UNIT 5 ~ DAY 8

Applications

WARM UP

❖**Find the derivative of each function.**

1. $f(x) = \sqrt[3]{(3x^2 + 4x)^5}$

2. $g(x) = x^4\sqrt{2x - 3}$

WARM UP ANSWERS

- **Find the derivative.**

1. $f(x)=\sqrt[3]{(3x^2+4x)^5}$

$$f'(x)=\frac{5}{3}(3x^2+4x)^{\frac{2}{3}}(6x+4)$$

$$=(10x+\frac{20}{3})(3x^2+4x)^{\frac{2}{3}}$$

$$=(10x+\frac{20}{3})\sqrt[3]{(3x^2+4x)^2}$$

2. $g(x)=x^4\sqrt{2x-3}$

$$g'(x)=(4x^3)(2x-3)^{\frac{1}{2}} + x^4(\frac{1}{2}(2x-3)^{-\frac{1}{2}}\bullet 2)$$

$$=(4x^3)(2x-3)^{\frac{1}{2}} + x^4(2x-3)^{-\frac{1}{2}}$$

$$=4x^3\sqrt{2x-3} + \frac{x^4}{\sqrt{2x-3}}$$

HW QUESTIONS?

TONIGHT'S HW

PACKET P. 10

OMIT #1 AND #9

#3, 7, AND 11 HAVE <u>NEGATIVE</u> VELOCITY

<u>Additional Hint</u>: If an object is **DROPPED**, then the initial velocity is 0 (#5 and #10)

NOTES: APPLICATIONS OF DERIVATIVES

Definitions

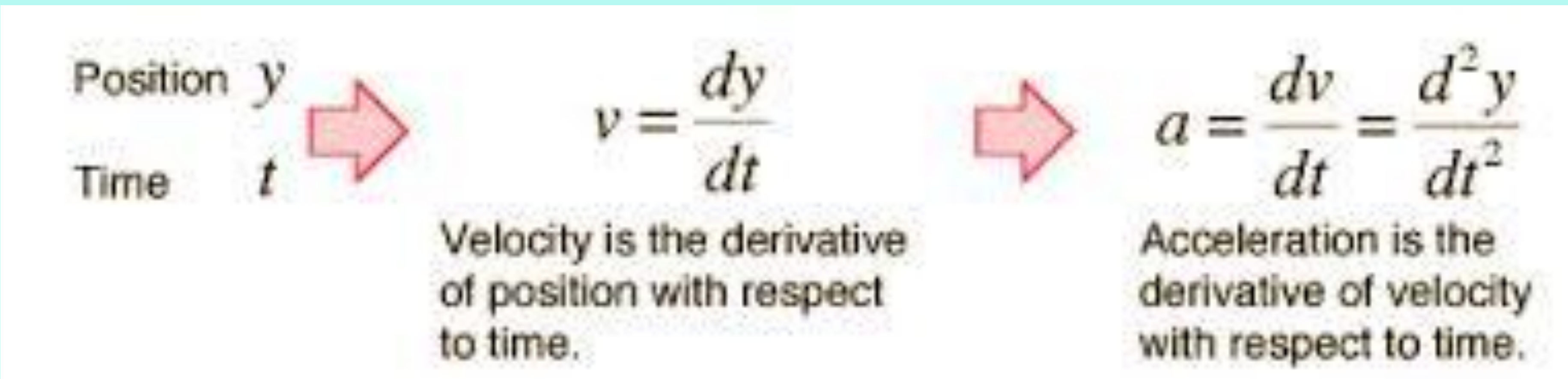
Velocity: the rate of motion in a specific **direction**
(also known as **Speed)**

Acceleration: the rate at which velocity (speed) is changing.

→ If an object is moving with a constant velocity, then its acceleration is zero, since the velocity never changes.

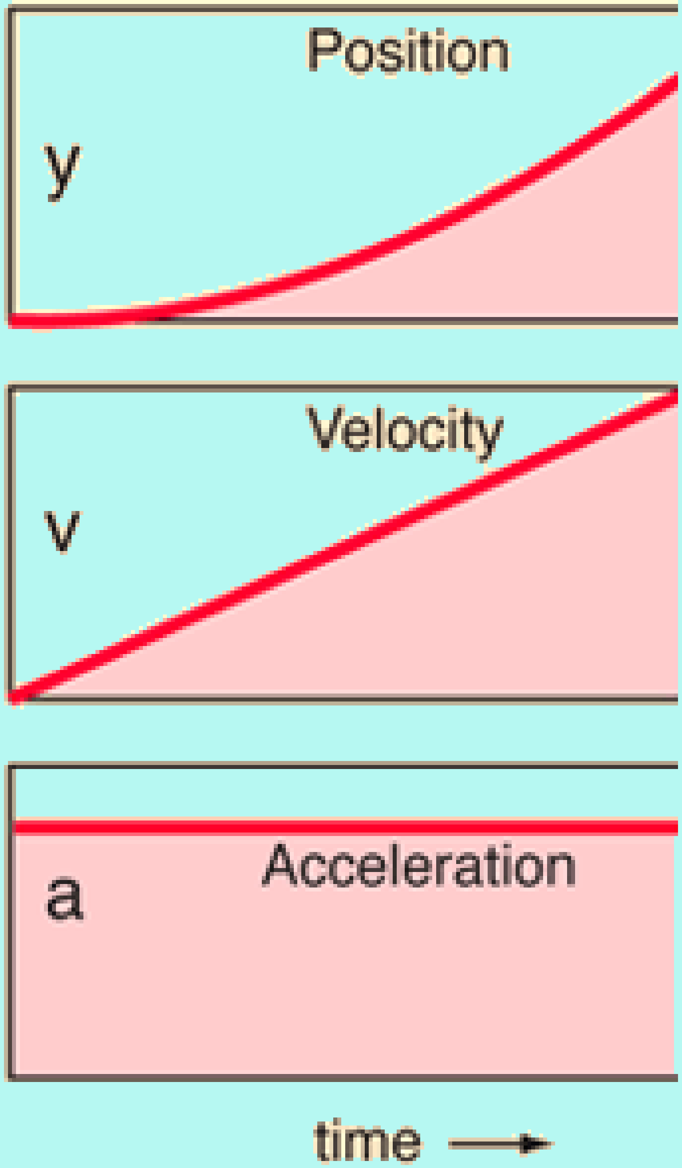
RECALL

- ❖Position vs Time graph (original function)
- ❖Velocity vs Time graph (first derivative)
- ❖Acceleration vs Time graph (second derivative)



position | *velocity* | *acceleration*

$= s$ | $v = s'$ | $a = v' = s''$



Motion relationships in one dimension.

$$y = y_0 + v_0 t + \frac{1}{2} a t^2$$

Derivative of position is velocity

$$v = \frac{dy}{dt}$$

$$v = v_0 + at$$

Derivitive of velocity is acceleration

$$a = \frac{dv}{dt} = a$$

position

$$= s$$

velocity

$$v = s'$$

acceleration

$$a = v' = s''$$

APPLICATIONS

- Maximums and Minimums – find in calculator

- **Instantaneous Velocity** – Use the velocity function (**first derivative**) of the position equation and find the value at the **specific x**.

- **Acceleration** – Use the **second derivative** of the position …the derivative of the velocity function.

- Read the problems **carefully!** Highlight what they're asking for AND what they're giving you!

FORMULA…

- The position of a free falling object (neglecting air resistance) under the influence of gravity can be given by the equation:

$$s(t) = \frac{1}{2}gt^2 + v_0 t + s_0$$

s_0 = initial height

v_0 = initial velocity (or speed for our purposes)

g = accelertation due to gravity (-32ft/sec^2 or -9.8m/sec^2)

EXAMPLE 1

- If a billiard ball is dropped from a height of 100ft, its height **s** at time **t** is given by the position function $\mathbf{s = -16t^2 + 100}$, where **s** is measured in feet and **t** is measured in seconds.

- Find the velocity at t = 1 and t = 2.

EXAMPLE 1 ANSWERS

- If a billiard ball is dropped from a height of 100ft, its height **s** at time **t** is given by the position function **$s = -16t^2 + 100$**, where **s** is measured in feet and **t** is measured in seconds.

- Find the velocity at t = 1 and t = 2.

$$s'(t) = -32t$$

$$s'(1) = -32 \; ft/\sec$$

$$s'(2) = -32(2) = -64 \; ft/\sec$$

EXAMPLE 2

❖At time t = 0, a diver jumps down from a platform diving board that is 50 feet above the water. The position of the diver is given by

$s(t) = -16t^2 + 18t + 50$, where **s** is the measured in feet and **t** is measured in seconds.

a. When does the diver hit the water?

b. What is the velocity at impact?

EXAMPLE 2 ANSWERS

❖At time t = 0, a diver jumps down from a platform diving board that is 50 feet above the water. The position of the diver is given by

$s(t) = -16t^2 + 18t + 50$, where **s** is the measured in feet and **t** is measured in seconds.

a. When does the diver hit the water? After 2.418 seconds

b. What is the velocity at impact? $s'(2.418) = -32(2.418) + 18$

$= -59.36\ ft/\sec$

EXAMPLE 3

❖An arrow is shot straight up in the air with a speed of 900m/s. The arrow is launched with an initial height of 2.4m.

$$s(t) = \frac{1}{2}gt^2 + v_0 t + s_0$$

a. What is the equation relating its height as a function of time?

b. Find the instantaneous velocity at t = 1.5s

EXAMPLE 3 ANSWERS

❖An arrow is shot straight up in the air with a speed of 900m/s. The arrow is launched with an initial height of 2.4m.

$$s(t) = \frac{1}{2}gt^2 + v_0 t + s_0$$

a. What is the equation relating its height as a function of time?

$$h(t) = -4.9t^2 + 900t + 2.4$$

b. Find the instantaneous velocity at t = 1.5s

$$h'(t) = -9.8t + 900$$

$$h'(1.5) = -9.8(1.5) + 900$$
$$= 885.3\ m/\sec$$

EXAMPLE 4

- The position of a particle is given by the equation $s(t)=\frac{2}{3}t^3-12t^2+54t-8,$ where **t** is measured in seconds and **s** in meters.

a. What is the particle's velocity function?

b. What is the particle's acceleration function?

3. When is the particle at rest **(velocity=0)?** ← **write down!**

EXAMPLE 4 ANSWERS

- The position of a particle is given by the equation $s(t) = \frac{2}{3}t^3 - 12t^2 + 54t - 8,$ where **t** is measured in seconds and **s** in meters.

a. What is the particle's velocity function?

$$s'(t) = 2t^2 - 24t + 54$$

b. What is the particle's acceleration function?

$$s''(t) = 4t - 24$$

c. When is the particle at rest **(velocity=0)?** ← **write down!**

$$0 = 2t^2 - 24t + 54, \quad so\ t = 3 \text{ and } 9\,\text{sec}$$

HW if you do not finish!

GET IN GROUPS TO WORK ON THE APPLICATION QUESTIONS PACKET P. 10

OMIT #1 AND #9

#3, 7, AND 11 HAVE NEGATIVE VELOCITY

Additional Hint: If an object is **DROPPED**, then the initial velocity is 0 (#5 and #10)