

**DAY 7**

**LIMITS PART 2 - WHAT ABOUT LIMITS  
IN THE MIDDLE OF THE FUNCTION?**

# WARM-UP DAY 7

**TIP: You may want to print “Day 7 Graphs for Notes” - in Power Point area of website - to help with notetaking :)**

Find the requested information.

**HINT: find extrema 1<sup>st</sup>**

• 1)  $g(x) = \frac{x^4}{x^4 - 1}$

Increasing:

Decreasing:

Domain:

Range:

Express End Behaviors using proper Limit Notation

• 2)  $f(x) = x^3 - 2x^2 - 3x$

Increasing:

Decreasing:

Domain:

Range:

Express End Behaviors using proper Limit Notation

# WARM-UP DAY 7

**TIP: You may want to print "Day 7 Graphs for Notes" - in Power Point area of website - to help with notetaking :)**

Find the requested information.

**HINT: find extrema 1<sup>st</sup>**

• 1)  $g(x) = \frac{x^4}{x^4 - 1}$

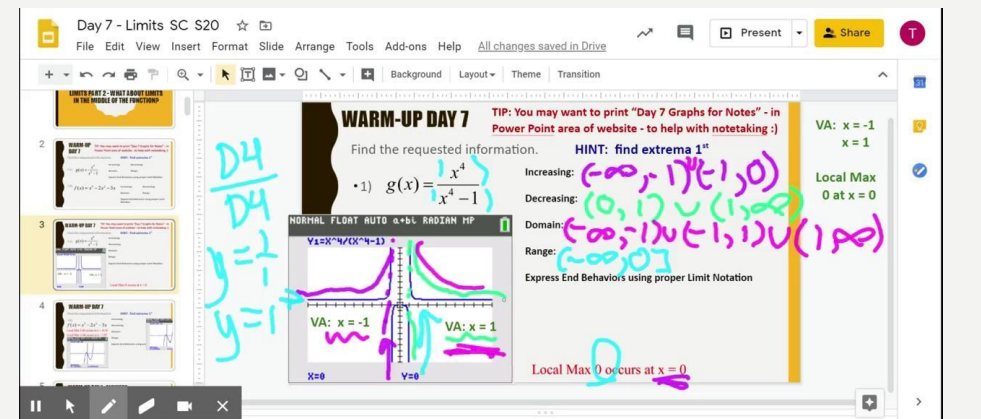
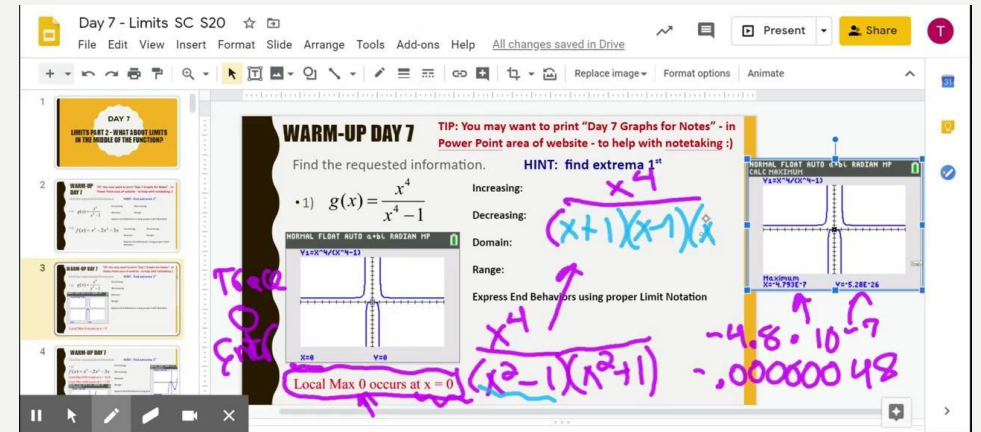
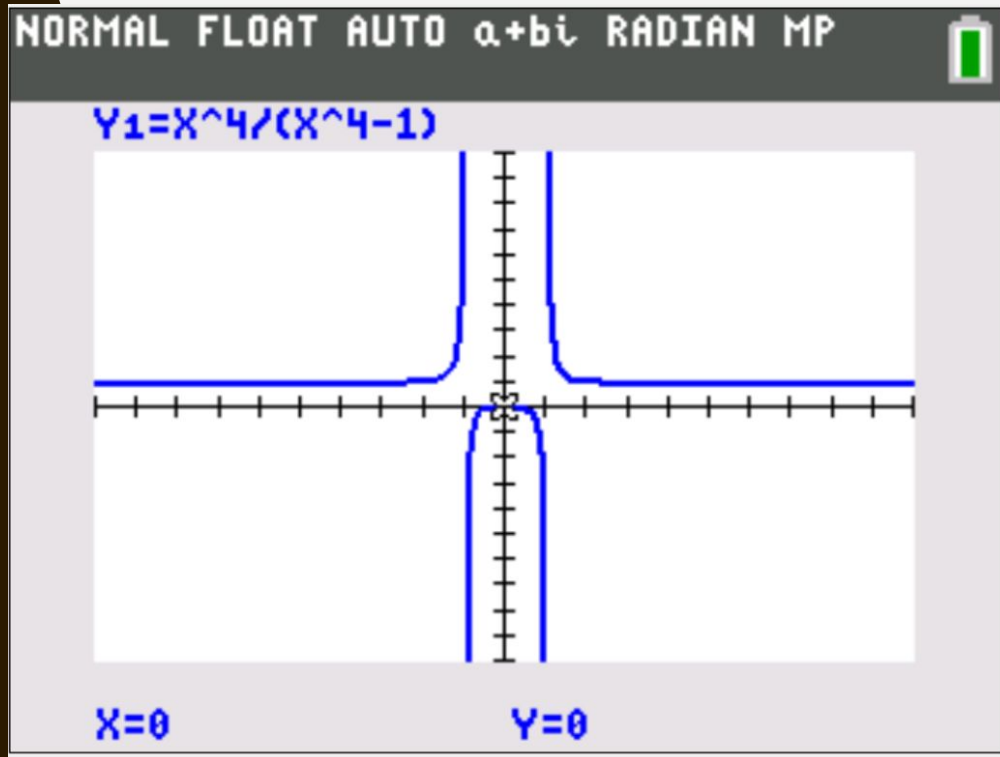
Increasing:

Decreasing:

Domain:

Range:

Express End Behaviors using proper Limit Notation



# WARM-UP DAY 7

Find the requested information.

**HINT: find extrema 1<sup>st</sup>**

• 2)

$$f(x) = x^3 - 2x^2 - 3x$$

Local Max 0.88 occurs at  $x = -0.54$

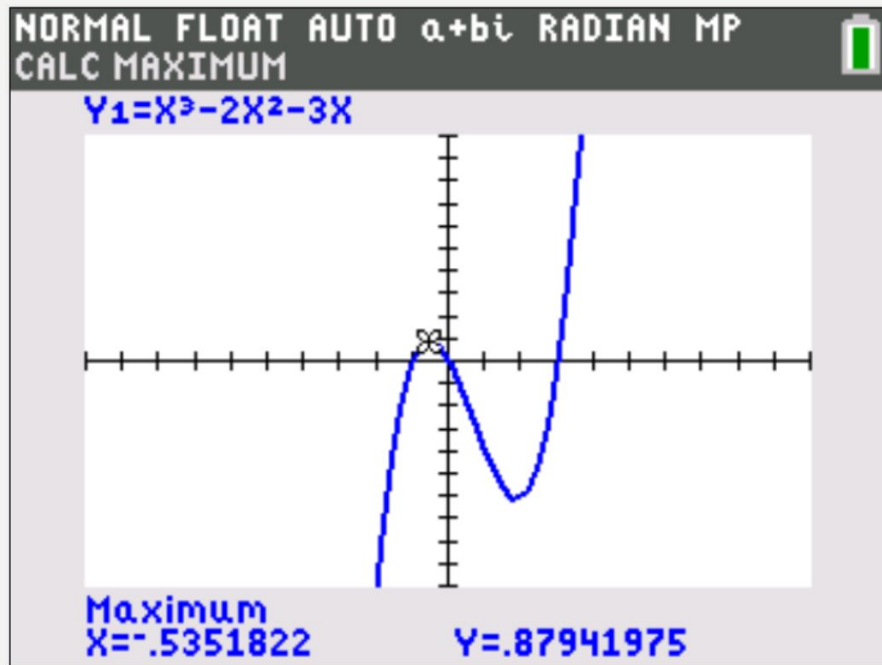
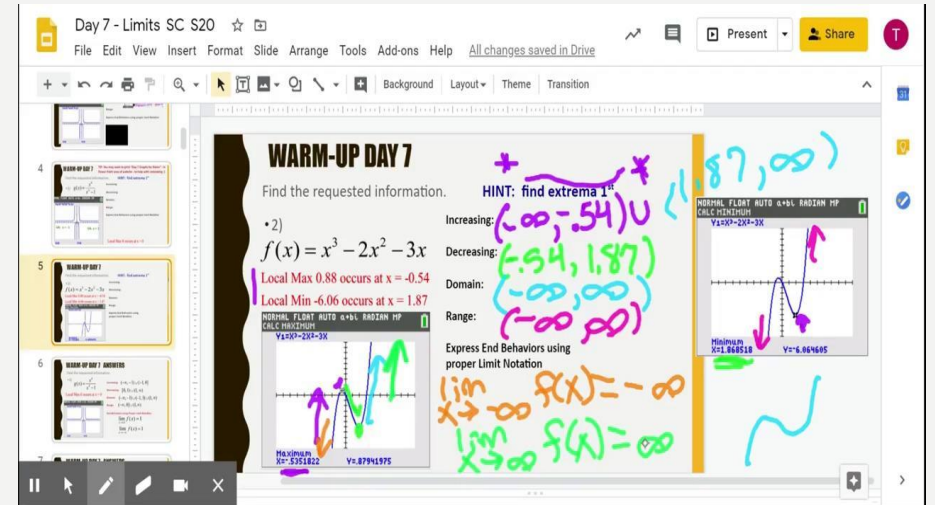
Local Min -6.06 occurs at  $x = 1.87$

Increasing:

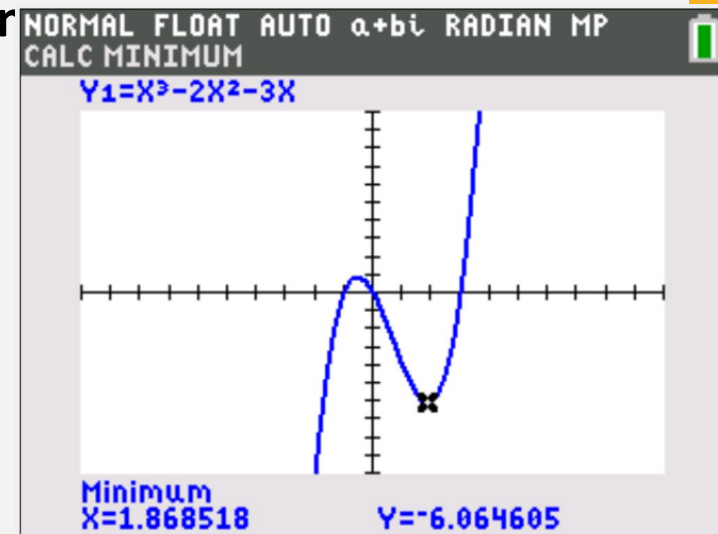
Decreasing:

Domain:

Range:



Express End Behaviors using proper Limit Notation

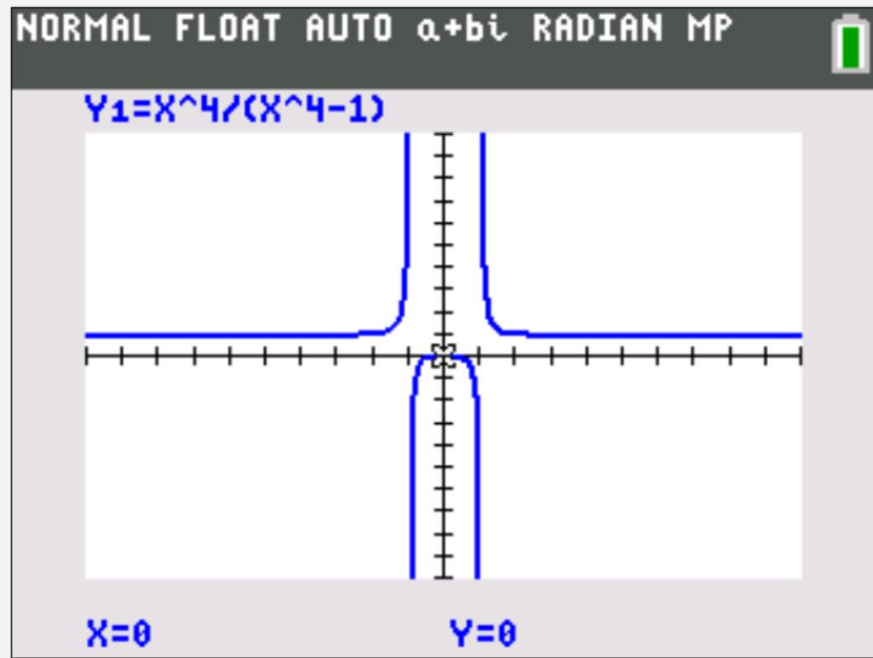


# WARM-UP DAY 7 ANSWERS

Find the requested information.

• 1) 
$$g(x) = \frac{x^4}{x^4 - 1}$$

Local Max 0 occurs at  $x = 0$



**Increasing:**  $(-\infty, -1) \cup (-1, 0)$

**Decreasing:**  $(0, 1) \cup (1, \infty)$

**Domain:**  $(-\infty, -1) \cup (-1, 1) \cup (1, \infty)$

**Range:**  $(-\infty, 0] \cup (1, \infty)$

**End Behaviors using Proper Limit Notation:**

$$\lim_{x \rightarrow \infty} f(x) = 1$$

$$\lim_{x \rightarrow -\infty} f(x) = 1$$

# WARM-UP DAY 7 ANSWERS

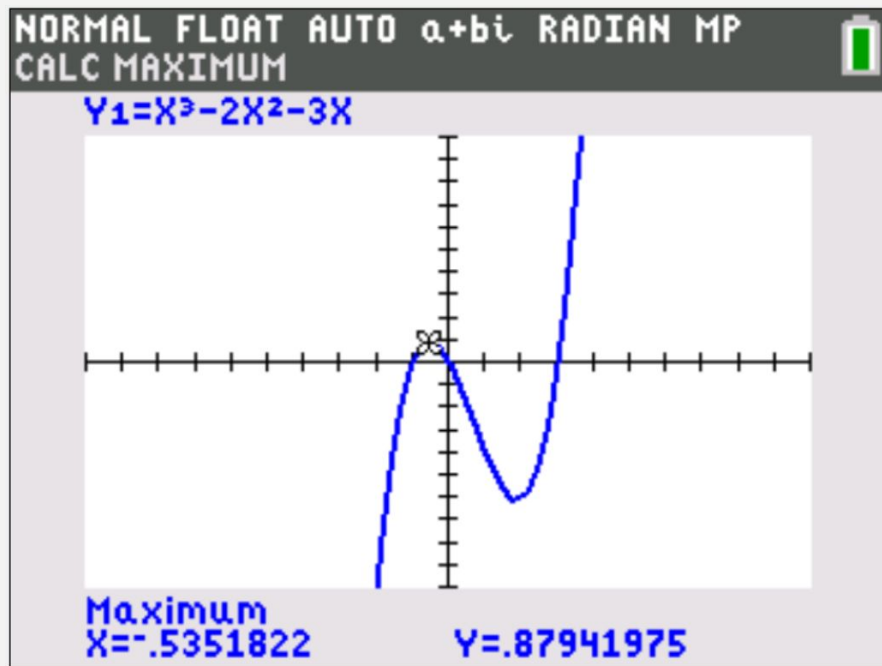
Find the requested information

• 2)

$$f(x) = x^3 - x^2 - 2x$$

Local Max 0.88 occurs at  $x = -0.54$

Local Min -6.06 occurs at  $x = 1.87$



**Increasing:**  $(-\infty, -0.54) \cup (1.87, \infty)$

**Decreasing:**  $(-0.54, 1.87)$

**Domain:**  $(-\infty, \infty)$

**Range:**  $(-\infty, \infty)$

**End Behavior using Proper Limit Notation:**

$$\lim_{x \rightarrow \infty} f(x) = \infty$$

$$\lim_{x \rightarrow -\infty} f(x) = -\infty$$



**HW DAY 7:**

PACKET P.9 AND HANDOUT  
INTRO TO LIMITS (ON  
WEBSITE)

# **NOTES DAY 7: LIMITS PART 2**

**LIMITS TO CERTAIN VALUES & LIMITS AT  
LEFT-HAND AND RIGHT-HAND SIDES**

**USE “Day 7 Graphs for Notes” handout in  
Power Point area of Website for help  
taking notes today 😊**



# REMEMBER THE DEFINITION OF A LIMIT

- If  $f(x)$  becomes arbitrarily close to a unique number  $L$  as  $x$  approaches  $c$  from either side, the limit of  $f(x)$  as  $x$  approaches  $c$  is  $L$ .
- $L$  is a  $y$ -value!  $c$  is an  $x$ -value!

$$\lim_{x \rightarrow c} f(x) = L$$

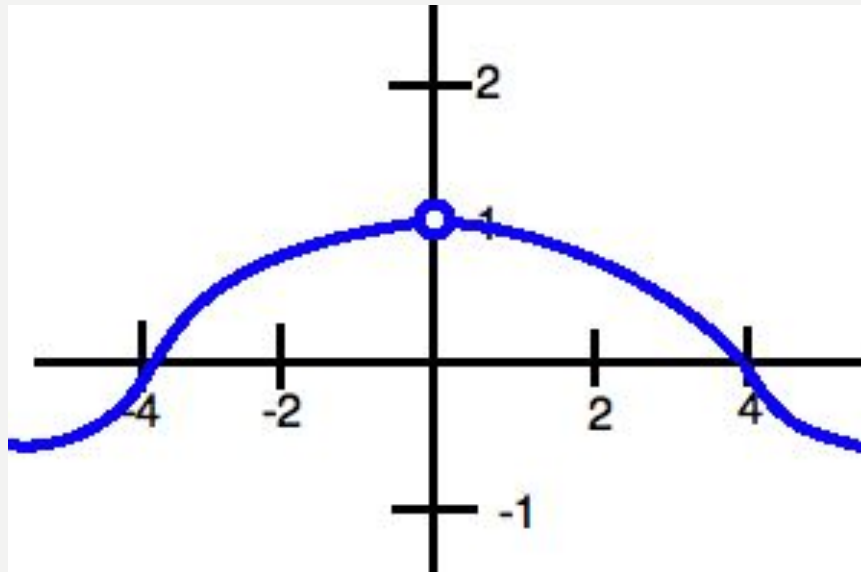
# TAKE A LOOK AT THIS GRAPH...

- Remember, a **limit is** asking what **y-value** the function is approaching as  $x$  gets close to some value.
- Using the definition of a limit, I could say:

$$\lim_{x \rightarrow 0} f(x) =$$

$$\lim_{x \rightarrow 4} f(x) =$$

$$\lim_{x \rightarrow -4} f(x) =$$



**TAKE A LOOK AT THIS GRAPH..**

- Remember, a **limit is** asking what **y-value** the function is approaching as  $x$  gets close to some value.
- Using the definition of a limit, I could say: *what y were we approaching?*

$\lim_{x \rightarrow 0} f(x) = 1$   
 $\lim_{x \rightarrow 4} f(x) = 0$   
 $\lim_{x \rightarrow -4} f(x) = 0$

*hole at (0, 1)*

**NOTICE:**  
The function itself **DOES NOT** exist at  $x = 0$ , but the limit does!

**NOTICE:**  
The function itself **DOES NOT** exist at  $x = 0$ , but the limit does!

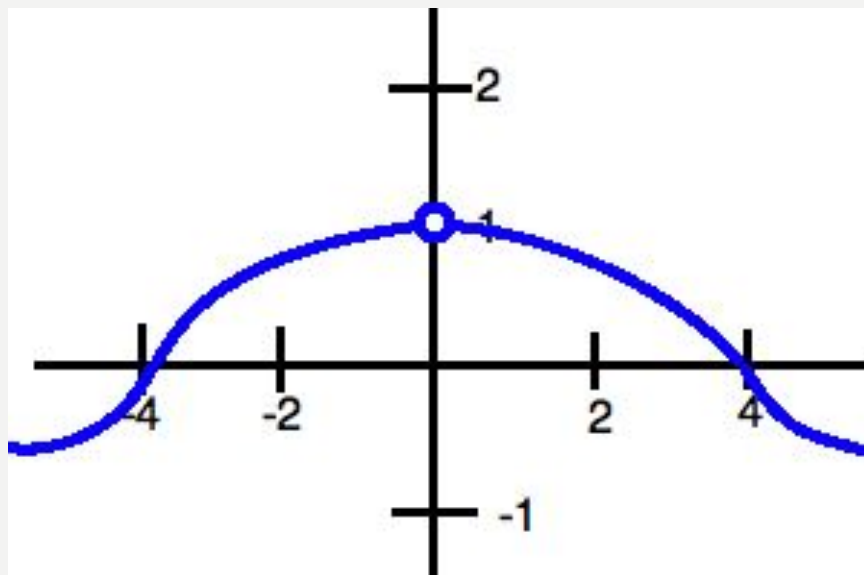
# TAKE A LOOK AT THIS GRAPH... ANSWERS

- Remember, a limit is asking what  $y$ -value the function is approaching as  $x$  gets close to some value.
- Using the definition of a limit, I could say:

$$\lim_{x \rightarrow 0} f(x) = 1$$

$$\lim_{x \rightarrow 4} f(x) = 0$$

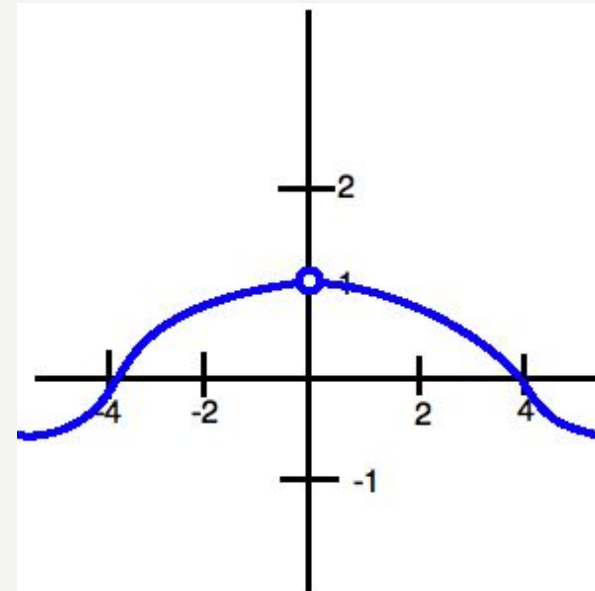
$$\lim_{x \rightarrow -4} f(x) = 0$$



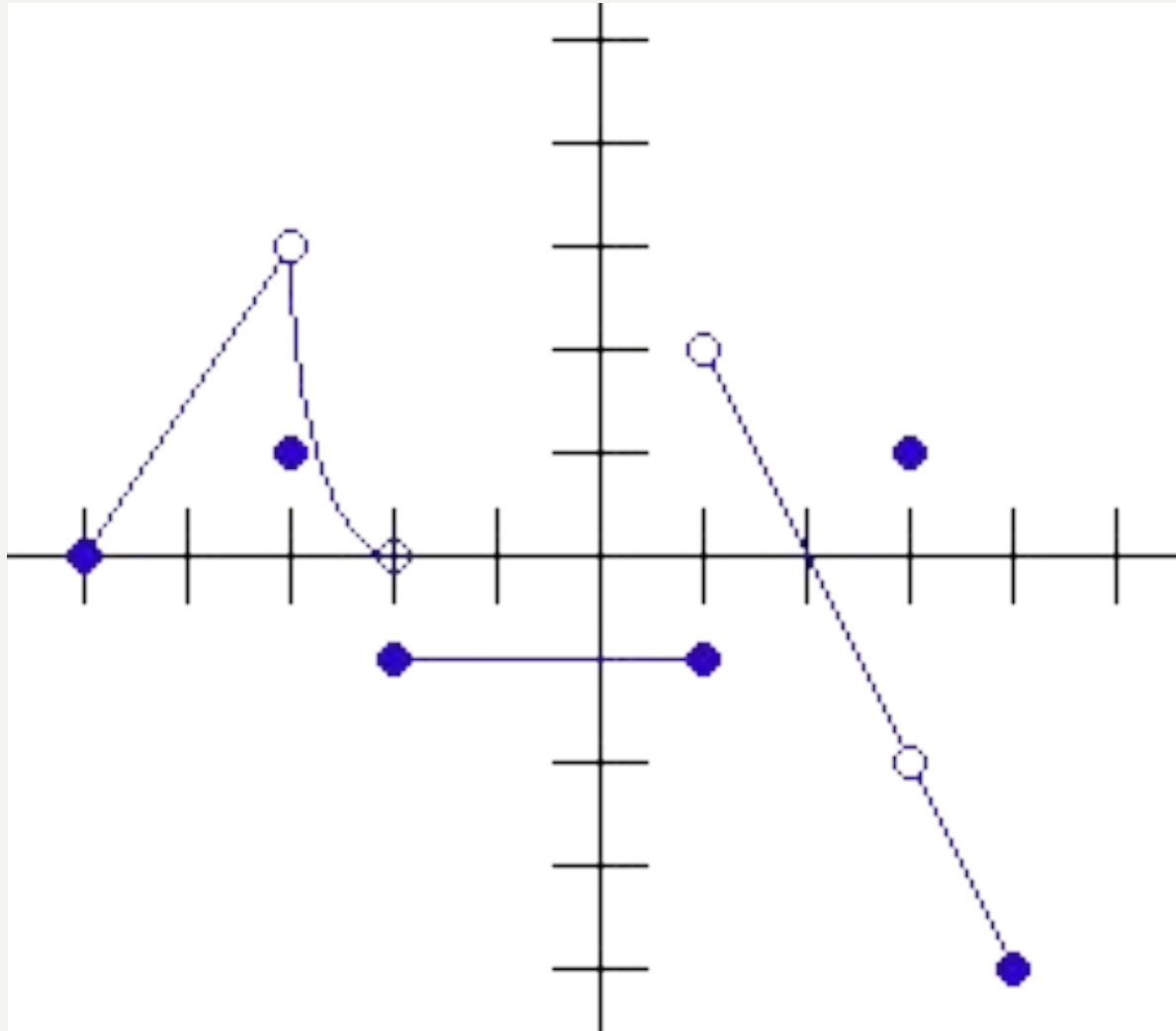
**NOTICE:**  
The function itself  
**DOES NOT** exist  
at  $x = 0$ , but the  
limit does!

# \$0000....

The **existence/non-existence** of  $f(x)$  at  $x = c$  has **NO BEARING** on the existence of the **limit** of  $f(x)$  as  $x$  approaches  $c$ .

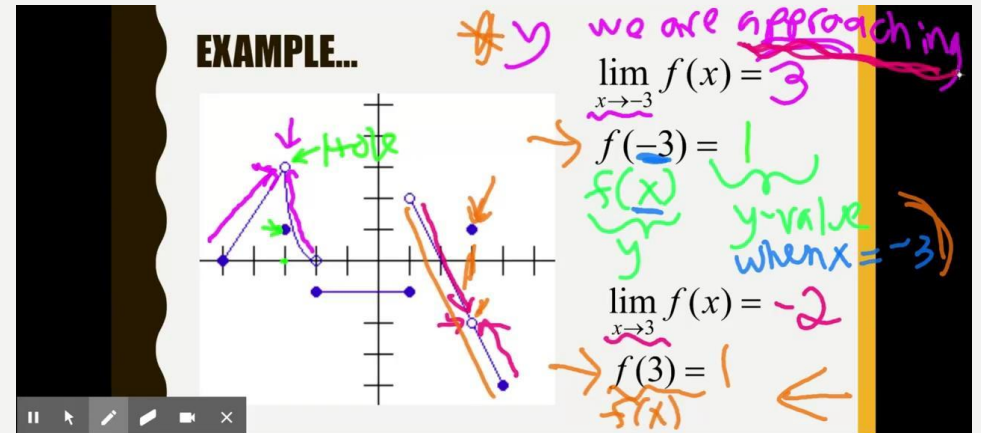


# EXAMPLE...



$$\lim_{x \rightarrow -3} f(x) =$$

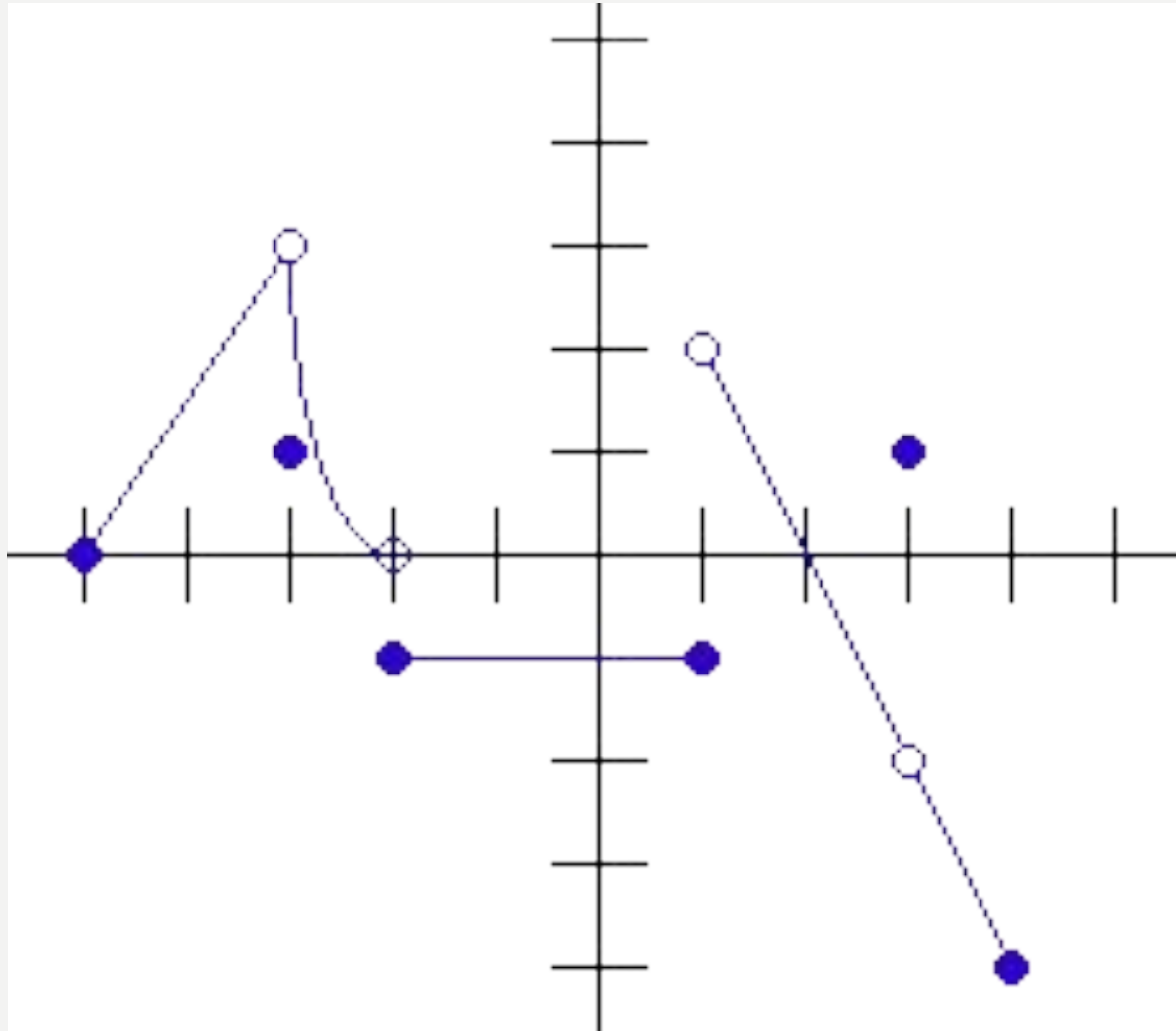
$$f(-3) =$$



$$\lim_{x \rightarrow 3} f(x) =$$

$$f(3) =$$

# EXAMPLE... ANSWERS



$$\lim_{x \rightarrow -3} f(x) = 3$$

$$f(-3) = 1$$

$$\lim_{x \rightarrow 3} f(x) = -2$$

$$f(3) = 1$$

# LIMITS AT "HOLE"

Remember, a hole occurs when there is a factor in the denominator that **can be canceled out** when algebraic steps are taken.

- Ex: Graph it, find the domain, and find value of the limit requested.

$$f(x) = \frac{(x+2)^2}{x+2}$$

*Domain:*

$$\lim_{x \rightarrow -2} f(x) =$$

**LIMITS AT "HOLE"**

• Ex: Graph it, find the domain, and find value of the limit requested.

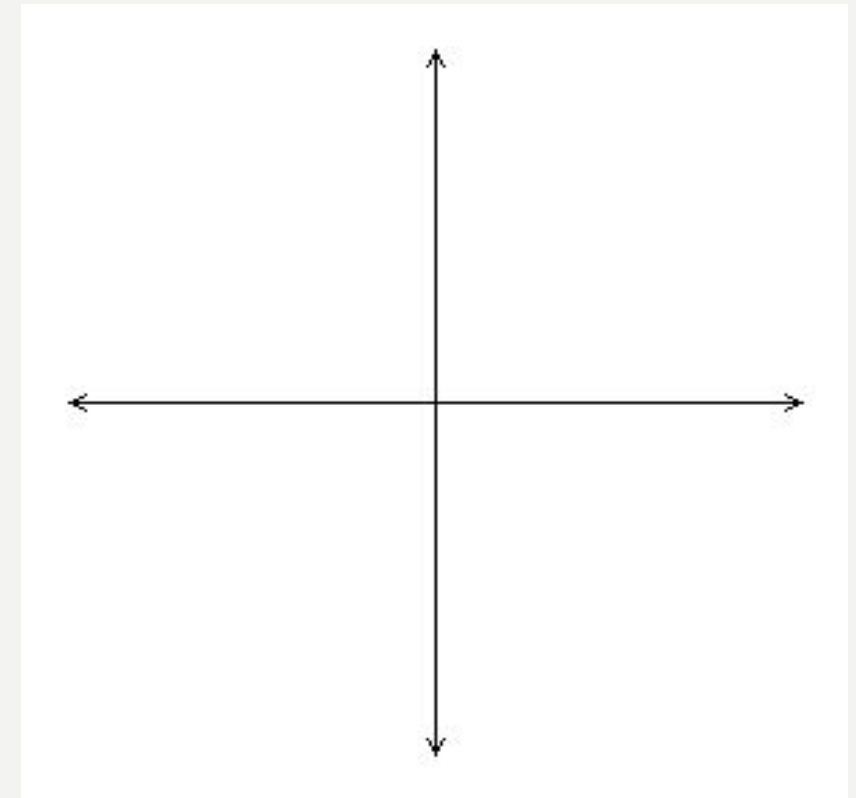
$$f(x) = \frac{(x+2)^2}{x+2} = \frac{(x+2)(x+2)}{x+2}$$

Hole  $(-2, 0)$  No PV

Remember, a hole occurs when there is a factor in the denominator that **can be canceled out** when algebraic steps are taken.

$$x+2=0 \quad y=x+2$$
$$y=-2+2$$
$$y=0$$

limit at a hole will always be the **y-value of the hole!!** ← Write this down!



**\*The limit at a hole will always be the **y-value of the hole!!****

**Write this down!**

# LIMITS AT "HOLE"

## ANSWERS

Ex: Graph it and write the domain.

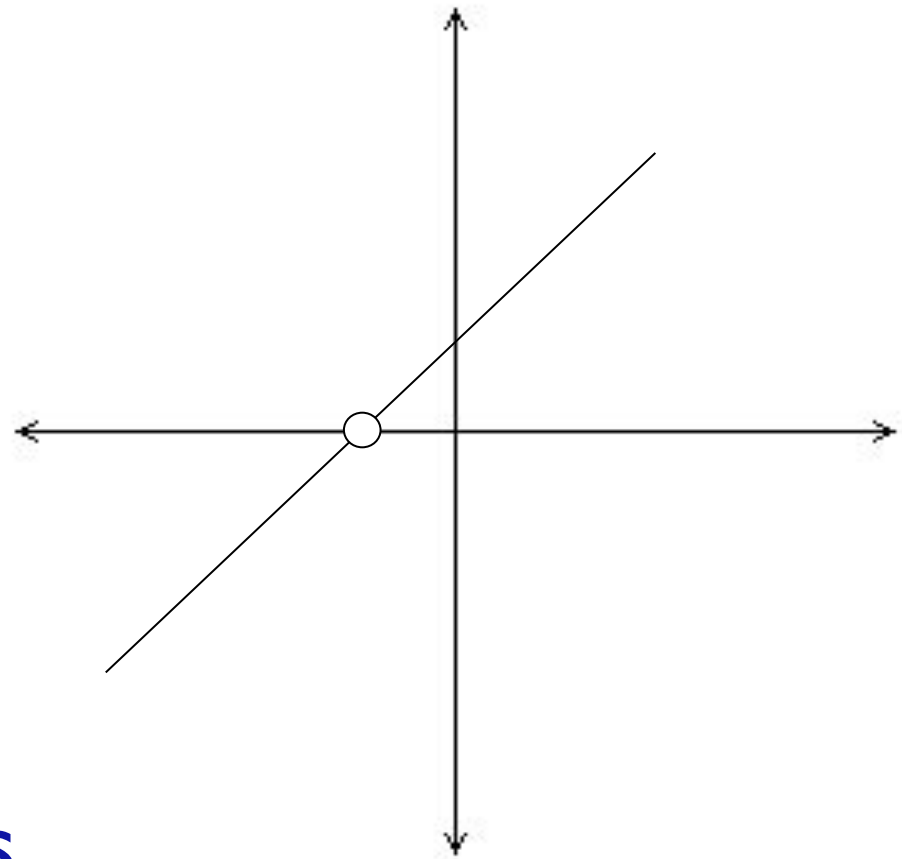
$$f(x) = \frac{(x+2)^2}{x+2}$$

*Domain:*  $(-\infty, -2) \cup (-2, \infty)$

$$\lim_{x \rightarrow -2} f(x) = 0$$

\*The limit at a hole will always be the **y-value of the hole!**

Remember, a hole occurs when there is a factor in the denominator that **can be canceled out** when algebraic steps are taken.

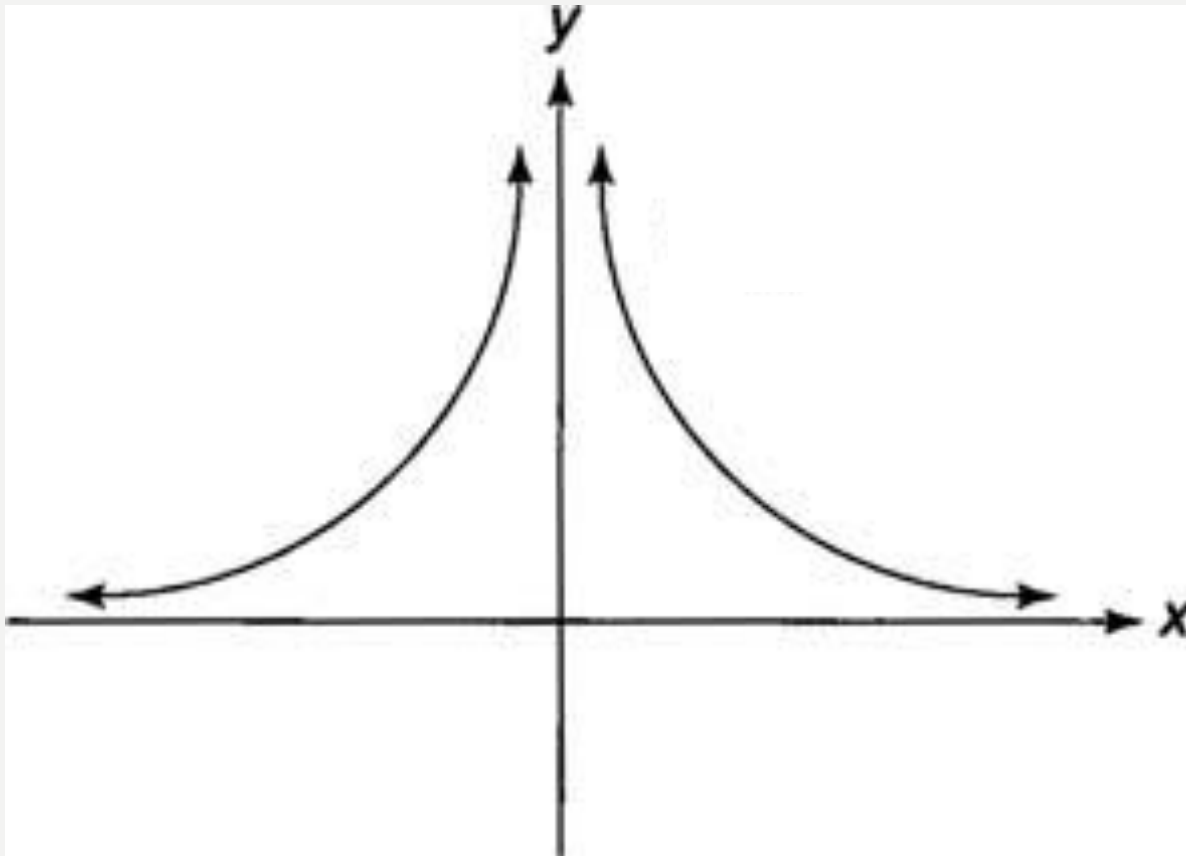


Write this down!

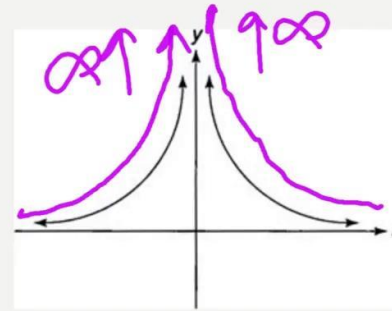


# WHAT ABOUT THIS...

$$\lim_{x \rightarrow 0} f(x) = ?$$



## WHAT ABOUT THIS...

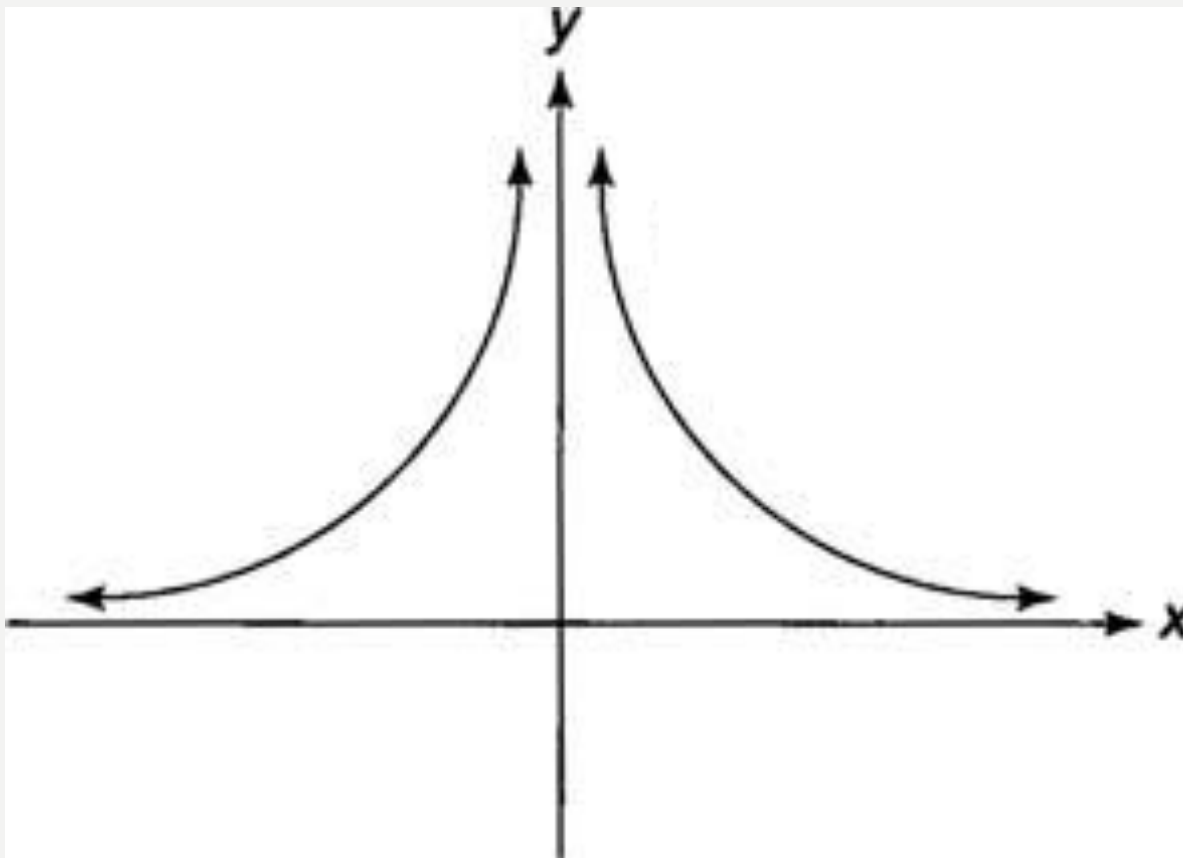


$$\lim_{x \rightarrow 0} f(x) = ?$$

Even though the two curves are on different sides of  $x = 0$ , since **both curves approach the same  $y$ -value**, as  $x$  approaches 0, we have a limit. 😊

Even though the two curves are on different sides of  $x = 0$ , since **both curves approach the same  $y$ -value**, as  $x$  approaches 0, we have a limit. 😊

# WHAT ABOUT THIS... ANSWERS



$$\lim_{x \rightarrow 0} f(x) = ? \quad \infty$$

Even though the two curves are on different sides of  $x = 0$ , since **both curves approach the same  $y$ -value**, as  $x$  approaches 0, we have a limit. 😊

# LIMIT AT VERTICAL ASYMPTOTE

$$f(x) = \frac{x + 3}{x - 2}$$

- Using this example, find the domain and graph it.

*D:*

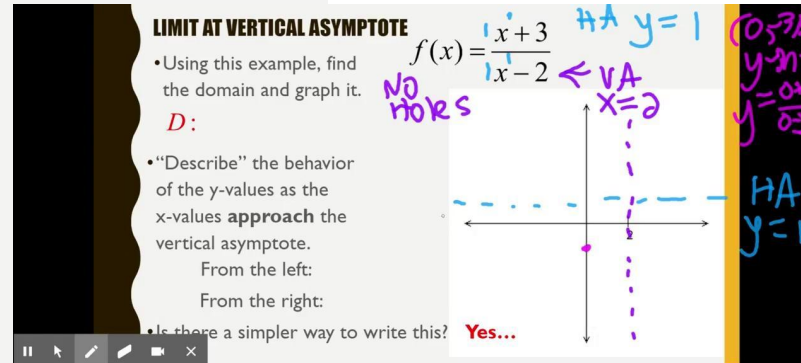
- “Describe” the behavior of the y-values as the x-values **approach** the vertical asymptote.

From the left:

From the right:

- Is there a simpler way to write this?

**Yes...**



# LIMIT AT VERTICAL ASYMPTOTE

## ANSWERS

- Using this example, find the domain and graph it.

$$D: (-\infty, 2) \cup (2, \infty)$$

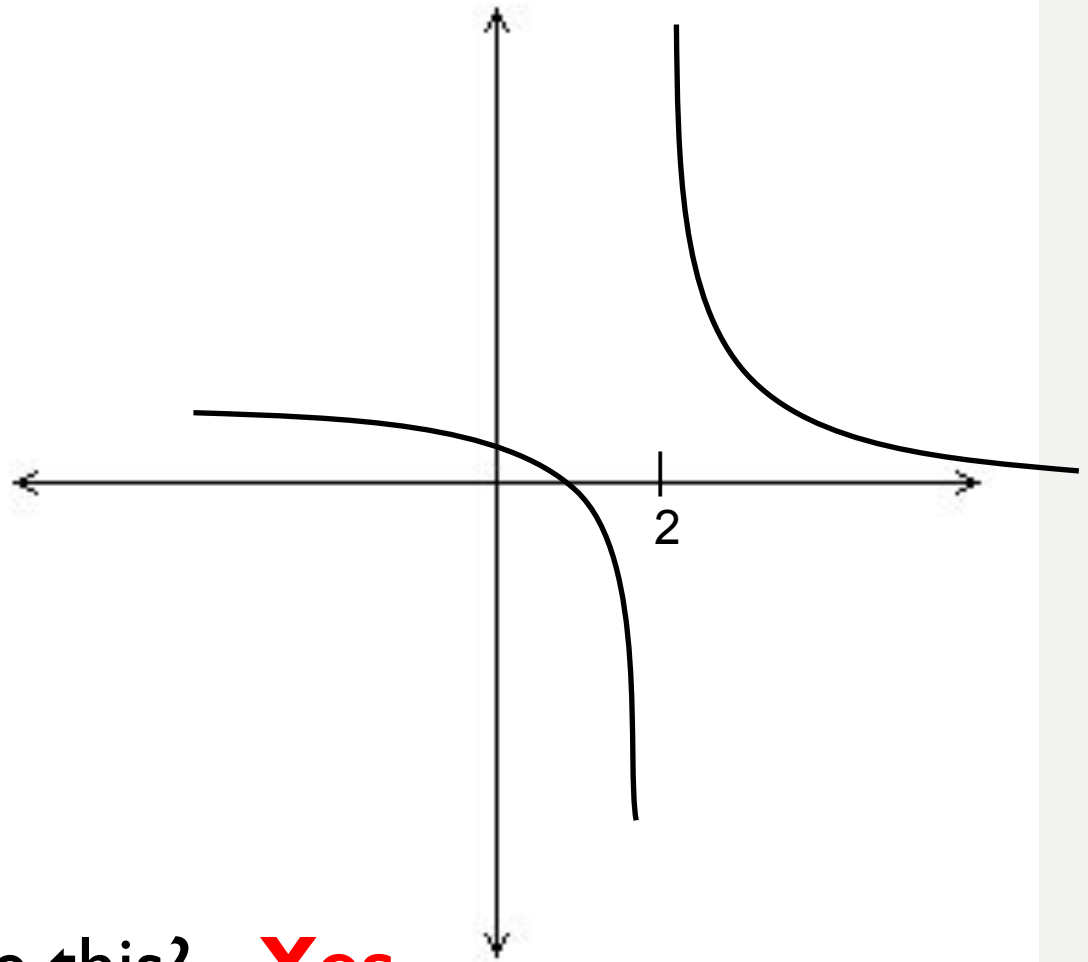
- “Describe” the behavior of the y-values as the x-values **approach** the vertical asymptote.

From the left:  $-\infty$

From the right:  $\infty$

Is there a simpler way to write this? **Yes...**

$$f(x) = \frac{x + 3}{x - 2}$$



# ONE-SIDED LIMITS

## ONE-SIDED LIMITS

• Let  $f(x)$  be defined on an interval  $(a, b)$ , where  $a < b$ . If  $f(x)$  approaches arbitrarily close to  $L$  as  $x$  approaches  $a$  from within that interval, then we say that  $f$  has a **right-hand limit  $L$**  at  $a$ , and we write:  $\lim_{x \rightarrow a^+} f(x) = L$

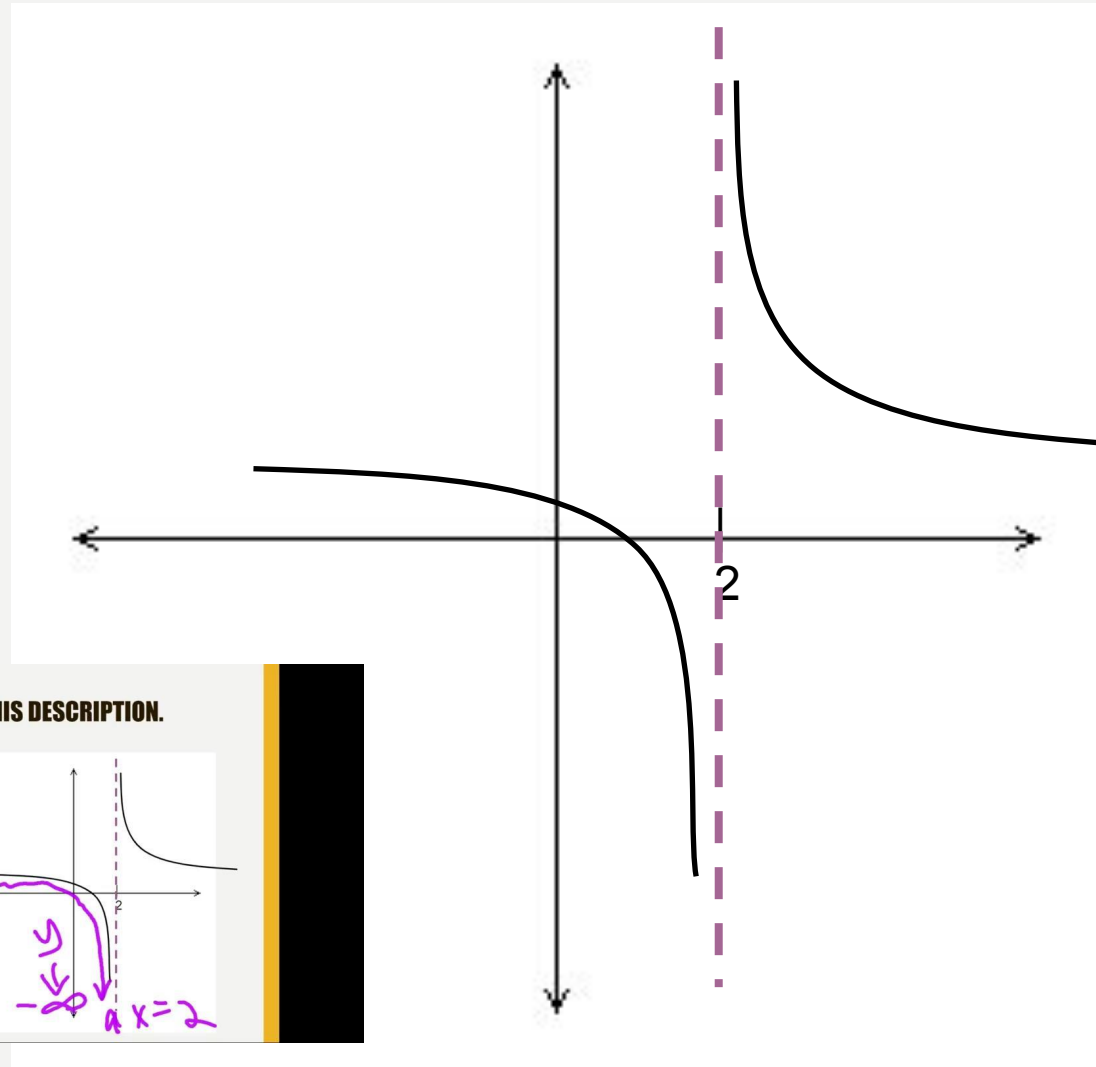
• Let  $f(x)$  be defined on an interval  $(c, a)$  where  $c < a$ . If  $f(x)$  approaches arbitrarily close to  $M$  as  $x$  approaches  $a$  from within the interval, then we say that  $f$  has a **left-hand limit  $M$**  at  $a$ , and we write:  $\lim_{x \rightarrow a^-} f(x) = M$

- Let  $f(x)$  be defined on an interval  $(a, b)$ , where  $a < b$ . If  $f(x)$  approaches arbitrarily close to  $L$  as  $x$  approaches  $a$  from within that interval, then we say that  $f$  has a **right-hand limit  $L$**  at  $a$ , and we write:  $\lim_{x \rightarrow a^+} f(x) = L$
- Let  $f(x)$  be defined on an interval  $(c, a)$  where  $c < a$ . If  $f(x)$  approaches arbitrarily close to  $M$  as  $x$  approaches  $a$  from within the interval, then we say that  $f$  has a **left-hand limit  $M$**  at  $a$ , and we write:  $\lim_{x \rightarrow a^-} f(x) = M$

# LIMIT NOTATION TO REPRESENT THIS DESCRIPTION.

Limit of  $f(x)$  as  $x$   
approaches 2 from the  
left (negative side):

$$\lim_{x \rightarrow 2^-} f(x) = ?$$



LIMIT NOTATION TO REPRESENT THIS DESCRIPTION.

Limit of  $f(x)$  as  $x$   
approaches 2 from the  
left (negative side):

$\lim_{x \rightarrow 2^-} f(x) = ?$

LHS

$-\infty$

$x=2$

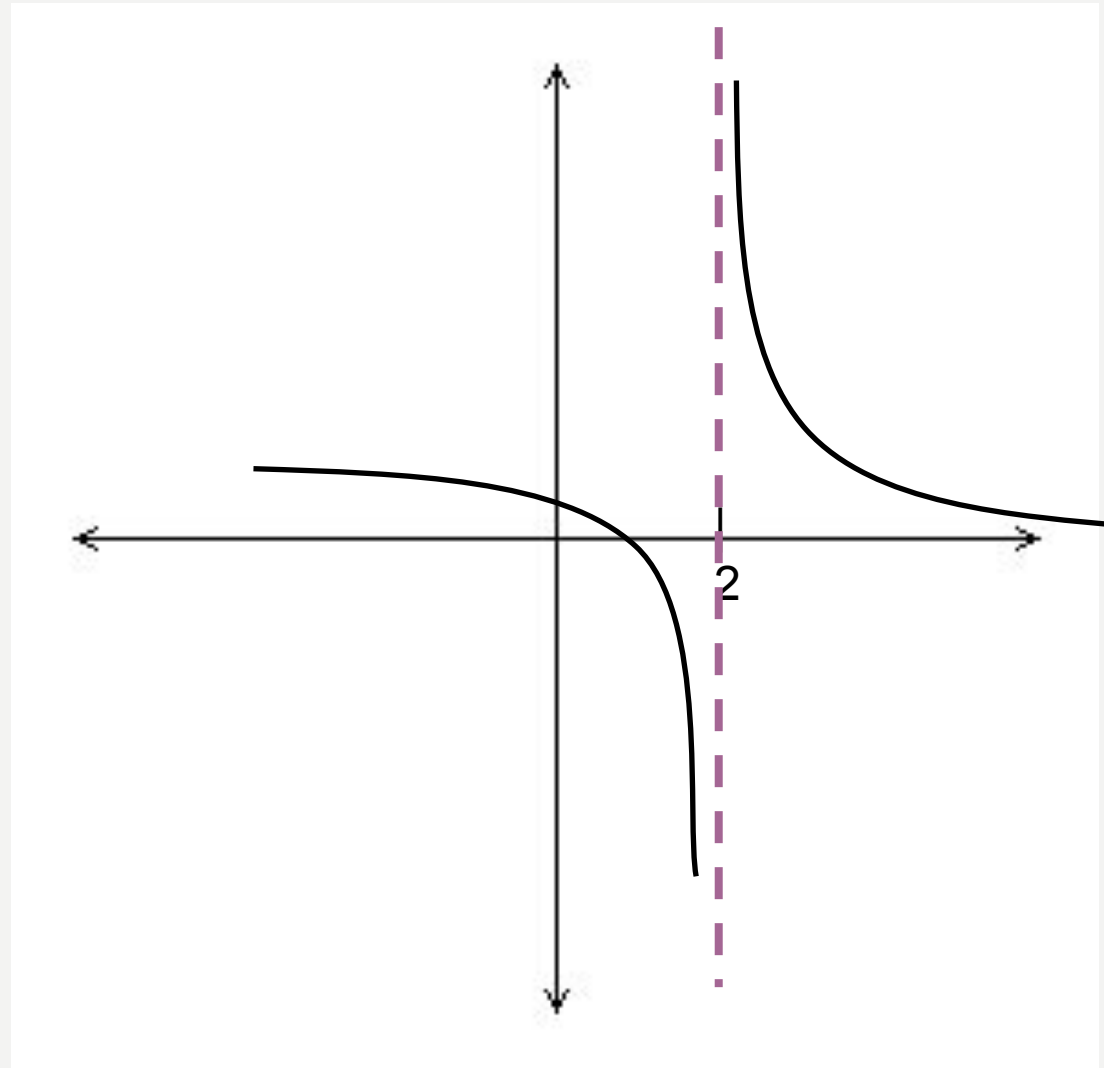
A screenshot of a video player showing the same content as the main slide. The video player interface includes a play button, a progress bar, and a volume icon. The content of the video is the same as the main slide, but with handwritten purple annotations. The text "LIMIT NOTATION TO REPRESENT THIS DESCRIPTION." is underlined. The text "Limit of  $f(x)$  as  $x$  approaches 2 from the left (negative side):" is underlined. The equation  $\lim_{x \rightarrow 2^-} f(x) = ?$  is underlined. The text "LHS" is written below the equation. The text " $-\infty$ " is written to the left of the graph. The text " $x=2$ " is written below the graph. The graph itself is also present in the screenshot.

# LIMIT NOTATION TO REPRESENT THIS DESCRIPTION. **ANSWER**

Limit of  $f(x)$  as  $x$   
approaches 2 from the  
left (negative side):

$$\lim_{x \rightarrow 2^-} f(x) = ?$$

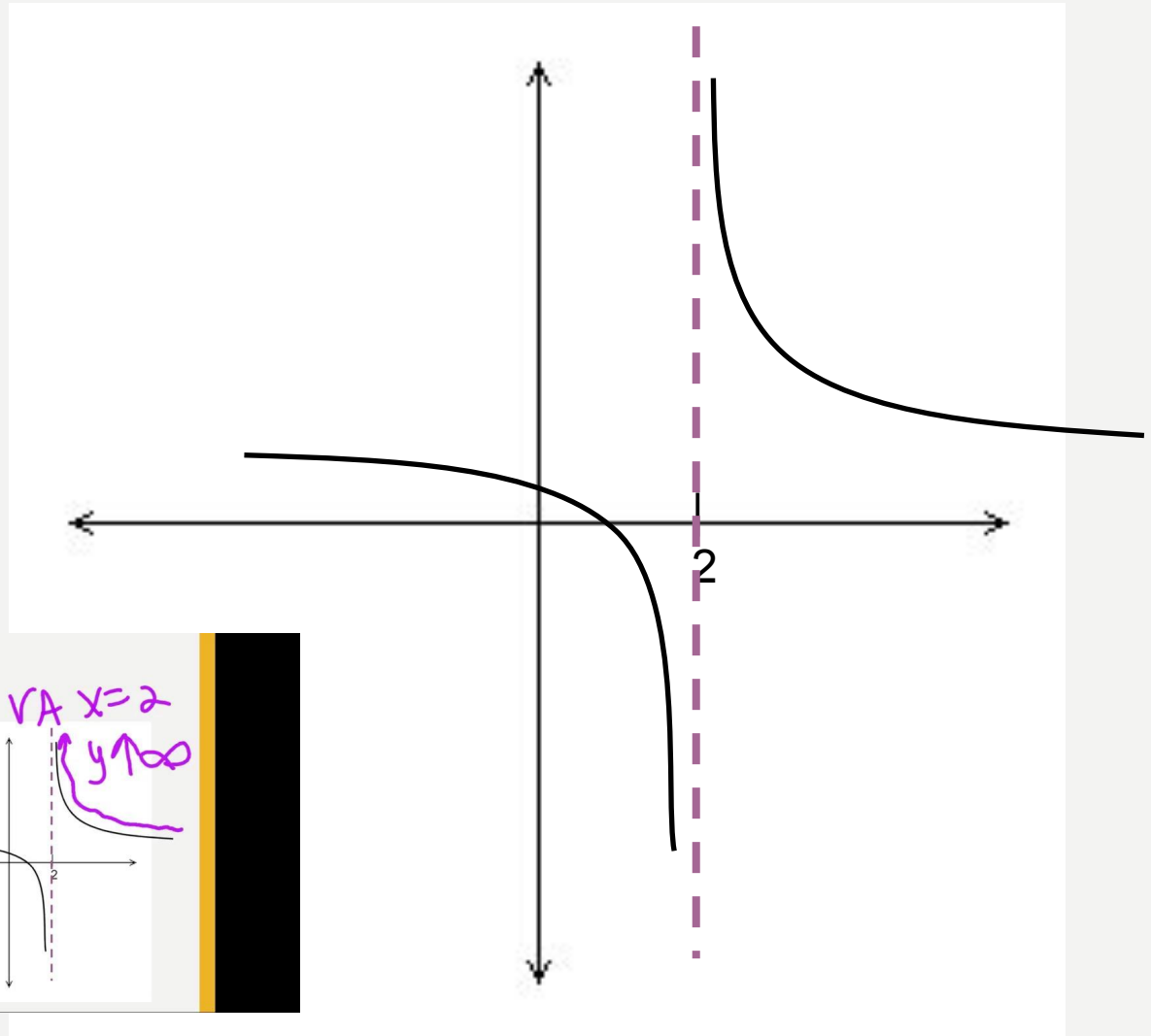
$$\lim_{x \rightarrow 2^-} f(x) = -\infty$$



# LIMIT NOTATION:

Limit of  $f(x)$  as  $x$   
approaches 2 from  
the right (positive  
side):

$$\lim_{x \rightarrow 2^+} f(x) = ?$$



**LIMIT NOTATION:**  
Limit of  $f(x)$  as  $x$   
approaches 2 from  
the right (positive  
side):

$\lim_{x \rightarrow 2^+} f(x) = ?$

\* RHS

VA  $x=2$   
 $y \uparrow \infty$

A smaller version of the graph from the main figure, with handwritten purple annotations. A vertical dashed line is labeled 'VA  $x=2$ '. An arrow points from the text ' $y \uparrow \infty$ ' to the curve as it approaches the asymptote from the right. The limit expression  $\lim_{x \rightarrow 2^+} f(x) = ?$  is circled in purple. The text '\* RHS' is written below the limit expression.



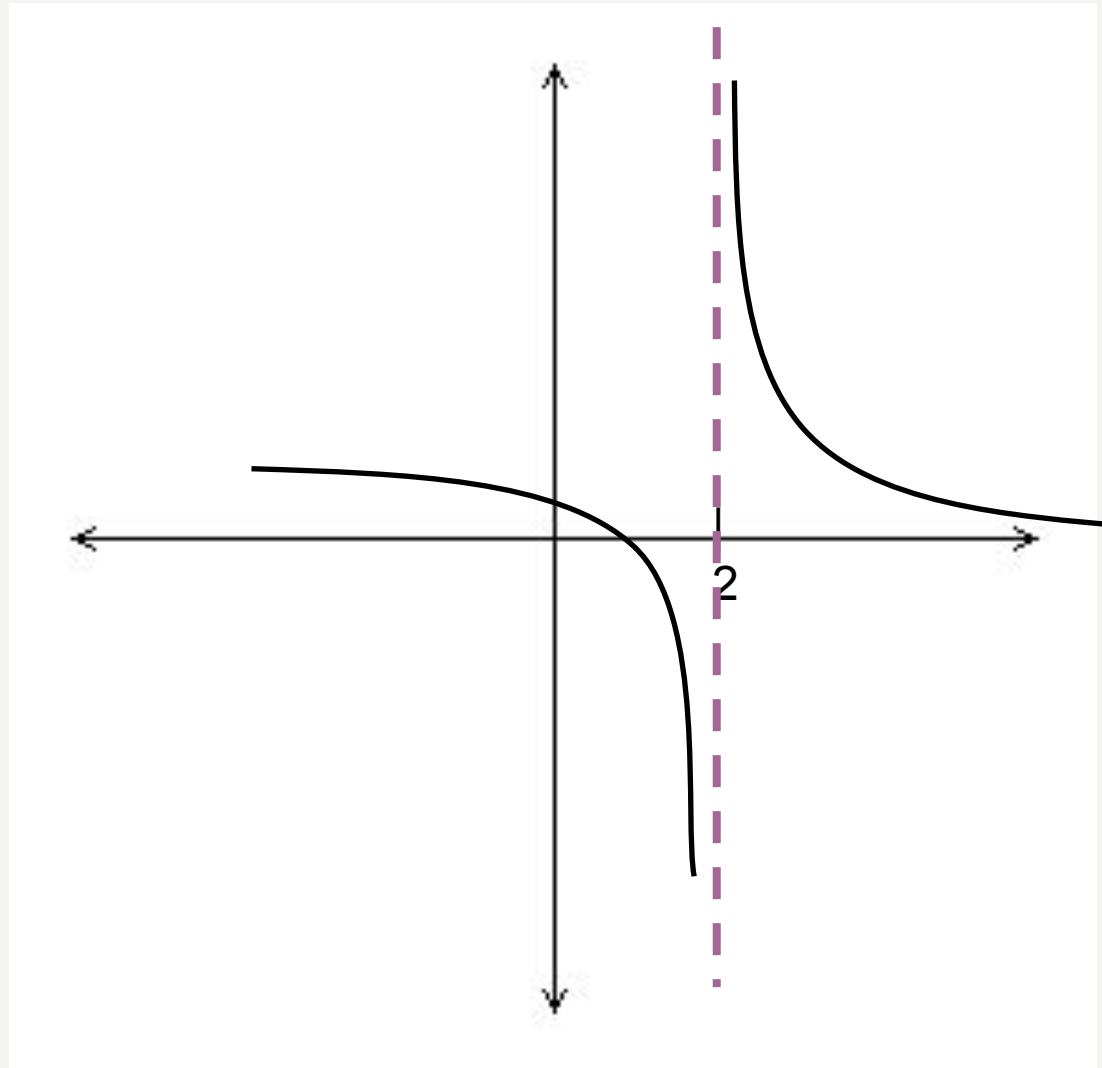
# LIMIT NOTATION:

# ANSWER

Limit of  $f(x)$  as  $x$   
approaches 2 from  
the right (positive  
side):

$$\lim_{x \rightarrow 2^+} f(x) = ?$$

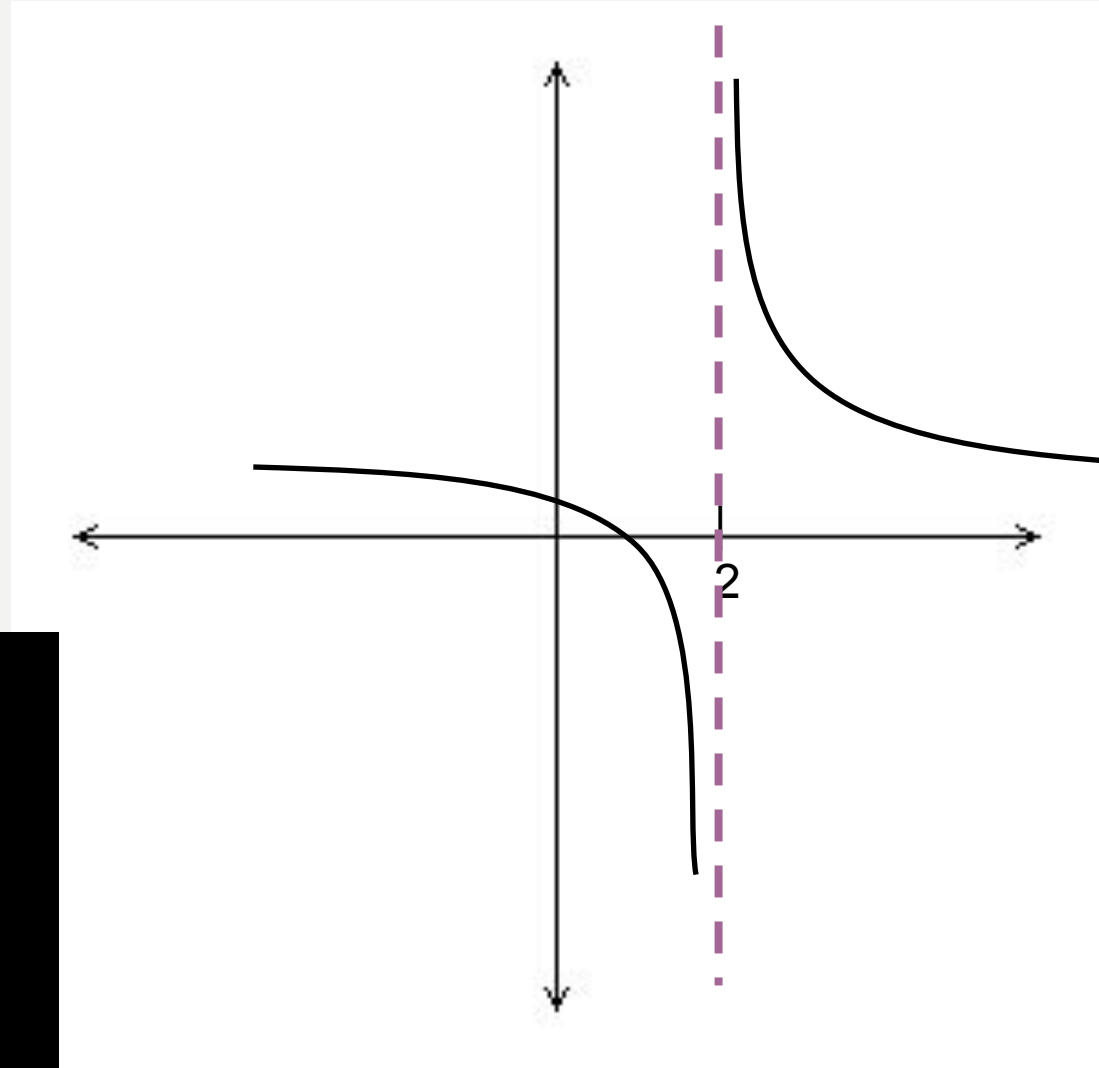
$$\lim_{x \rightarrow 2^+} f(x) = \infty$$



# SO THE RESULT????

Limit of  $f(x)$  as  $x$  approaches 2:

$$\lim_{x \rightarrow 2} f(x) = ?$$



SO THE RESULT????

Limit of  $f(x)$  as  $x$  approaches 2:

$$\lim_{x \rightarrow 2} f(x) = ?$$

*Handwritten notes:*  $\forall x = 2$  (with a circled 2), and a pink arrow pointing to the  $x \rightarrow 2$  in the limit expression.

A smaller version of the graph from the main slide, showing the function and the vertical asymptote at  $x=2$ .

Navigation icons: ||, <, >, ✎, 📺, ✕

**SO THE RESULT?????**

**ANSWER**

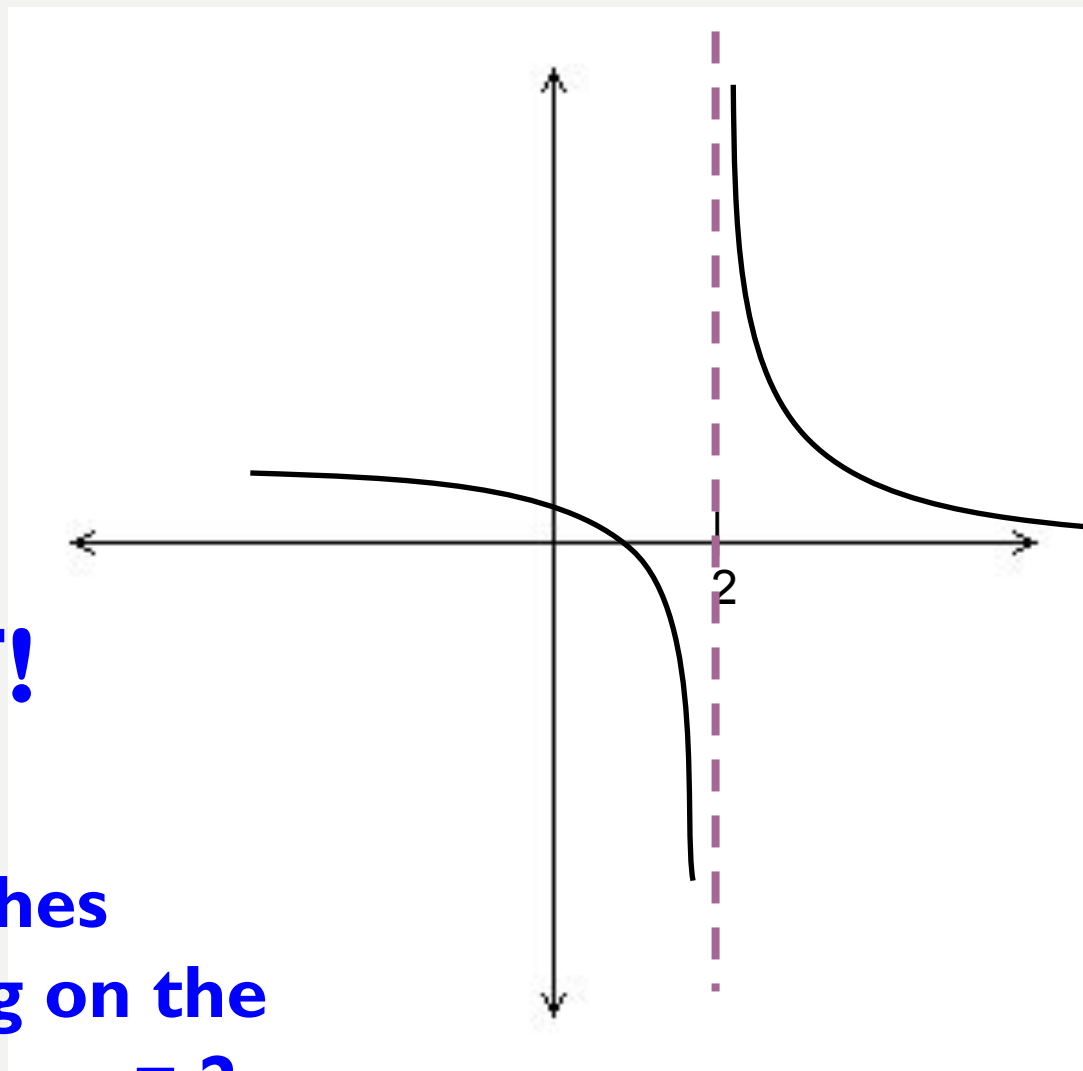
Limit of  $f(x)$  as  $x$   
approaches 2:

$$\lim_{x \rightarrow 2} f(x) = ?$$

**DOES NOT EXIST!**

**(a.k.a. “DNE”)**

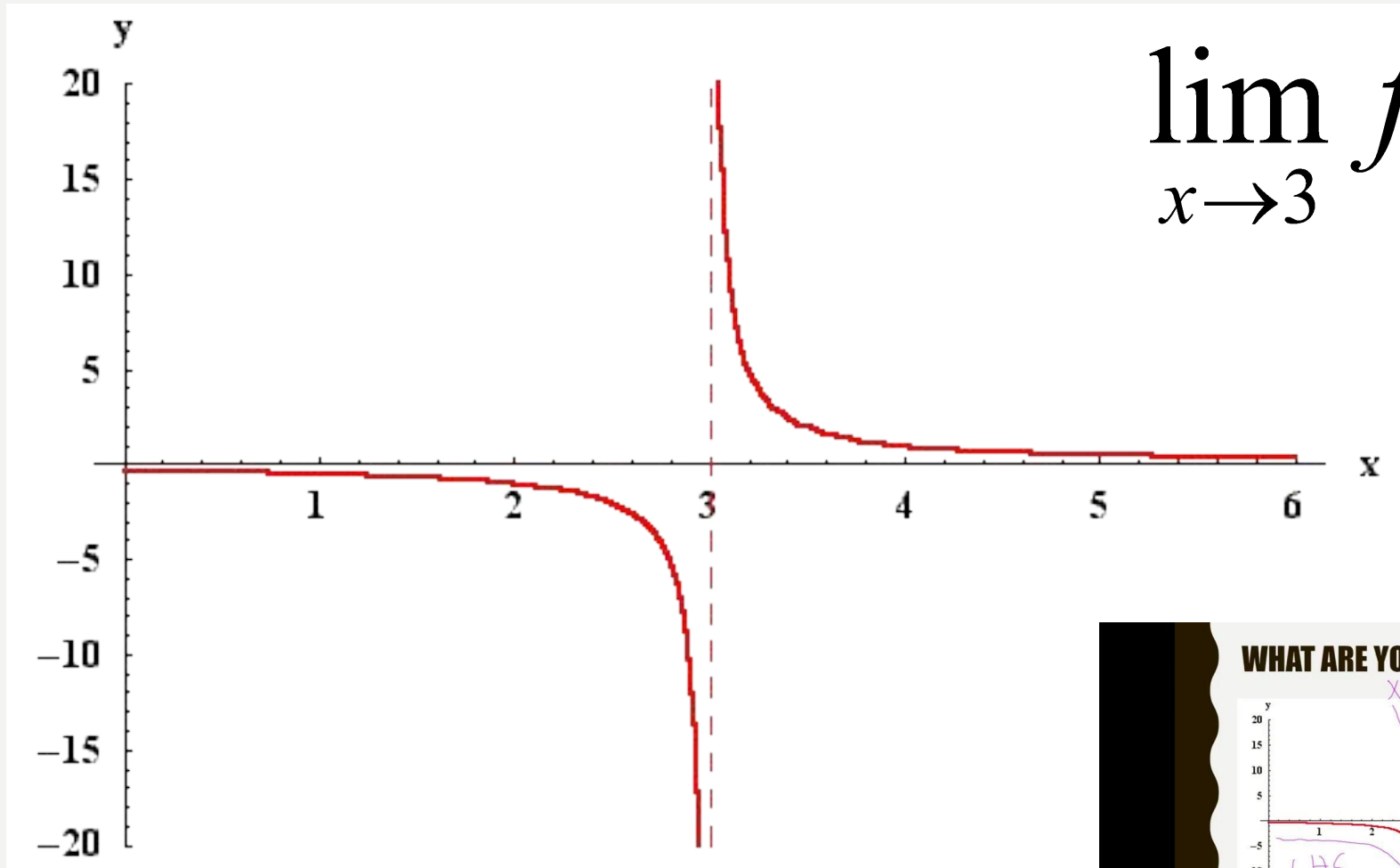
**Because the graph approaches  
different  $y$ -values depending on the  
direction you're approaching  $x = 2$**



# SO, A LIMIT DOES NOT EXIST (DNE) IF:

- $f(x)$  approaches a different value from the right side than from the left.
- $f(x)$  oscillates between two fixed values as  $x$  approaches  $c$ .
- Ex: Graph  $y = \sin(x)$ . Find limit as  $x$  approaches infinity. **DOES NOT EXIST!**

# WHAT ARE YOUR THOUGHTS ON THIS?

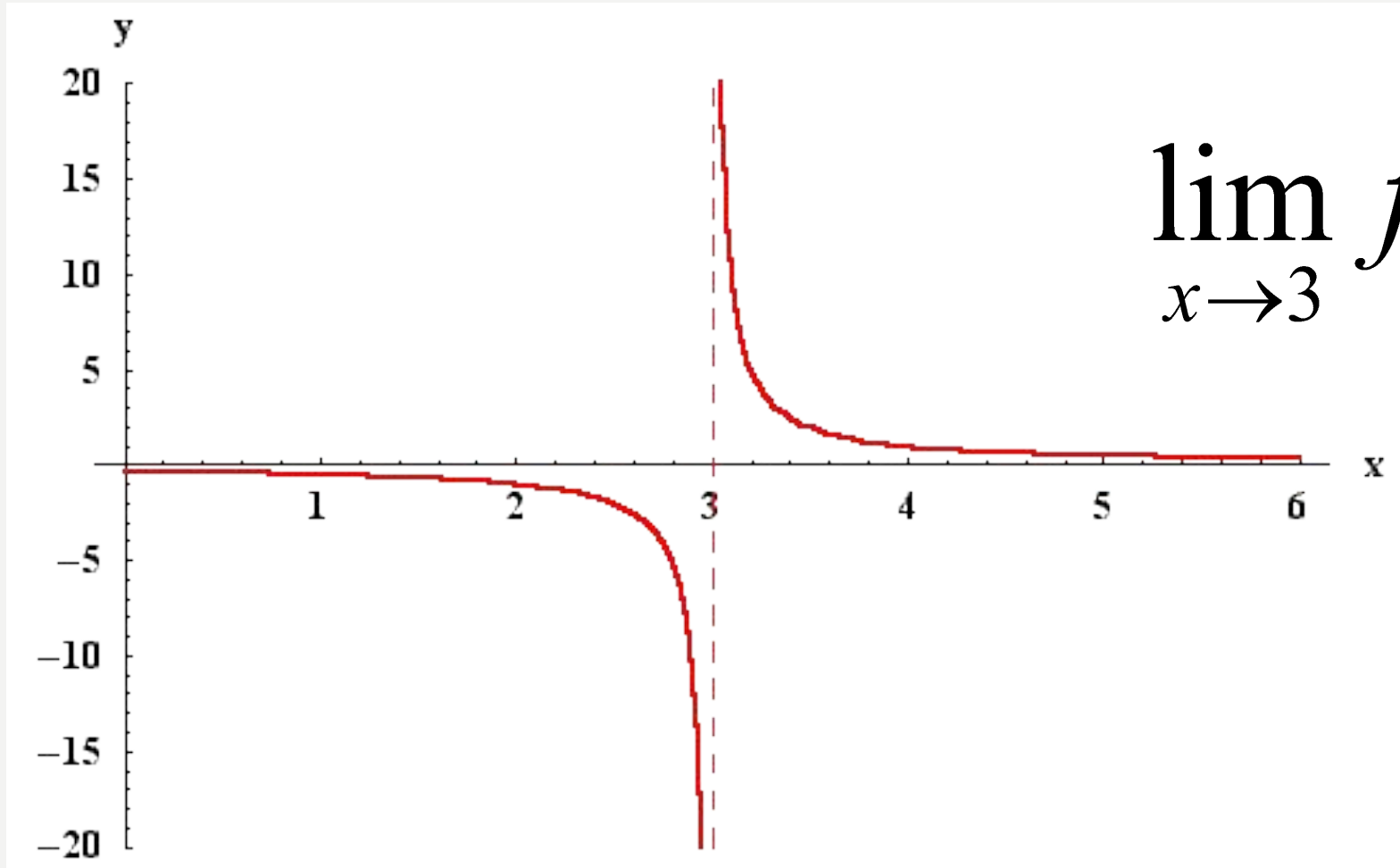


$$\lim_{x \rightarrow 3} f(x) = ?$$

A video player interface showing a smaller version of the graph from the main image. The video title is "WHAT ARE YOUR THOUGHTS ON THIS?". The graph has handwritten purple annotations: "x=3 VA" with an arrow pointing to the vertical asymptote, "LHS" with an arrow pointing to the left side of the asymptote, and "approach" written vertically on the right. The limit expression  $\lim_{x \rightarrow 3} f(x) = ?$  is also present. The video player controls at the bottom show a play button, a progress bar, and a close button.

# WHAT ARE YOUR THOUGHTS ON THIS?

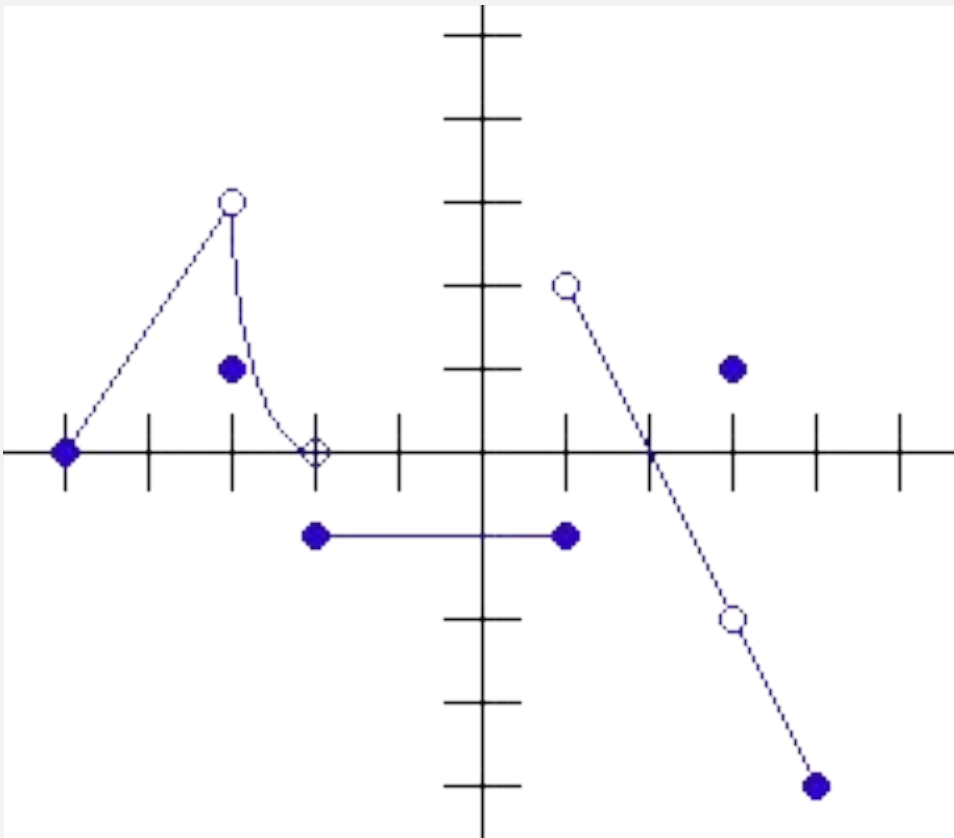
**ANSWER**



$$\lim_{x \rightarrow 3} f(x) = ?$$

**DNE**

# WHAT ABOUT THIS??



$$\lim_{x \rightarrow 0} f(x) =$$

$$\lim_{x \rightarrow 2} f(x) =$$

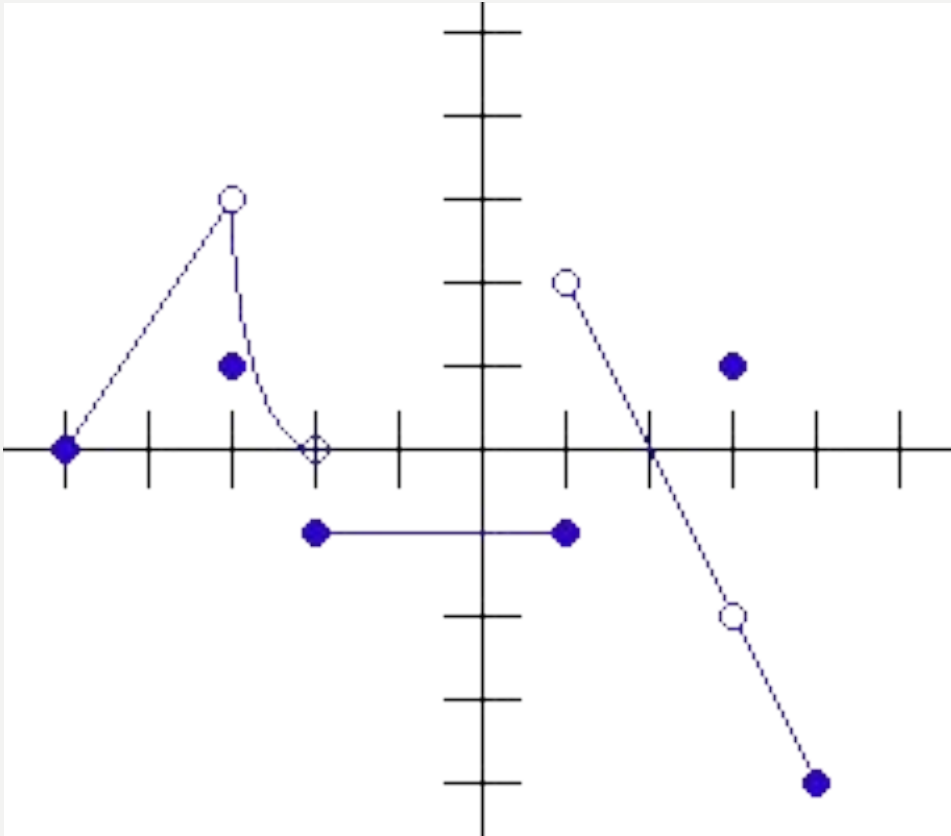
$$\lim_{x \rightarrow 1} f(x) =$$

$$\lim_{x \rightarrow -2} f(x) =$$

$$\lim_{x \rightarrow -5} f(x) =$$



# WHAT ABOUT THIS??



$$\lim_{x \rightarrow 0} f(x) = -1$$

$$\lim_{x \rightarrow 2} f(x) = 0$$



## ANSWERS

$$\lim_{x \rightarrow 1} f(x) = \text{DNE}$$

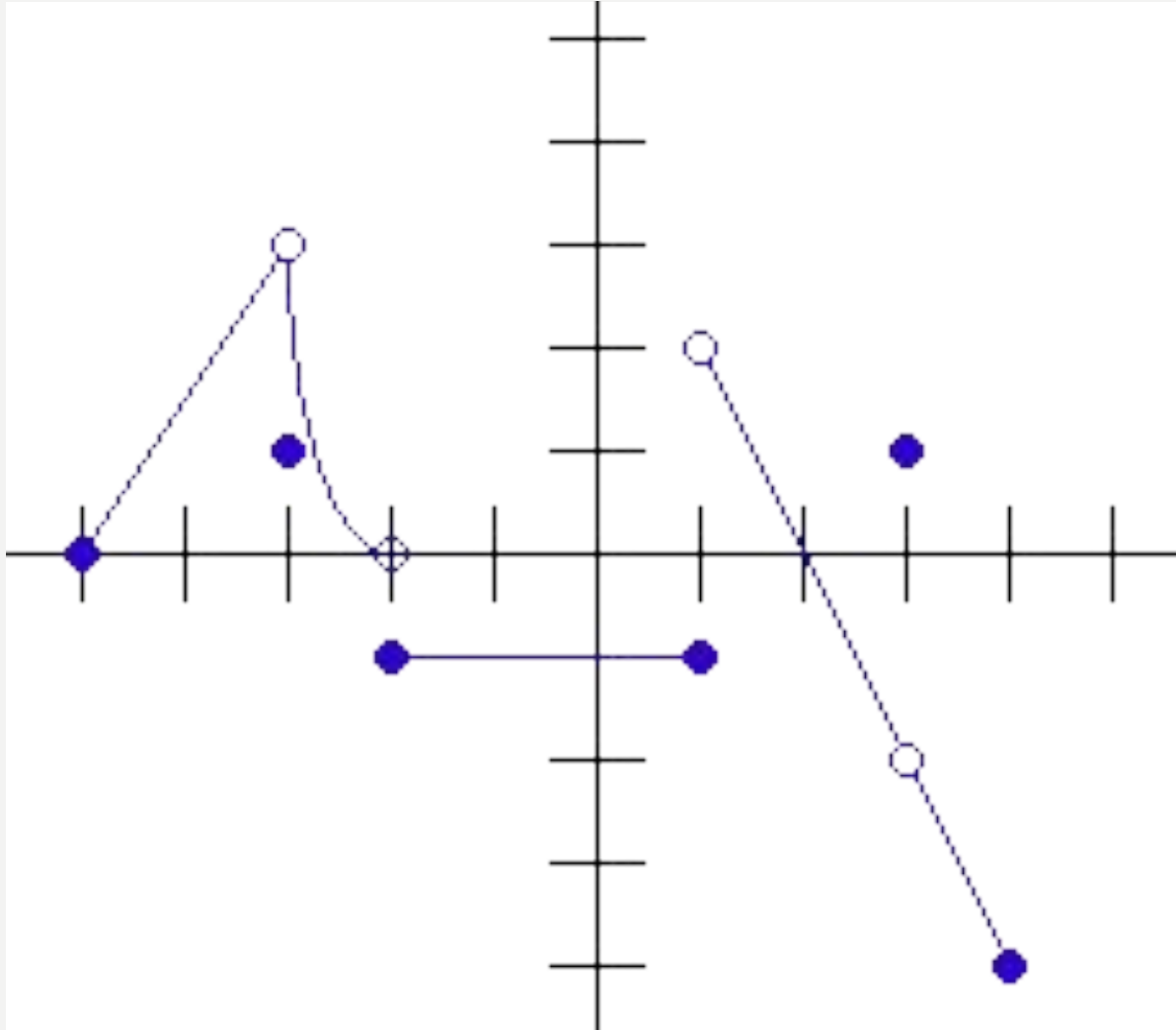
$$\lim_{x \rightarrow -2} f(x) = \text{DNE}$$

$$\lim_{x \rightarrow -5} f(x) = 0$$





# YOU TRY! EVALUATE.



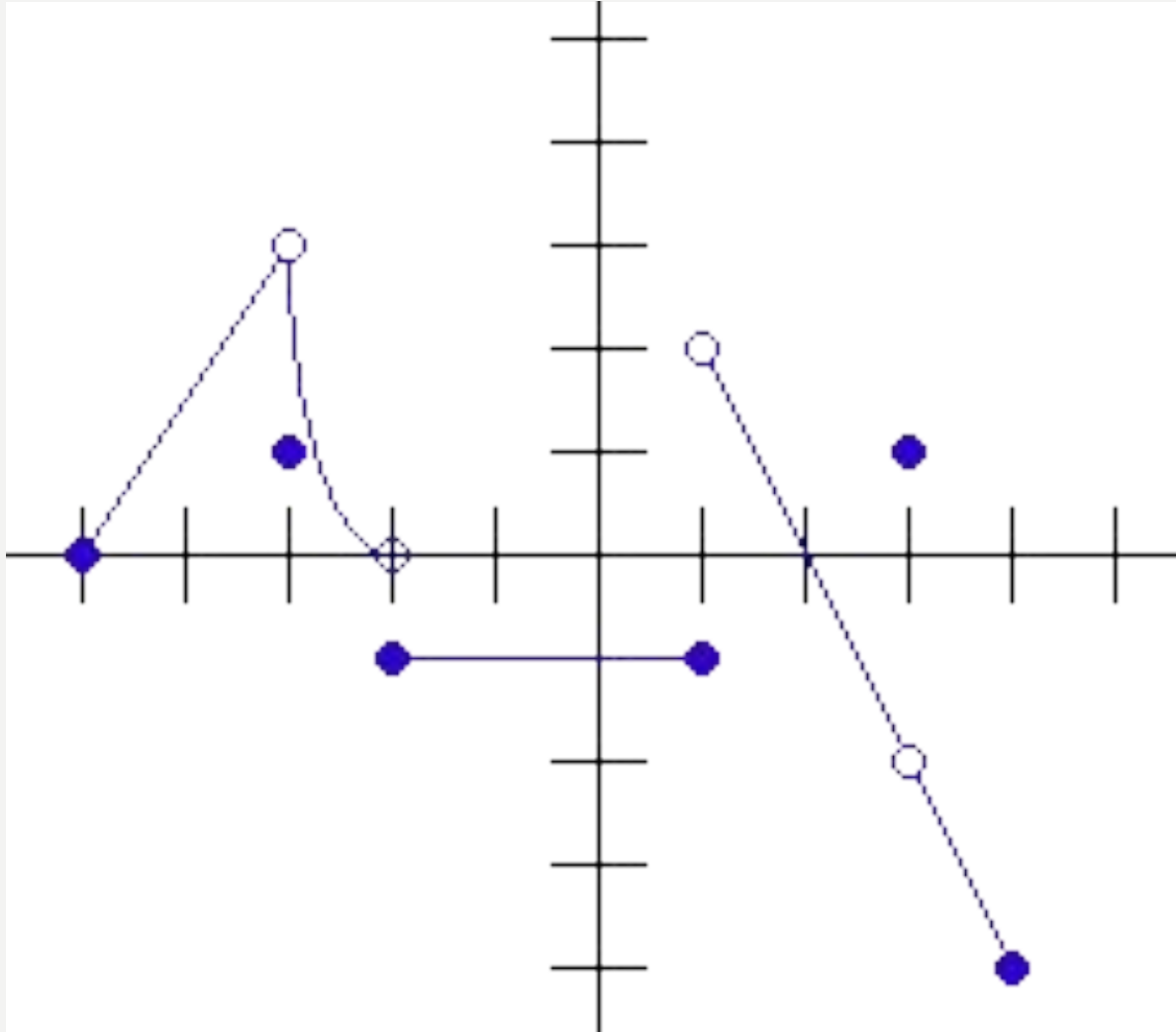
$$\lim_{x \rightarrow -2^-} f(x) =$$

$$\lim_{x \rightarrow -2^+} f(x) =$$

$$\lim_{x \rightarrow 1^-} f(x) =$$

$$\lim_{x \rightarrow 1^+} f(x) =$$

# YOU TRY! EVALUATE.



\*\*\*

## ANSWERS

$$\lim_{x \rightarrow -2^-} f(x) = 0$$

$$\lim_{x \rightarrow -2^+} f(x) = -1$$

$$\lim_{x \rightarrow 1^-} f(x) = -1$$

$$\lim_{x \rightarrow 1^+} f(x) = 2$$

YOU TRY! EVALUATE. \*\*\*

$\lim_{x \rightarrow 2^-} f(x) = 0$

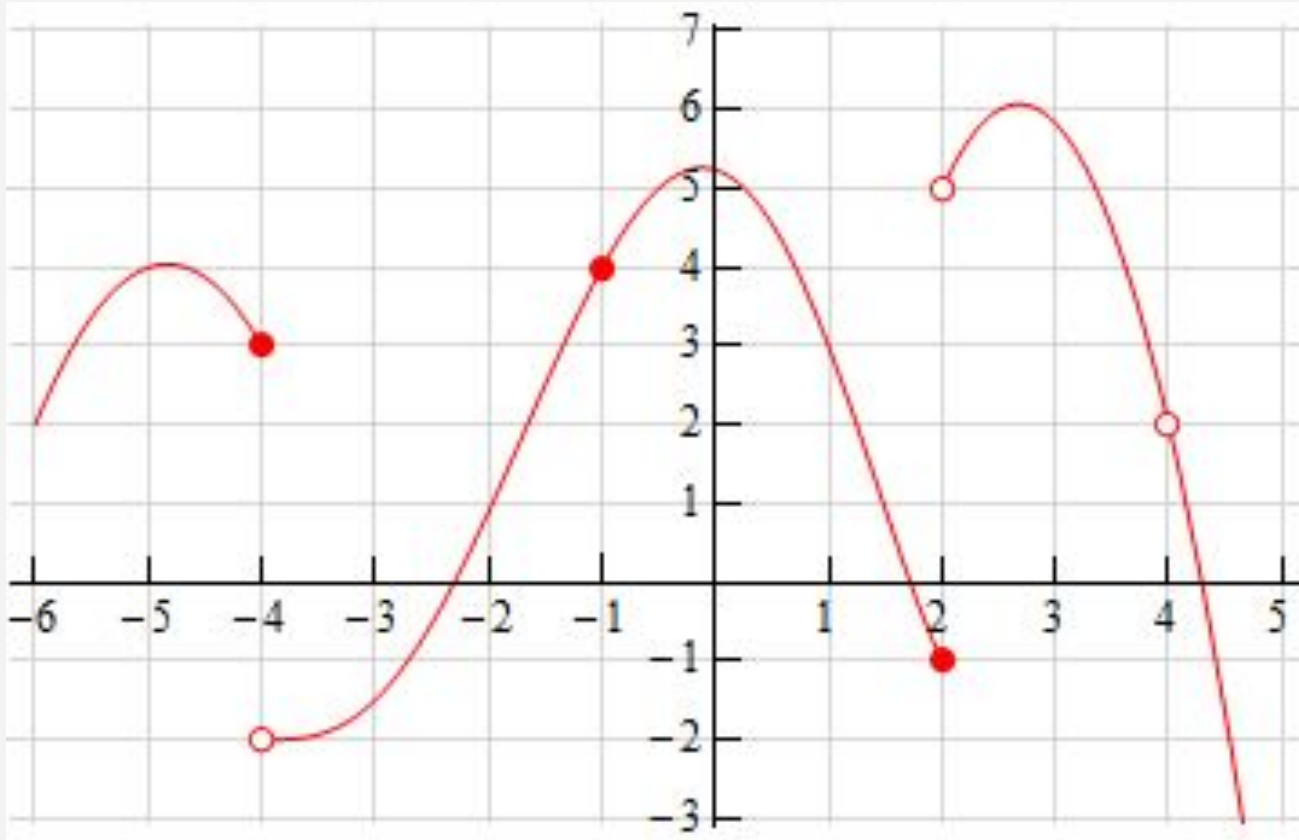
$\lim_{x \rightarrow 2^+} f(x) = -1$

$\lim_{x \rightarrow 1^-} f(x) = -1$

$\lim_{x \rightarrow 1^+} f(x) = 2$

$\lim_{x \rightarrow 2} f(x) = \text{DNE}$

# REVIEW



$$\lim_{x \rightarrow -4} f(x) =$$

$$\lim_{x \rightarrow -4^-} f(x) =$$

$$\lim_{x \rightarrow -1} f(x) =$$

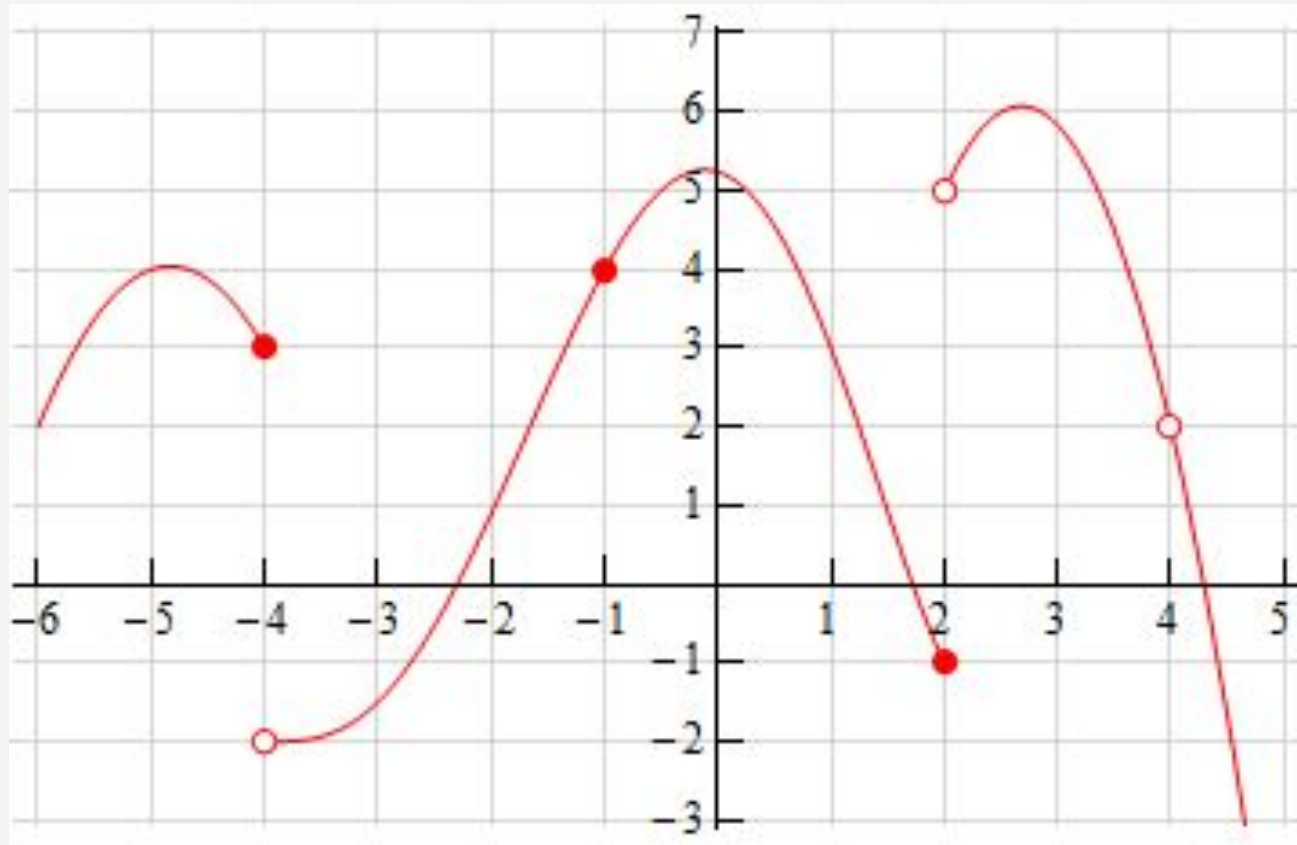
$$\lim_{x \rightarrow 4} f(x) =$$

$$f(4) =$$

$$f(2) =$$

# REVIEW

# ANSWERS



$$\lim_{x \rightarrow -4} f(x) = \text{DNE}$$

$$\lim_{x \rightarrow -4^-} f(x) = 3$$

$$\lim_{x \rightarrow -1} f(x) = 4$$

$$\lim_{x \rightarrow 4} f(x) = 2$$

$$f(4) = \text{DNE}$$

$$f(2) = -1$$

**REVIEW**

$\lim_{x \rightarrow -4} f(x) = \text{DNE}$

$\lim_{x \rightarrow -4^-} f(x) = 3$

$\lim_{x \rightarrow -1} f(x) = 4$

$\lim_{x \rightarrow 4} f(x) = 2$

$f(4) = \text{DNE}$

$f(2) = -1$

# CONTINUOUS DEFINITION AGAIN...

- Function  $f$  is continuous at a point  $a$  if the following conditions are satisfied.

$f(a)$  is defined

$\lim_{x \rightarrow a} f(x)$  exists

$\lim_{x \rightarrow a} f(x) = f(a)$

The screenshot shows a presentation slide with the title "CONTINUOUS DEFINITION AGAIN..." in bold black text. To the right of the title is a hand-drawn graph of a function with a jump discontinuity at a point, drawn in purple and green. Below the title is a bulleted list of conditions for continuity, with the first condition underlined in green. The conditions are: "Function  $f$  is continuous at a point  $a$  if the following conditions are satisfied.", " $f(a)$  is defined" (underlined in green), " $\lim_{x \rightarrow a} f(x)$  exists", and " $\lim_{x \rightarrow a} f(x) = f(a)$ ". At the bottom of the slide is a control bar with icons for back, forward, search, and close.

**CONTINUOUS DEFINITION AGAIN...**

- Function  $f$  is continuous at a point  $a$  if the following conditions are satisfied.

$f(a)$  is defined

$\lim_{x \rightarrow a} f(x)$  exists

$\lim_{x \rightarrow a} f(x) = f(a)$

# PRACTICE

- Find the values requested for

$$g(x) = \frac{3x^2 + 16x - 12}{x^2 + 2x - 24}$$

D:

R:

Removable Disc:

NonRemovable Disc:

Horizontal Asymptote:

Increasing:

Decreasing:

End Behavior, written as limits

The following limits

$$\lim_{x \rightarrow -6} g(x) =$$

$$\lim_{x \rightarrow 4} g(x) =$$

$$\lim_{x \rightarrow 4^-} g(x) =$$

$$\lim_{x \rightarrow 4^+} g(x) =$$

# PRACTICE ANSWERS

- Find the values requested for

D:  $(-\infty, -6) \cup (-6, 4) \cup (4, \infty)$

R:  $(-\infty, 2) \cup (2, 3) \cup (3, \infty)$

Removable Disc: Hole at  $(-6, 2)$

NonRemovable Disc: VA at  $x = 4$

Horizontal Asymptote:  $y = 3$

Increasing: None

Decreasing:  $(-\infty, -6) \cup (-6, 4) \cup (4, \infty)$

End Behavior, written as limits.

The following limits

$$\lim_{x \rightarrow -6} g(x) = 2$$

$$\lim_{x \rightarrow 4} g(x) = DNE$$

$$\lim_{x \rightarrow 4^-} g(x) = -\infty$$

$$\lim_{x \rightarrow 4^+} g(x) = \infty$$

$$g(x) = \frac{3x^2 + 16x - 12}{x^2 + 2x - 24}$$

**PRACTICE**

Find the values requested for  $g(x) = \frac{3x^2 + 16x - 12}{x^2 + 2x - 24}$

D:  $(-\infty, -6) \cup (-6, 4) \cup (4, \infty)$

R:  $(-\infty, 2) \cup (2, 3) \cup (3, \infty)$

Removable Disc: Hole at  $(-6, 2)$

NonRemovable Disc: VA at  $x = 4$

Horizontal Asymptote:  $y = 3$

Increasing: None

Decreasing:  $(-\infty, -6) \cup (-6, 4) \cup (4, \infty)$

End Behavior, written as limits

The following limits

$\lim_{x \rightarrow -6} g(x) = 2$     $\lim_{x \rightarrow 4} g(x) = DNE$     $\lim_{x \rightarrow 4^-} g(x) = -\infty$     $\lim_{x \rightarrow 4^+} g(x) = \infty$

**PRACTICE**

Find the values requested for  $g(x) = \frac{3x^2 + 16x - 12}{x^2 + 2x - 24}$

D:  $(-\infty, -6) \cup (-6, 4) \cup (4, \infty)$

R:  $(-\infty, 2) \cup (2, 3) \cup (3, \infty)$

Removable Disc: Hole at  $(-6, 2)$

NonRemovable Disc: VA at  $x = 4$

Horizontal Asymptote:  $y = 3$

Increasing: None

Decreasing:  $(-\infty, -6) \cup (-6, 4) \cup (4, \infty)$

End Behavior, written as limits:  $\lim_{x \rightarrow -\infty} g(x) = 3$     $\lim_{x \rightarrow \infty} g(x) = 3$

The following limits

$\lim_{x \rightarrow -6} g(x) = 2$     $\lim_{x \rightarrow 4} g(x) = DNE$     $\lim_{x \rightarrow 4^-} g(x) = -\infty$     $\lim_{x \rightarrow 4^+} g(x) = \infty$



# LIMIT PRACTICE HANDOUT

THIS EXTRA HANDOUT IS ON THE  
WEBSITE, IF YOU'D LIKE MORE  
PRACTICE 😊





**HW DAY 7:**

PACKET P.9 AND HANDOUT  
INTRO TO LIMITS (ON  
WEBSITE)