Day 7

At least one and combining events

Day 7 Warm-up

1. A Quiz Bowl Team must choose who to send to the championships. They must choose a captain and co-captain from 5 members, 2 math reps from their 6 math whizzes, and 3 history reps from their 7 history buffs. In how many ways can they choose the team of reps?

2. On a fair die, what is the probability of rolling a multiple of 3 or a number greater than 4? Are these events mutually inclusive or exclusive?

3. A study of 3 children families is conducted looking at the combinations of boys and girls.

   a) Write the sample space S for this experiment.

   b) List the Event A that there are 2 girls and 1 boy.

4. Find the following, given S = {4, 5, 6, 7, 8, 9, 10}, A = {4, 7, 8}, and B = {4, 6, 8, 10}

   a) $P(A \cap B)$ b) P(A∪B) c) $B \cap A^c$ d) P(A∪B)^c

5. Simplify f(x – 5) – 17 given f(x) = x^2 + 13

Warm-up Answers

1. **The Quiz Bowl Team must choose who to send to the championships. They must choose a captain and co-captain from 5 members, 2 math reps from their 6 math whizzes, and 3 history reps from their 7 history buffs. In how many ways can they choose the team of reps?**

$${}_5P_2 \bullet {}_6C_2 \bullet {}_7C_3 = 20 \bullet 15 \bullet 35 = \mathbf{10500}$$

2. **On a fair die, what is the probability of rolling a multiple of 3 or a number greater than 4? Are these events mutually exclusive?**

$2/6 + 2/6 - 1/6 = 3/6 = \frac{1}{2}$

Mutually Inclusive because the events have an **In**tersection

Warm-up Answers

3. **A study of 3 children families is conducted looking at the combinations of boys and girls.**

   a) **Write the sample space S for this experiment.**

   S = {BBB, BGB, BBG, BGG, GGG, GBG, GBB, GGB}

   b) **List the Event A that there are 2 girls and 1 boy.**

   A = {GGB}, {GBG}, {BGG}

4. **Find the following, given S = {4, 5, 6, 7, 8, 9, 10}, A = {4, 7, 8}, and B = {4, 6, 8, 10}**

   a) $P(A \cap B)$ =2/7

   b) P(A∪B) =5/7

   c) $B \cap A^c$ ={6, 10}

   d) P(A∪B)^c =2/7

Warm-up Answers

5. Simplify $f(x-5)-17$ given $f(x) = x^2 + 13$

$x^2 - 10x + 21$

Homework Questions?!

Notes Day 7

At least one and combining events

Notes: Probabilities for Combining Events

Union/OR events - Adding

Intersection/AND events – Multiply

IF you have ***multiple events***

Two cards are selected at random from a well-shuffled deck of 52 playing cards. What is the probability that…

- They are both aces?

$$P(Ace, Ace) = \frac{4}{52} \bullet \frac{3}{51} = \frac{1}{221} = .005$$

- Neither of them is an ace?

$$P(NotAce, NotAce) = \frac{48}{52} \bullet \frac{47}{51} = \frac{188}{221} = .851$$

- The first card is an ace and the second is not?

$$P(Ace, NotAce) = \frac{4}{52} \bullet \frac{48}{51} = \frac{192}{2652} = .0724$$

Practice..

An AP Statistics class is made up of 16 seniors, 14 juniors, 3 sophomores, and 1 freshman. What is the probability that Ms. Jarvis will randomly select…

- A. a senior? $\frac{16}{34} = .4706$
- B. a senior and then a junior? $\frac{16}{34} \bullet \frac{14}{33} = .1996$
- C. one upperclassman and then one underclassman? $\frac{30}{34} \bullet \frac{4}{33} = .10695$

Combining Independent Events

Section 7.4 and 7.5

- **<u>Independent Events</u>** – when the outcome of one event **does not** affect the outcome of the other.
  - **Ex:** Flipping a coin or rolling a die or people's choices

- **<u>Test for Independent Events</u>** – events **A** and **B** are independent events IFF (if and only if)

$$P(A \cap B) = P(A) * P(B)$$

- ***Note: This generalizes to more than two independent events***

Note: IFF means that the equation goes both ways (forwards & backwards)

- EX. If a die is rolled twice, show that rolling a 5 on the 1st roll and rolling a 4 on the 2nd roll are independent events.

$$P(5 \text{ on 1st die}) = \frac{1}{6}$$

$$P(4 \text{ on } 2nd \text{ die}) = \frac{1}{6}$$

$$P(5 \cap 4) = \frac{1}{36}$$

$$S\left\{\begin{array}{l}(1,1)\ (2,1)\ (3,1)\ (4,1)(5,1)\ (6,1)\\(1,2)(2,2)(3,2)(4,2)(5,2)(6,2)\\(1,3)(2,3)(3,3)(4,3)\ (5,3)(6,3)\\(1,4)(2,4)(3,4)(4,4)(5,4)(6,4)\\(1,5)(2,5)(3,5)(4,5)\ \ (5,5)(6,5)\\(1,6)(2,6)(3,6)(4,6)(5,6)(6,6)\end{array}\right\}$$

Yes they are independent
because $P(A \cap B) = P(A) * P(B)$

You Try! Find the probability of...

1) Rolling a even number on a six-sided die and then drawing a queen from a standard deck. $\frac{3}{6} * \frac{4}{52} = 0.038$

2) "One in eight win," the lottery says. What is the probability that you and your friend win? $\frac{1}{8} \bullet \frac{1}{8} = \frac{1}{64}$

3) Groundhog Punxsutawney Phil has seen his shadow 100 out of 116 times. What is the probability he will see his shadow, then not see it, then see it again over the next 3 independent years? $\frac{100}{116} \bullet \frac{16}{116} \bullet \frac{100}{116} = 0.1025$

Notes: Probability of At Least One

Sometimes it really only matters if something occurs once. Examples include floods, hurricanes, natural disasters.

Suppose the probability of an event A occurring in one trial is P(A). If all trials are independent, the probability that event A occurs at least once in ***n*** trials is the same as 1 minus the probability of the event never occurring.

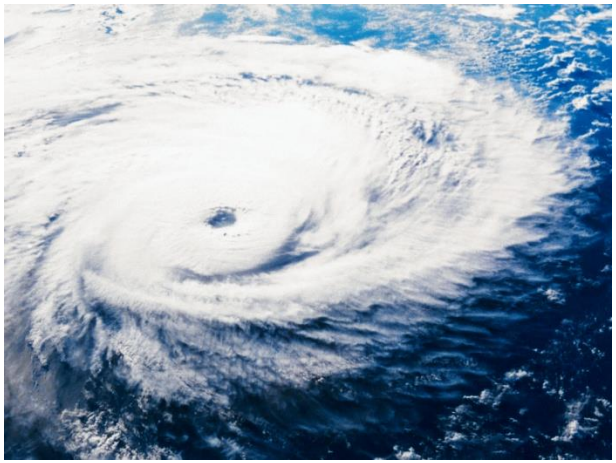
Therefore, the probability is:

P(at least 1) = 1 - P(none)

Example: Probability of At Least One

What is the probability that a region will experience at least one hurricane during the next 50 years if the probability of a hurricane is 0.07 per year?

$$P(at\ least\ one\ hurricane) = 1 - P(no\ Hurricanes)$$



Example: Prob. of At Least One ANSWER

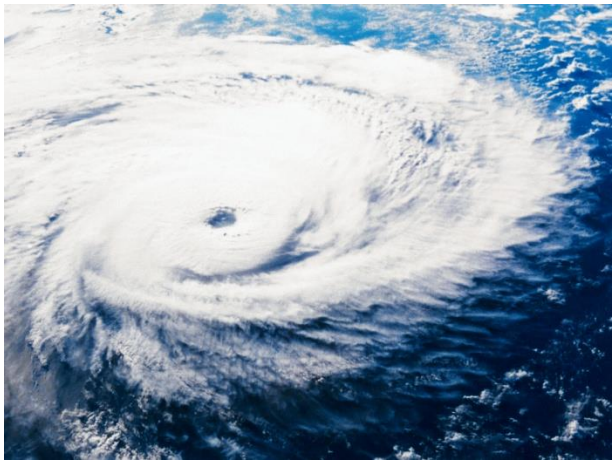
What is the probability that a region will experience at least one hurricane during the next 50 years if the probability of a hurricane is 0.07 per year?

$$P(\textit{at least one hurricane}) = 1 - P(\textit{no Hurricanes})$$

$$= 1 - 0.93^{50}$$

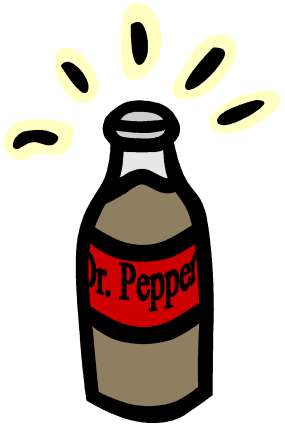
$$= 1 - 0.02656$$

$$= 0.9734$$



You Try!

For a sales promotion, the manufacturer places winning symbols under the caps of 10% of all Dr. Pepper bottles. You buy a six-pack. What is the probability that you win something?



You Try! ANSWERS

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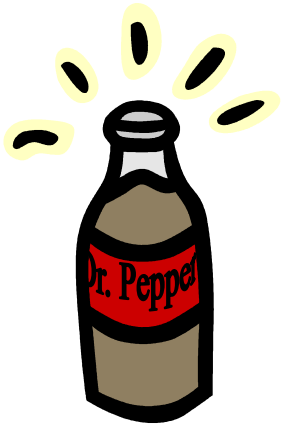

P(at least one winning symbol) =

$1 - P(\text{no winning symbols})$

$1 - (0.9)^6$

$= 0.4686$

Combining it all!!!

Unions and Intersections!

More Practice

A certain brand of light bulbs are defective five percent of the time. You randomly pick a package of two such bulbs off the shelf of a store. What is the probability that exactly one bulb is defective?

Let D=Defective

$$P(\text{exactly one}) = P(D \ \& \ D^C) \text{ or } P(D^C \ \& \ D)$$

$$= (0.05)(0.95) + (0.95)(0.05)$$

$$= 0.095$$

A certain brand of light bulbs are defective five percent of the time. You randomly pick a package of two such bulbs off the shelf of a store. What is the probability that **at least one** bulb is defective?

$$P(\text{at least one}) = P(D \,\&\, D^C) \text{ or } P(D^C \,\&\, D) \text{ or } (D \,\&\, D)$$

$$= (.05)(.95) + (.95)(.05) + (.05)(.05)$$

$$= 0.0975$$

Or your can do:

$P(\text{at least one}) = 1 - P(\text{None}) = 1 - (0.95)(0.95)$

$= 0.0975$

You Try!

- The GHHS PTSA has a goodie bag of gift cards for outstanding teachers to win. There are 14 Target gift cards and 18 Dunkin Donuts gift cards. If each department gets to select two goodies, what is the probability that the math department

  a) will get at least 1 DD card?

  b) will get exactly 1 DD card?

You Try! ANSWERS

- The GHHS PTSA has a goodie bag of gift cards for outstanding teachers to win. There are 14 Target gift cards and 18 Dunkin Donuts gift cards. If each department gets to select two goodies, what is the probability that the math department will

  a) get at least 1 DD card?

$$P(\geq 1\,DD) = 1 - P(none)$$

$$1 - \left(\frac{14}{32} \bullet \frac{13}{31}\right)$$

$$1 - \frac{182}{992}$$

$$= 0.8165$$

$$P(DD, DD) + P(DD, T) + P(T, DD)$$

$$\frac{18}{32} \bullet \frac{17}{31} + \frac{18}{32} \bullet \frac{14}{31} + \frac{14}{32} \bullet \frac{18}{31}$$

$$0.308 + 0.254 + 0.254$$

$$= 0.8165$$

You Try! ANSWERS

- The GHHS PTSA has a goodie bag of gift cards for outstanding teachers to win. There are 14 Target gift cards and 18 Dunkin Donuts gift cards. If each department gets to select two goodies, what is the probability that the math department will

b) get exactly 1 DD card?

$$P(DD,T) + P(T,DD)$$

$$\frac{18}{32}\cdot\frac{14}{31} + \frac{14}{32}\cdot\frac{18}{31}$$

$$0.254 + 0.254$$

$$= 0.508$$

You Try! Solving problems from a table…

	Comedy(C)	Drama(D)	Horror(H)	Sci-Fi (S)	Totals
Oscar (O)	28	53	17	22	120
Golden Globe(G)	32	43	9	21	105
Total	60	96	26	43	225

The table above gives the number of Best Picture wins by movie genre. Using the single letters as set names, answer the following questions.

1. $P(O \cap C)$
2. $(G \cup H)^c$
3. $(C \cup D) \cap O$
4. $P(G \cap S) \cup P(O \cap S)$

You Try! ANSWERS Prob. from a table...

	Comedy(C)	Drama(D)	Horror(H)	Sci-Fi (S)	Totals
Oscar (O)	28	53	17	22	120
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1. $P(O \cap C)$
2. $(G \cup H)^c$
3. $(C \cup D) \cap O$
4. $P(G \cap S) \cup P(O \cap S)$

1. $\frac{28}{225} = 0.1244$
2. $225 - 122 = 103$
3. $28 + 53 = 81$
4. $\frac{21}{225} + \frac{22}{225} = \frac{43}{225} = 0.191$

Practice from yesterday's material!! Let **E** and **F** be two *mutually exclusive* events and suppose P(E) = 0.25 and P(F) = 0.65. Compute:

1) $P(E \cup F)$

2) $P(E \cap F)$

3) $P(E^c)$

4) $P(E^c \cup F^c)$

5) $P(E^c \cap F^c)$

ANSWERS to Practice from yesterday's material!! Let **E** and **F** be two *mutually exclusive* events and suppose P(E) = 0.25 and P(F) = 0.65. Compute:

1) $P(E \cup F)$ = 0.9

2) $P(E \cap F)$ = 0 (mutually exclusive)

3) $P(E^c)$ = 0.75

4) $P(E^c \cup F^c)$ (DeMorgan's Law)

$$= P[(E \cap F)^c]$$

$$= 1 - P(E \cap F) = 1 - 0 = 1$$

OR Remember, $P(E^c \cup F^c) =$

$P(E^c) + P(F^c) - P(E^c \cap F^c)$

$P(E^c) = 1 - P(E) = 1 - 0.25 = 0.75$

$P(F^c) = 1 - P(F) = 1 - 0.65 = 0.35$

$P(E^c \cap F^c) = 0.1$

$= 0.75 + 0.35 - 0.1 = 1$

5) $P(E^c \cap F^c)$ (DeMorgan's Law)

$$= P[(E \cup F)^c]$$

$$= 1 - P(E \cup F)$$

$$= 1 - 0.9$$

$$= 0.1$$

Homework Day 7

Packet p. 11-12

Work on PreAssessment Corrections

AND Work on Quiz 1 Corrections