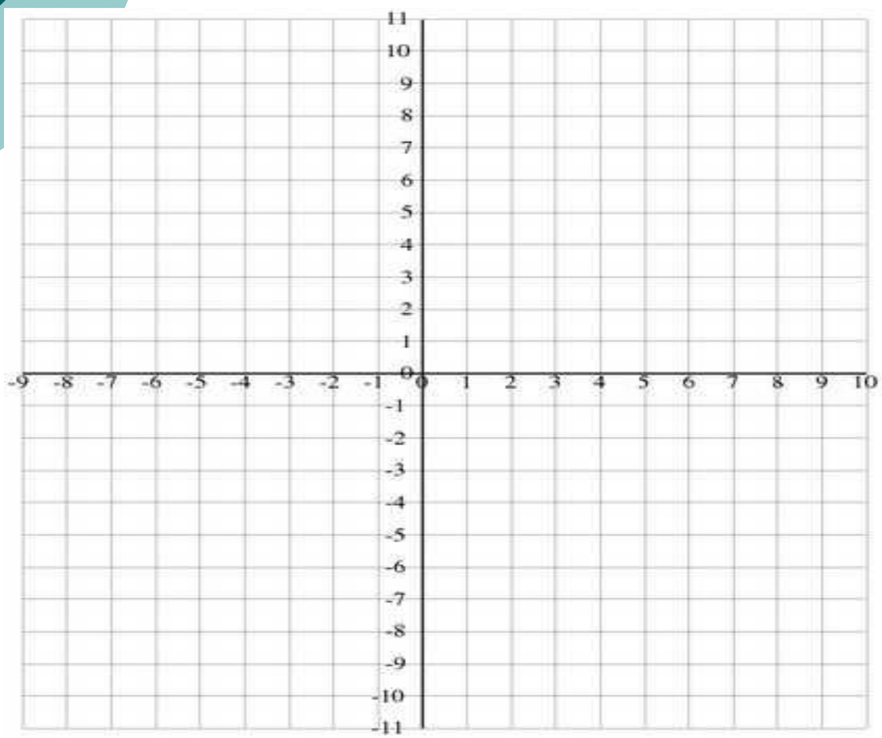


Algebraic

Is the function symmetric? ANSWER
If so, determine the symmetry.

$$g(x) = 3x - 2x^3$$

Graph



Algebraic

$$g(-x) = -g(x)$$

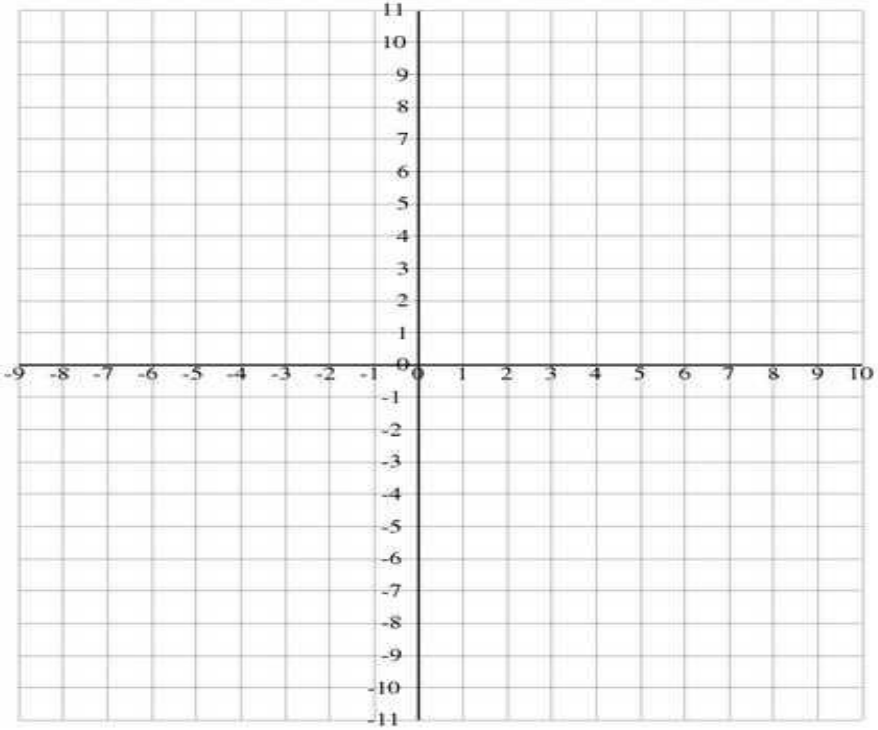
$$\therefore Odd$$

Symmetric about the origin

Is the function symmetric? If so, determine the symmetry.

$$h(x) = x^7 - 2$$

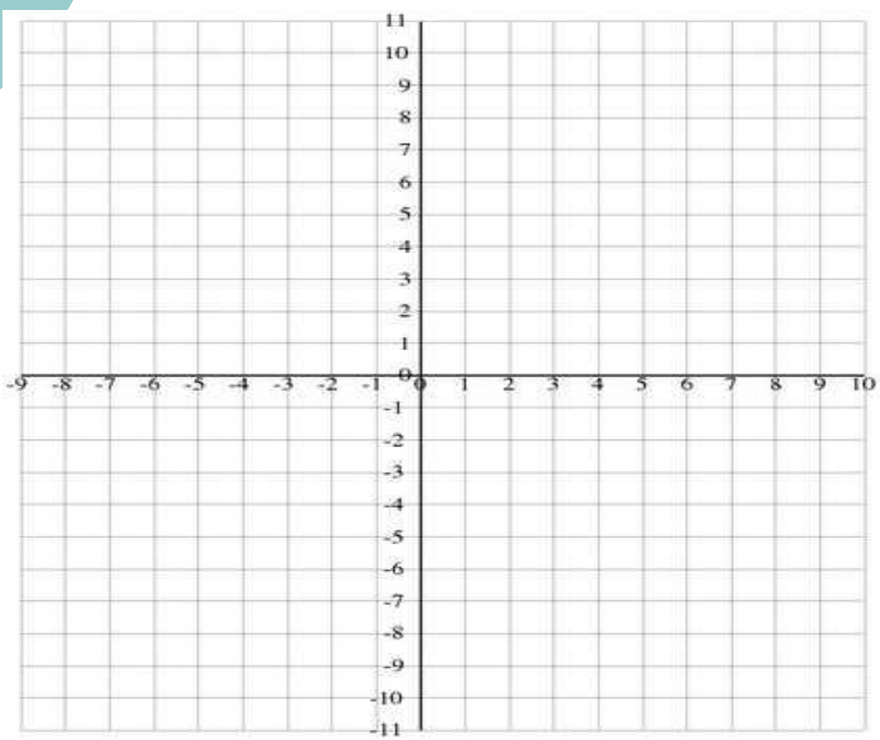
Graph **Algebraic**



Is the function symmetric? ANSWER
If so, determine the symmetry.

$$h(x) = x^7 - 2$$

Graph



Algebraic

$$h(-x) \neq -h(x)$$

$$h(-x) \neq h(x)$$

$$\therefore \textit{Neither}$$

You Try!

Is the function symmetric?
If so, determine the symmetry.

★ 3. $g(x) = \dfrac{x^3}{x^2 - 9}$

4. $f(x) = \dfrac{2x^3}{x - 5}$

5. $f(x) = \dfrac{x}{x^3 - x}$

You Try! Is the function symmetric? If so, determine the symmetry.

ANSWER

3. $g(x) = \dfrac{x^3}{x^2 - 9}$

$g(-x) = -g(x)$

$\therefore Odd$

4. $f(x) = \dfrac{2x^3}{x-5}$

$f(-x) \neq -f(x)$

$f(-x) \neq f(x)$

$\therefore Neither$

5. $f(x) = \dfrac{x}{x^3 - x}$

$f(-x) = f(x)$

$\therefore Even$

Now let's combine functions

Recall function notation!

Let's step it up a notch!

- If $f(x) = 3x$, (Simplify when possible)
- Find: $f(4) - f(x)$
- Find: $f(x+1) - x$
- Find: $\frac{f(x+h) - f(x)}{2}$

Let's step it up a notch!

- If $f(x) = 3x$, (Simplify when possible)
- Find: $f(4) - f(x)$ $\quad 12 - 3x$
- Find: $f(x+1) - x$ $\quad 3x+3 - x = 2x + 3$
- Find: $\dfrac{f(x+h) - f(x)}{2}$ $\quad \dfrac{3x+3h - 3x}{2} = \dfrac{3h}{2}$

Combinations of Functions

When you add, subtract, multiply, or divide two functions.

- Add: $(f + g)(x) = f(x) + g(x)$
- Subtract: $(f - g)(x) = f(x) - g(x)$
- Multiply: $(f \bullet g)(x) = f(x) \bullet g(x)$
- Divide: $(f/g)(x) = f(x) / g(x)$

Examples:

$$f(x) = \frac{3}{x-2} \qquad g(x) = \frac{3+x}{3}$$

Add and find the domain:

(f + g) (x) = f(x) + g(x)

Subtract and find the domain:

(f - g) (x) = f(x) - g(x)

Multiply and find the domain:

(f •g) (x) = f(x)•g(x)

Divide and find the domain:

(f/g) (x) = f(x)/g(x)

Examples:

$$f(x)=\frac{3}{x-2} \qquad g(x)=\frac{3+x}{3}$$

Add and find the domain:

(f + g) (x) = f(x) + g(x)

$$(f+g)(x)=\frac{3}{x-2}+\frac{3+x}{3}$$

$$D:(-\infty,2)\cup(2,\infty)$$

Subtract and find the domain:

(f - g) (x) = f(x) - g(x)

$$(f-g)(x)=\frac{3}{x-2}-\left(\frac{3+x}{3}\right)$$

$$D:(-\infty,2)\cup(2,\infty)$$

Multiply and find the domain:

(f •g) (x) = f(x)•g(x)

$$(f\bullet g)(x)=\left(\frac{3}{x-2}\right)\left(\frac{3+x}{3}\right)$$

$$D:(-\infty,2)\cup(2,\infty)$$

Divide and find the domain:

(f/g) (x) = f(x)/g(x)

$$(f/g)(x)=\frac{\left(\frac{3}{x-2}\right)}{\left(\frac{3+x}{3}\right)}$$

$$D:(-\infty,-3)\cup(-3,2)\cup(2,\infty)$$

Examples:

$$f(x) = \sqrt{x+4} \qquad g(x) = |x+2|$$

Add and find the domain:

(f + g) (x) = f(x) + g(x)

Subtract and find the domain:

(f - g) (x) = f(x) - g(x)

Examples:

$$f(x) = \sqrt{x+4} \qquad g(x) = |x+2|$$

Add and find the domain:

(f + g) (x) = f(x) + g(x)

$$(f+g)(x) = \sqrt{x+4} + |x+2|$$

$$D:[-4, \infty)$$

Subtract and find the domain:

(f - g) (x) = f(x) - g(x)

$$(f-g)(x) = \sqrt{x+4} - |x+2|$$

$$D:[-4, \infty)$$

Examples:

$$f(x) = \sqrt{x+4} \qquad\qquad g(x) = |x+2|$$

Multiply and find the domain:

(f •g) (x) = f(x)•g(x)

Divide and find the domain:

(f/g) (x) = f(x)/g(x)

Examples:

$$f(x)=\sqrt{x+4} \qquad g(x)=|x+2|$$

Multiply and find the domain:

(f •g) (x) = f(x)•g(x)

$$(f \bullet g)(x)=\sqrt{x+4}\bullet|x+2|$$

$$D:[-4,\infty)$$

Divide and find the domain:

(f/g) (x) = f(x)/g(x)

$$(f/g)(x)=\frac{\sqrt{x+4}}{|x+2|}$$

$$D:[-4,-2)\cup(-2,\infty)$$

Compositions

Let *f* & *g* be two functions such that the domain of *f* intersects the range of *g*.

$(f \circ g)(x) = f(g(x))$

$(g \circ f)(x) = g(f(x))$

Domain of $(f \circ g)(x) =$

- Intersection of the domain of g(x) with the domain of f(g(x)).

Domain of $(g \circ f)(x) =$

- Intersection of the domain of f(x) with the domain of g(f(x)).

Compositions

$f(x) = x^2$ $g(x) = 4 - 3x$

$(f \circ g)(x) = f(g(x))$ $(g \circ f)(x) = g(f(x))$

Domain: Domain:

Compositions

$f(x) = x^2$ $\qquad$ $g(x) = 4 - 3x$

$(f \circ g)(x) = f(g(x))$ $\qquad$ $(g \circ f)(x) = g(f(x))$

$f(g(x)) = (4 - 3x)^2$ $\qquad$ $g(f(x)) = 4 - 3x^2$

Domain:

$D: (-\infty, \infty)$

Domain:

$D: (-\infty, \infty)$

Find the domain of g(f(x))?

$f(x) = \sqrt{x+5}$ $\qquad$ $g(x) = x^2 - 3$

What numbers can't you substitute in to g(f(x))? Does your domain agree?

Find the domain of g(f(x))?

$$f(x) = \sqrt{x+5} \qquad g(x) = x^2 - 3$$

$$g(f(x)) = (\sqrt{x+5})^2 - 3$$

$$D:[-5,\infty)$$

$$Evaluate: g(f(-9))$$

What numbers can't you substitute in to g(f(x))? Does your domain agree?

Compositions

$$f(x) = x^2 - 1$$

$$g(x) = \frac{1}{x-1}$$

$$(f \circ g)(x) = f\left(g(x)\right)$$

$$(g \circ f)(x) = g\left(f(x)\right)$$

Domain:

Domain:

Compositions

$$f(x) = x^2 - 1 \qquad g(x) = \frac{1}{x-1}$$

$$(f \circ g)(x) = f\left(g(x)\right) \qquad (g \circ f)(x) = g\left(f(x)\right)$$

$$(f(g(x)) = \left(\frac{1}{x-1}\right)^2 - 1 \qquad (g(f(x)) = \frac{1}{(x^2-1)-1}$$

Domain:

$$D: (-\infty, 1) \cup (1, \infty)$$

Domain:

$$D: (-\infty, -\sqrt{2}) \cup (-\sqrt{2}, \sqrt{2}) \cup (\sqrt{2}, \infty)$$

$$f(x)=\sqrt{x+5} \qquad g(x)=x^2-3$$

Evaluate:

$$f(g(2)+g(-1))=$$

$$f(g(h+1))=$$

$$f(x)=\sqrt{x+5} \qquad g(x)=x^2-3$$

Evaluate:

$$f(g(2)+g(-1)) = \quad 2$$

$$f(g(h+1)) = \sqrt{h^2+2h+3}$$

Decompositions

We are now going to “undo” compositions by breaking a function back into its two pieces so that **h(x)=f(g(x)).**

$$1.\ h(x) = (x+1)^2 - 3(x+1) + 4$$

$$2.\ h(x) = \sqrt{x^3+1}$$

Decompositions

We are now going to "undo" compositions by breaking a function back into its two pieces so that **h(x)=f(g(x)).**

---

1. $h(x)=(x+1)^2-3(x+1)+4$

$$f(x)=x^2-3x+4$$

$$g(x)=x+1$$

2. $h(x)=\sqrt{x^3+1}$

$$f(x)=\sqrt{x+1}$$

$$g(x)=x^3$$

Extra Practice – up next

Warm Up ~

- State the following and sketch a graph
- Domain:
- Range:
- x & y intercepts:
- Max and Min:
- Increasing:
- Decreasing:
- Limits at discontinuities:
- End Behavior using limits:

$$g(x) = \frac{\sqrt[3]{x}}{x^2 - x}$$

Warm Up ANSWERS ~

- State the following and sketch a graph
- Domain: (-¥,0)È(0,1)È(1,¥)
- Range: (-¥,0)È(0,¥)
- x & y intercepts: NONE
- Max and Min: $\max of\ -3.07\ at\ x = .4$
- Increasing: $(0, .4]$
- Decreasing: $(-\infty, 0) \cup [.4, 1) \cup (1, \infty)$
- Limits at discontinuities:
- End Behavior using limits:

$$g(x) = \frac{\sqrt[3]{x}}{x^2 - x}$$

$$\lim_{x \to 0} f(x) = -\infty$$

$$\lim_{x \to 1} f(x) = DNE$$

$$\lim_{x \to \infty} f(x) = 0$$

$$\lim_{x \to -\infty} f(x) = 0$$