DAY 7

LIMITS PART 2 - WHAT ABOUT LIMITS IN THE MIDDLE OF THE FUNCTION?

WARM-UP DAY 7

TIP: You may want to print "Day 7 Graphs for Notes" - in Power Point area of website - to help with notetaking :)

Find the requested information.

HINT: find extrema 1st

- 1) $g(x)=\dfrac{x^4}{x^4-1}$

Increasing:

Decreasing:

Domain:

Range:

Express End Behaviors using proper Limit Notation

- 2) $f(x)=x^3-2x^2-3x$

Increasing:

Decreasing:

Domain:

Range:

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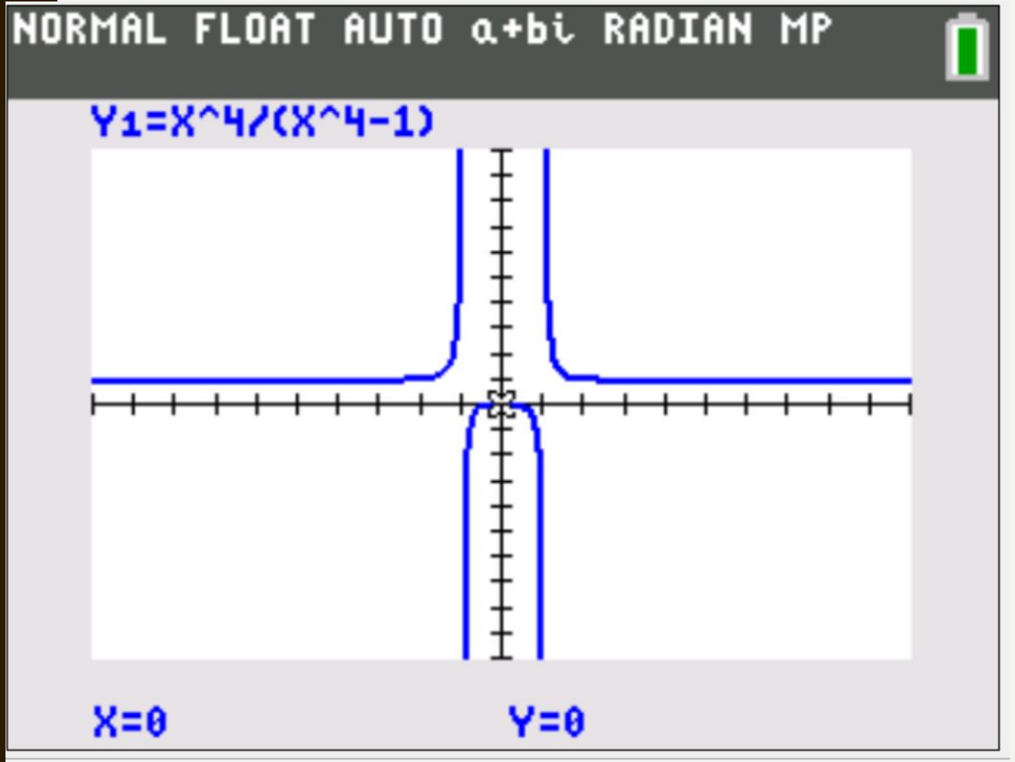
- 1) $g(x)=\dfrac{x^4}{x^4-1}$



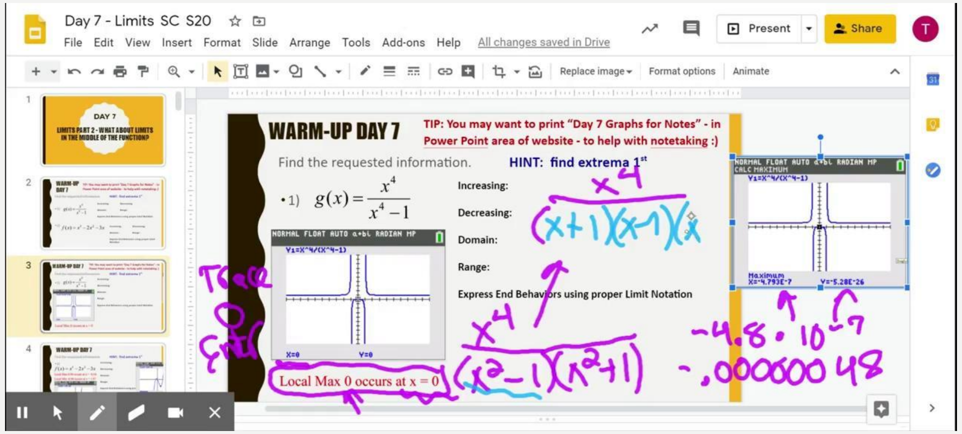
Increasing:

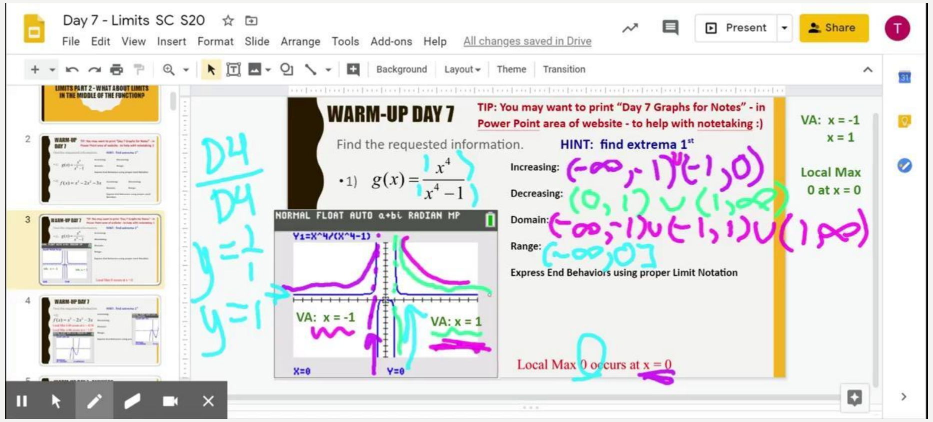
Decreasing:

Domain:

Range:

Express End Behaviors using proper Limit Notation





WARM-UP DAY 7

Find the requested information.

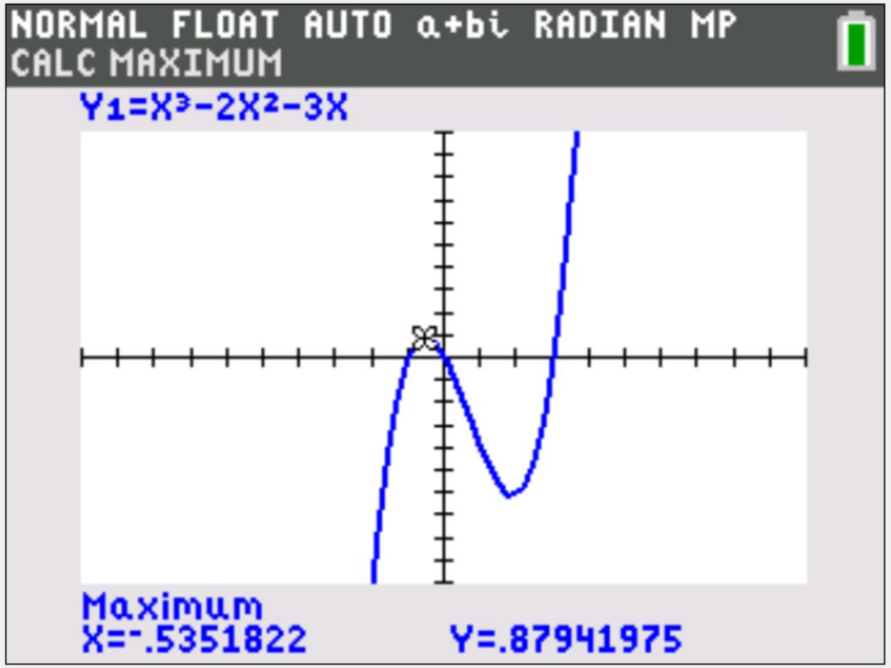
HINT: find extrema 1st

- 2)

$$f(x) = x^3 - 2x^2 - 3x$$

Local Max 0.88 occurs at x = -0.54

Local Min -6.06 occurs at x = 1.87

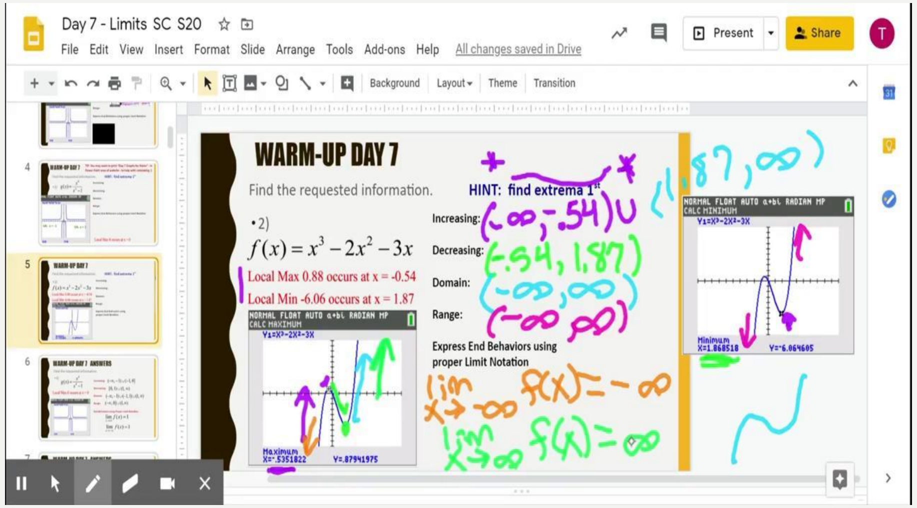


Increasing:

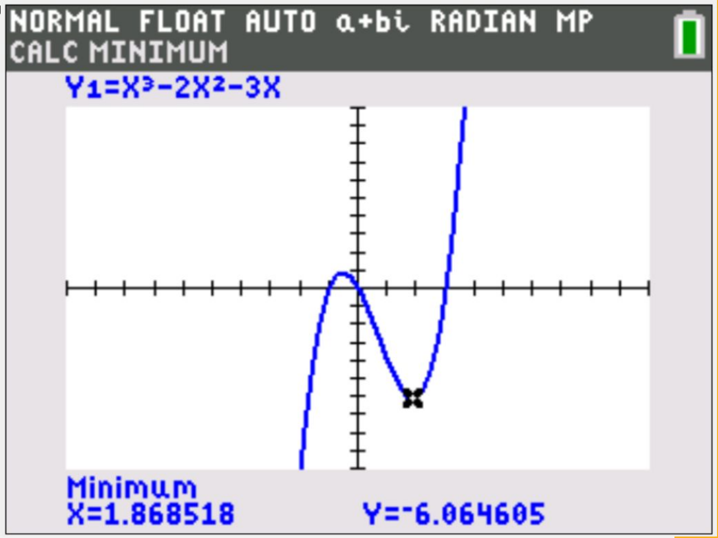
Decreasing:

Domain:

Range:



Express End Behaviors using proper Limit Notation

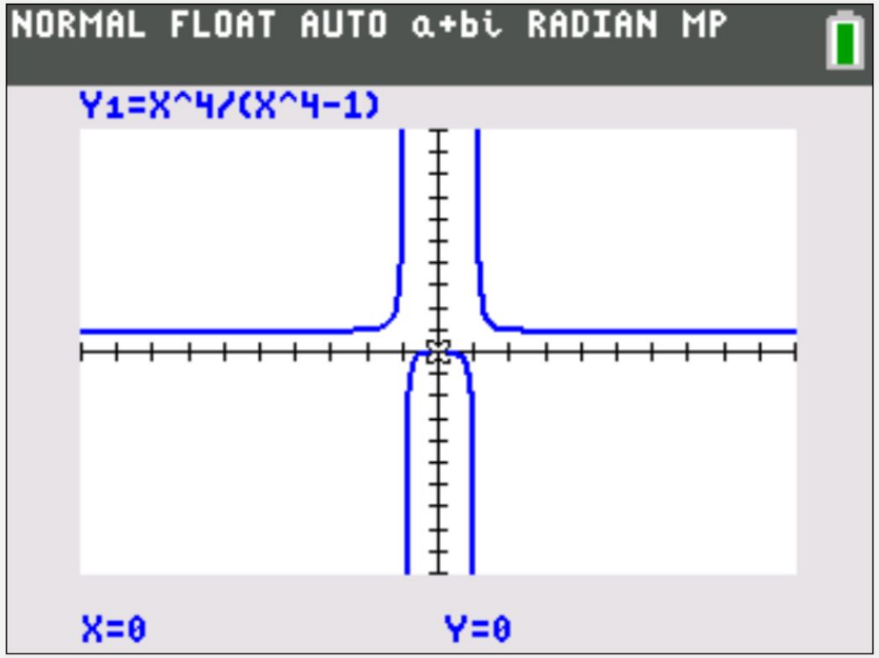


WARM-UP DAY 7 ANSWERS

Find the requested information.

- 1) $$g(x) = \frac{x^4}{x^4 - 1}$$

Local Max 0 occurs at x = 0



Increasing: $(-\infty, -1) \cup (-1, 0)$

Decreasing: $(0, 1) \cup (1, \infty)$

Domain: $(-\infty, -1) \cup (-1, 1) \cup (1, \infty)$

Range: $(-\infty, 0] \cup (1, \infty)$

End Behaviors using Proper Limit Notation:

$$\lim_{x \to \infty} f(x) = 1$$

$$\lim_{x \to -\infty} f(x) = 1$$

WARM-UP DAY 7 ANSWERS

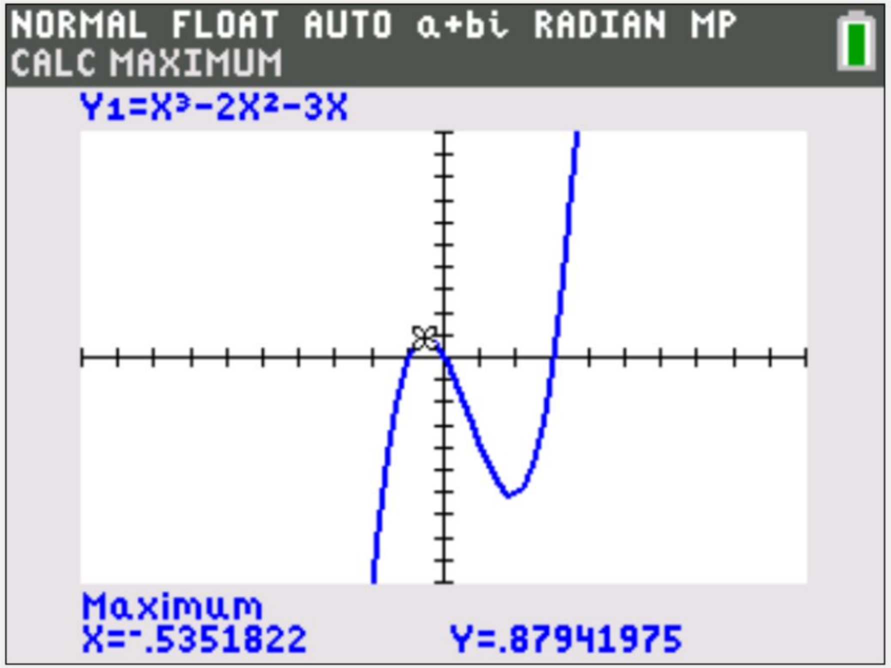
Find the requested information

- 2)

$$f(x) = x^3 - x^2 - 2x$$

Local Max 0.88 occurs at x = -0.54

Local Min -6.06 occurs at x = 1.87



Increasing: $(-\infty, -0.54) \cup (1.87, \infty)$

Decreasing: $(-0.54, 1.87)$

Domain: $(-\infty, \infty)$

Range: $(-\infty, \infty)$

End Behavior using Proper Limit Notation:

$$\lim_{x \to \infty} f(x) = \infty$$

$$\lim_{x \to -\infty} f(x) = -\infty$$

HW DAY 7:

PACKET P.9 AND HANDOUT INTRO TO LIMITS (ON WEBSITE)

NOTES DAY 7: LIMITS PART 2

LIMITS TO CERTAIN VALUES & LIMITS AT LEFT-HAND AND RIGHT-HAND SIDES

USE “Day 7 Graphs for Notes” handout in Power Point area of Website for help taking notes today ☺

REMEMBER THE DEFINITION OF A LIMIT

- If f(x) becomes arbitrarily close to a unique number L as x approaches c from either side, the **limit of f(x)** as x approaches c is L.

- L is a y-value! c is an x-value!

$$\lim_{x \to c} f(x) = L$$

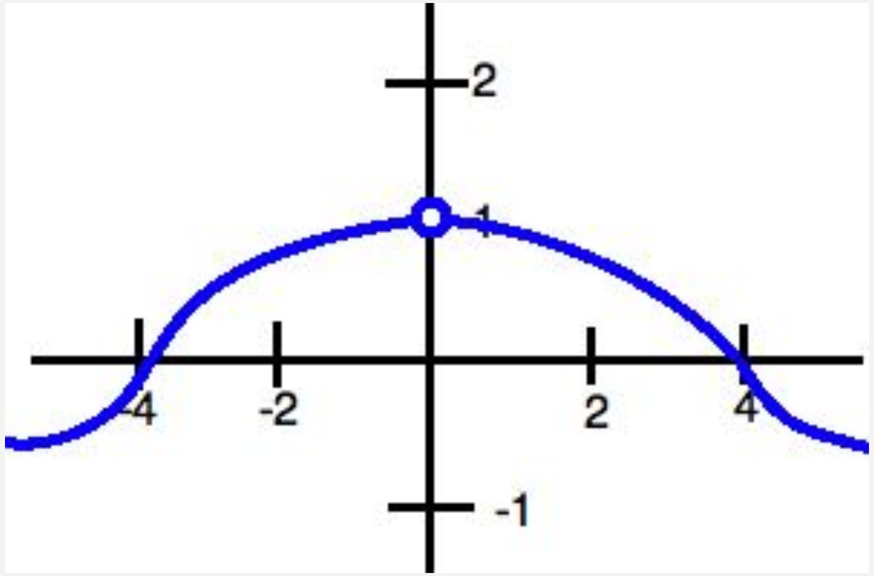
TAKE A LOOK AT THIS GRAPH…

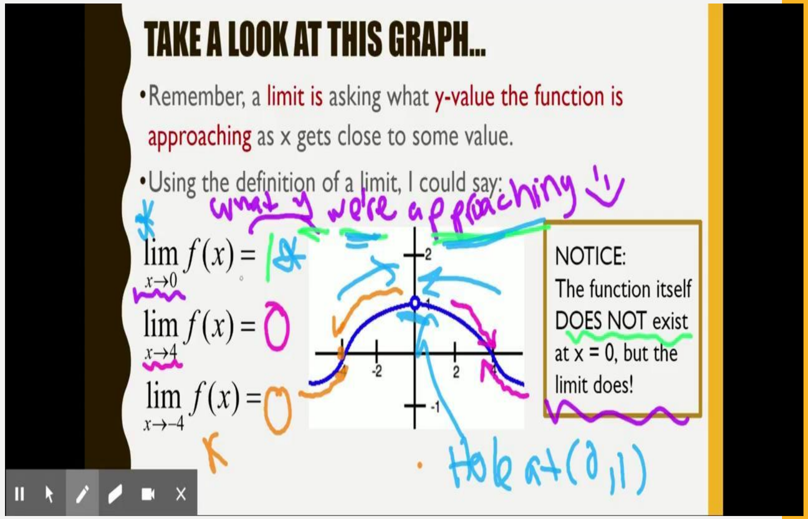
- Remember, a **limit is** asking what **y-value the function is approaching** as x gets close to some value.
- Using the definition of a limit, I could say:

$$\lim_{x \to 0} f(x) =$$

$$\lim_{x \to 4} f(x) =$$

$$\lim_{x \to -4} f(x) =$$





NOTICE:
The function itself DOES NOT exist at x = 0, but the limit does!

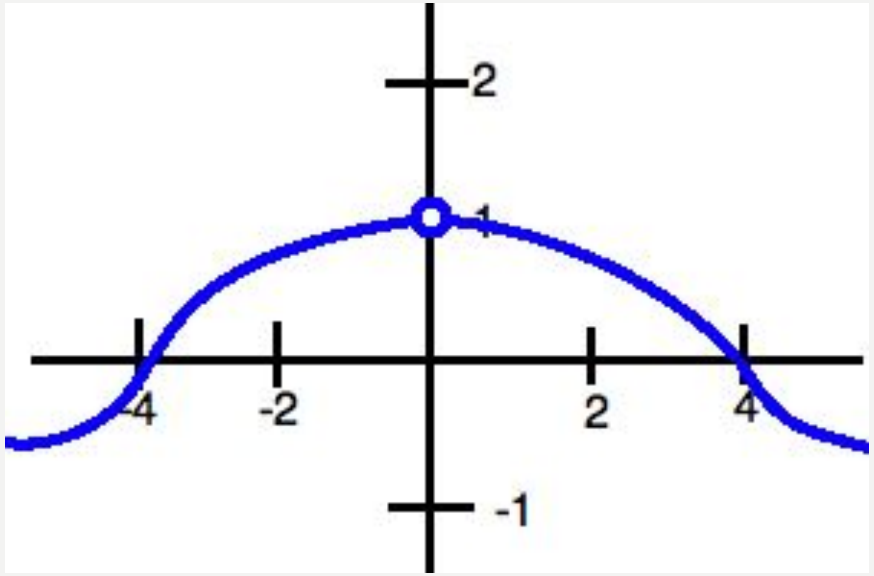
TAKE A LOOK AT THIS GRAPH… ANSWERS

- Remember, a limit is asking what y-value the function is approaching as x gets close to some value.
- Using the definition of a limit, I could say:

$$\lim_{x \to 0} f(x) = 1$$

$$\lim_{x \to 4} f(x) = 0$$

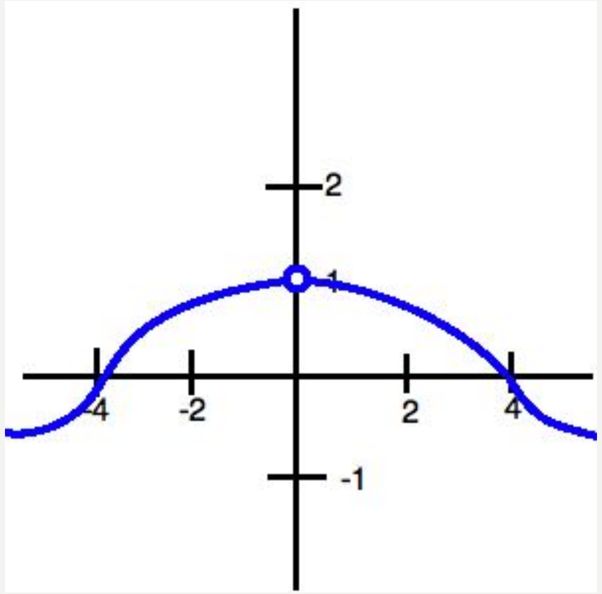
$$\lim_{x \to -4} f(x) = 0$$



NOTICE:
The function itself DOES NOT exist at x = 0, but the limit does!

SOOOO....

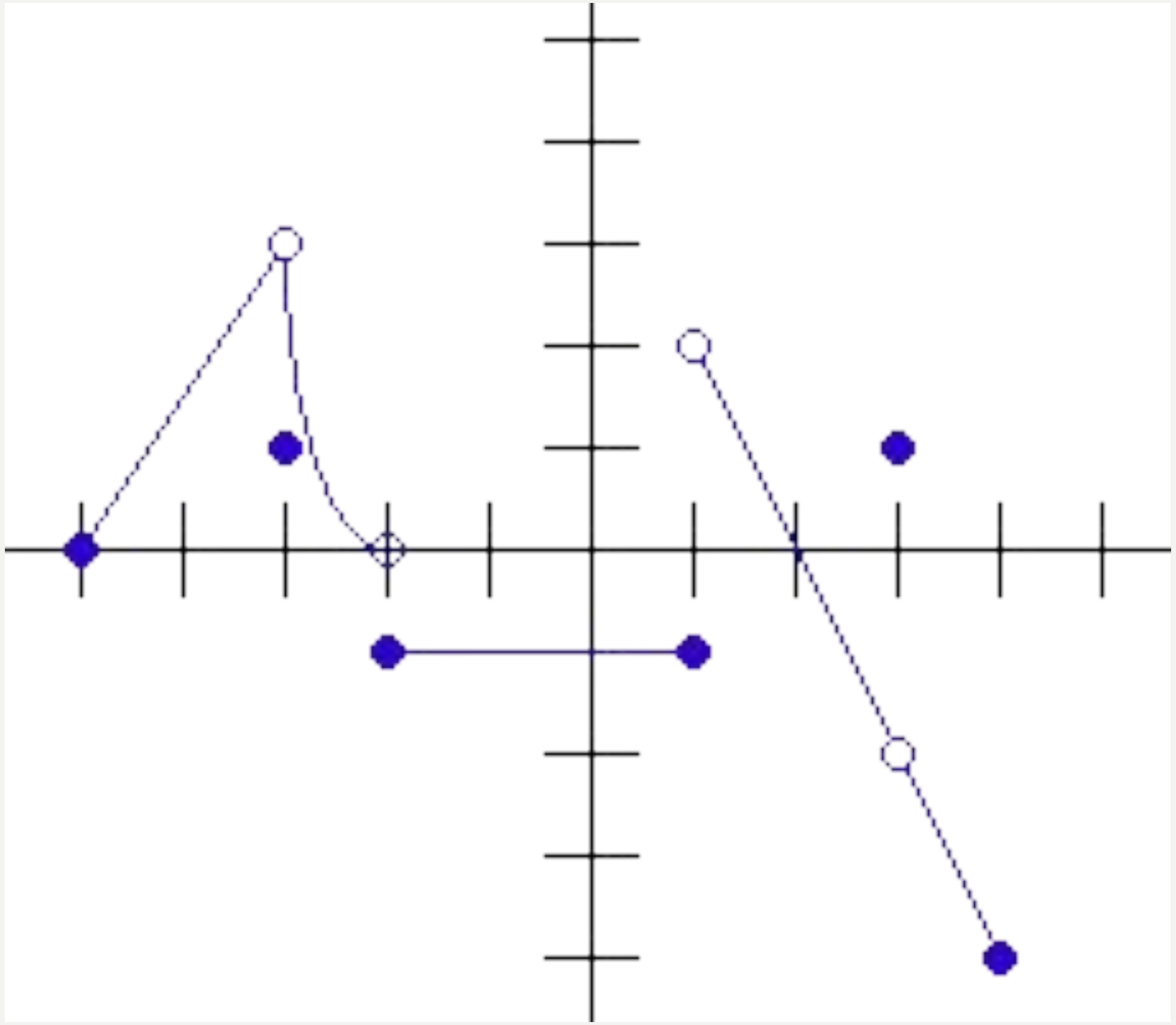
The existence/non-existence of f(x) at x = c has NO BEARING on the existence of the limit of f(x) as x approaches c.

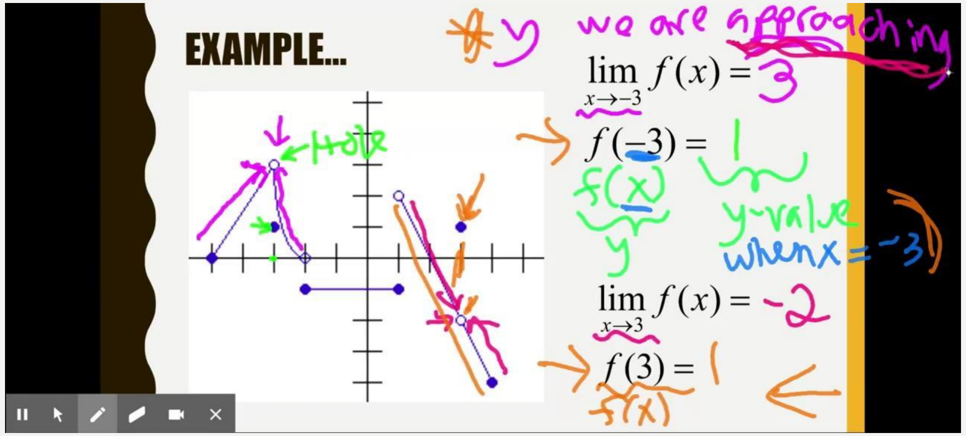


EXAMPLE…

$$\lim_{x \to -3} f(x) =$$

$$f(-3) =$$

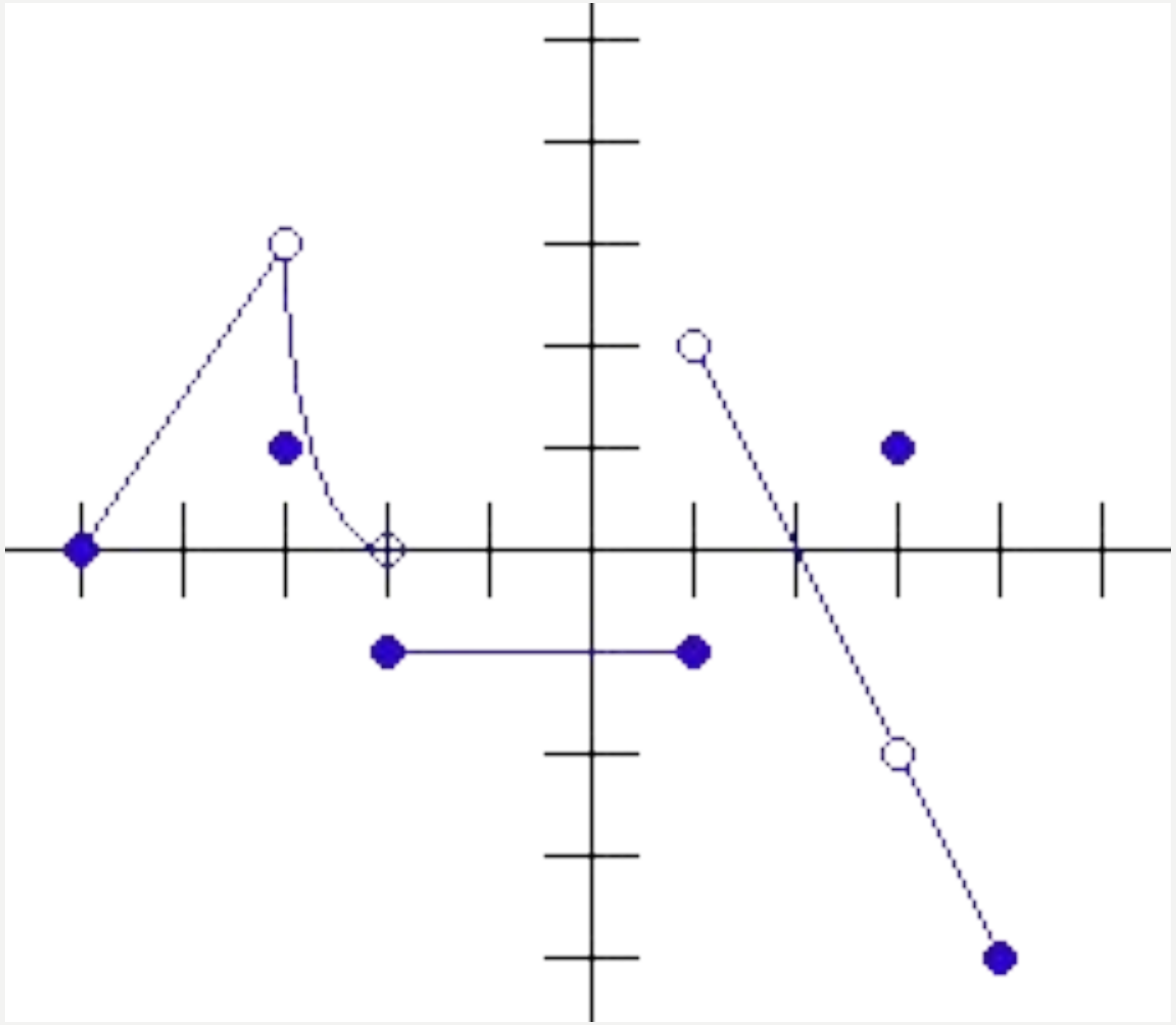




$$\lim_{x \to 3} f(x) =$$

$$f(3) =$$

EXAMPLE… ANSWERS



$$\lim_{x \to -3} f(x) = 3$$

$$f(-3) = 1$$

$$\lim_{x \to 3} f(x) = -2$$

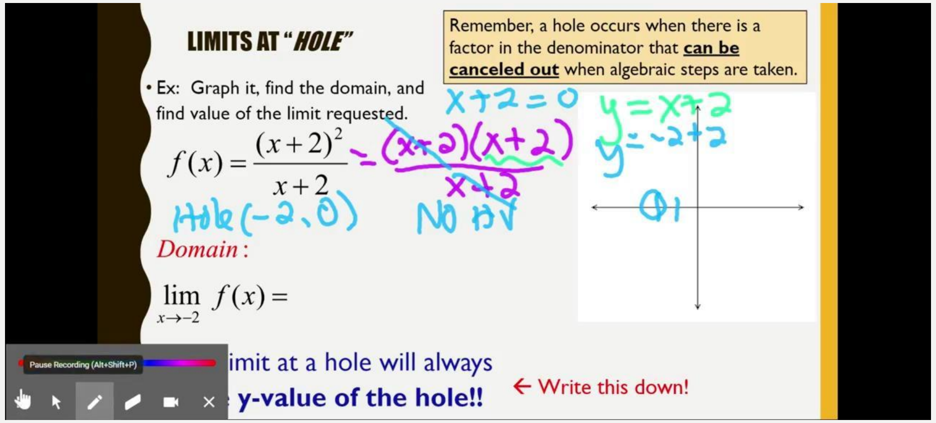
$$f(3) = 1$$

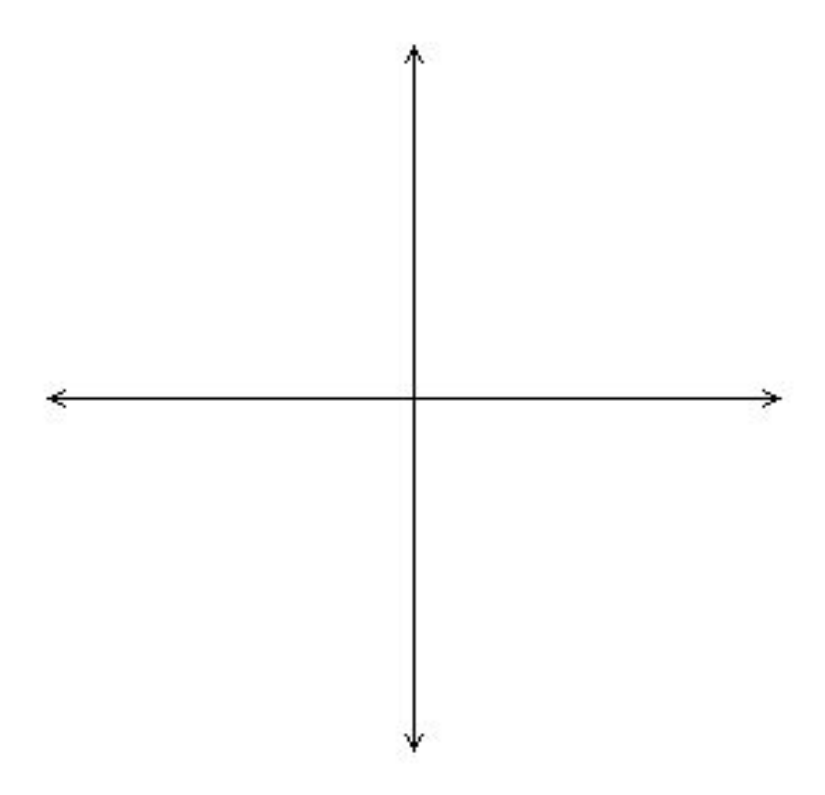
LIMITS AT "*HOLE*"

Remember, a hole occurs when there is a factor in the denominator that **<u>can be canceled out</u>** when algebraic steps are taken.

- Ex: Graph it, find the domain, and find value of the limit requested.

$$f(x)=\frac{(x+2)^2}{x+2}$$





Domain :

$$\lim_{x\to -2} f(x)=$$

*The limit at a hole will always be the **y-value of the hole!!**

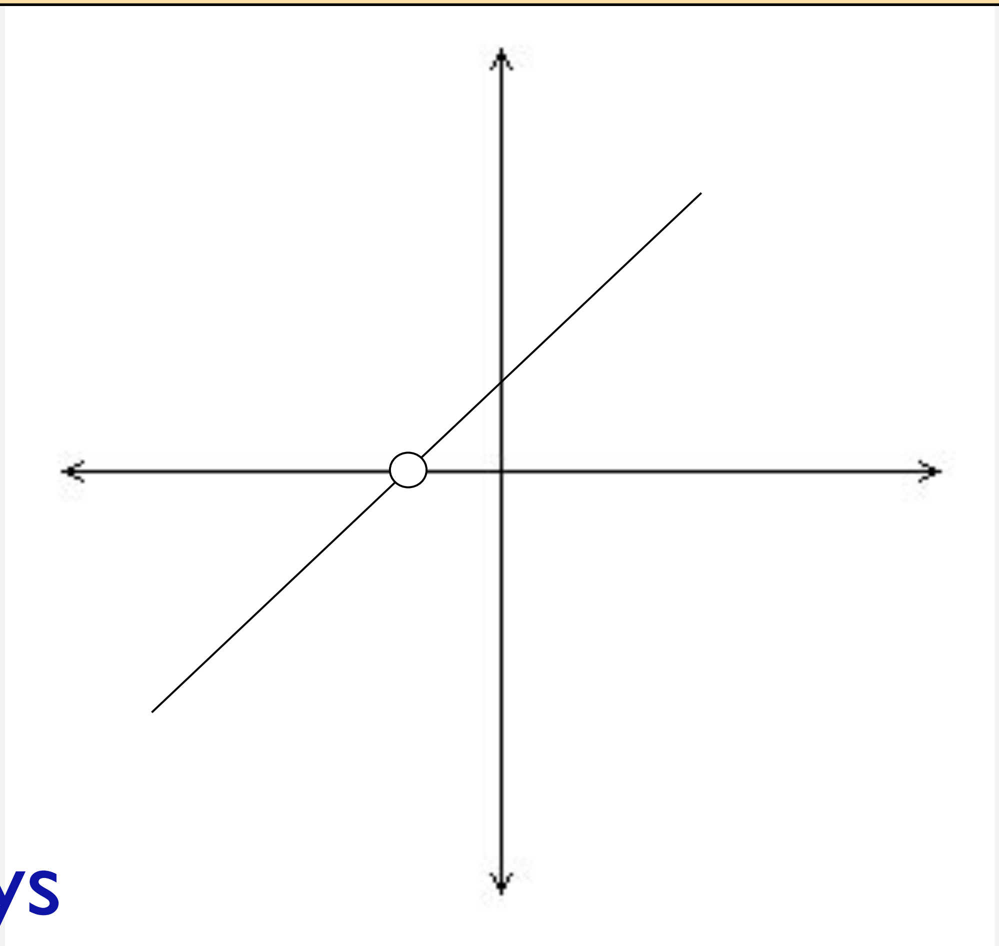
□ Write this down!

LIMITS AT "*HOLE*"
ANSWERS

Remember, a hole occurs when there is a factor in the denominator that **can be canceled out** when algebraic steps are taken.

Ex: Graph it and write the domain.

$$f(x) = \frac{(x+2)^2}{x+2}$$



$$Domain: (-\infty, -2) \cup (-2, \infty)$$

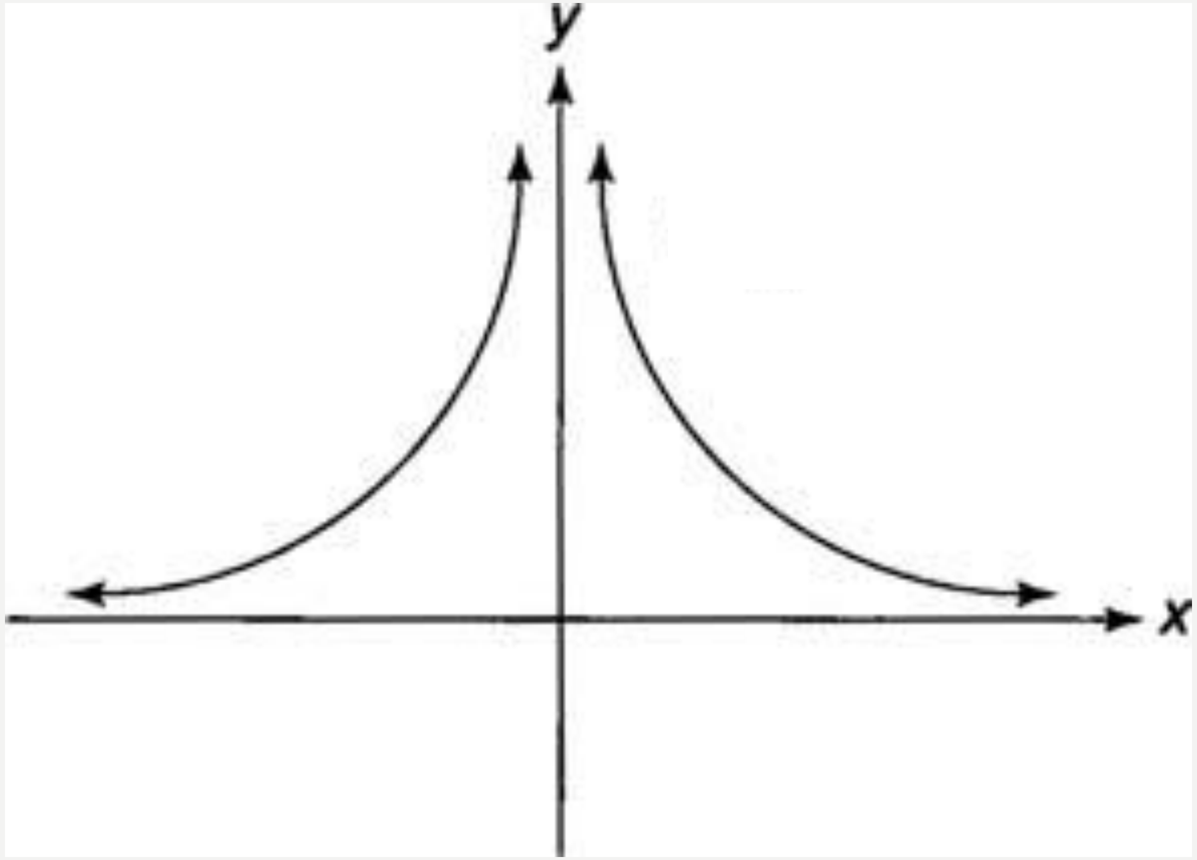
$$\lim_{x \to -2} f(x) = 0$$

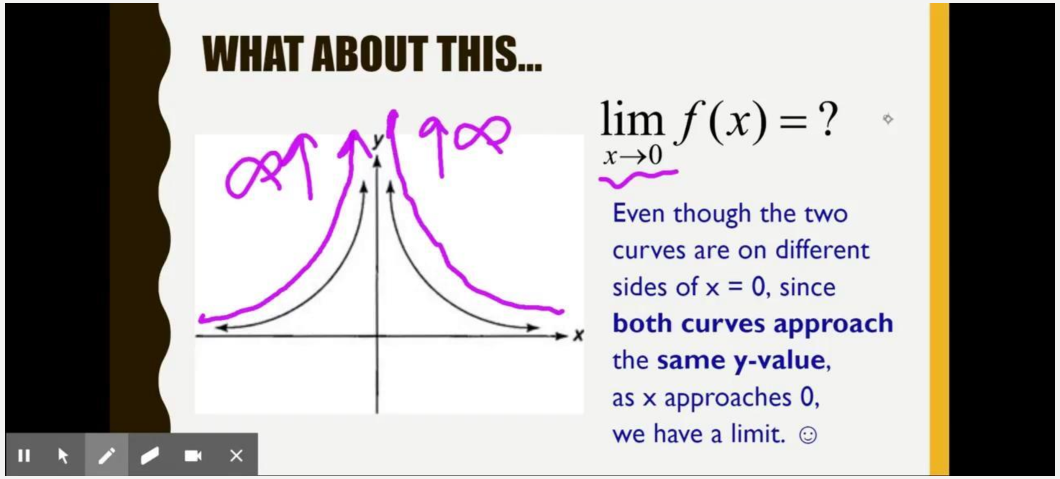
*The limit at a hole will always be the **y-value of the hole!!**

 Write this down!

WHAT ABOUT THIS…

$$\lim_{x \to 0} f(x) = ?$$

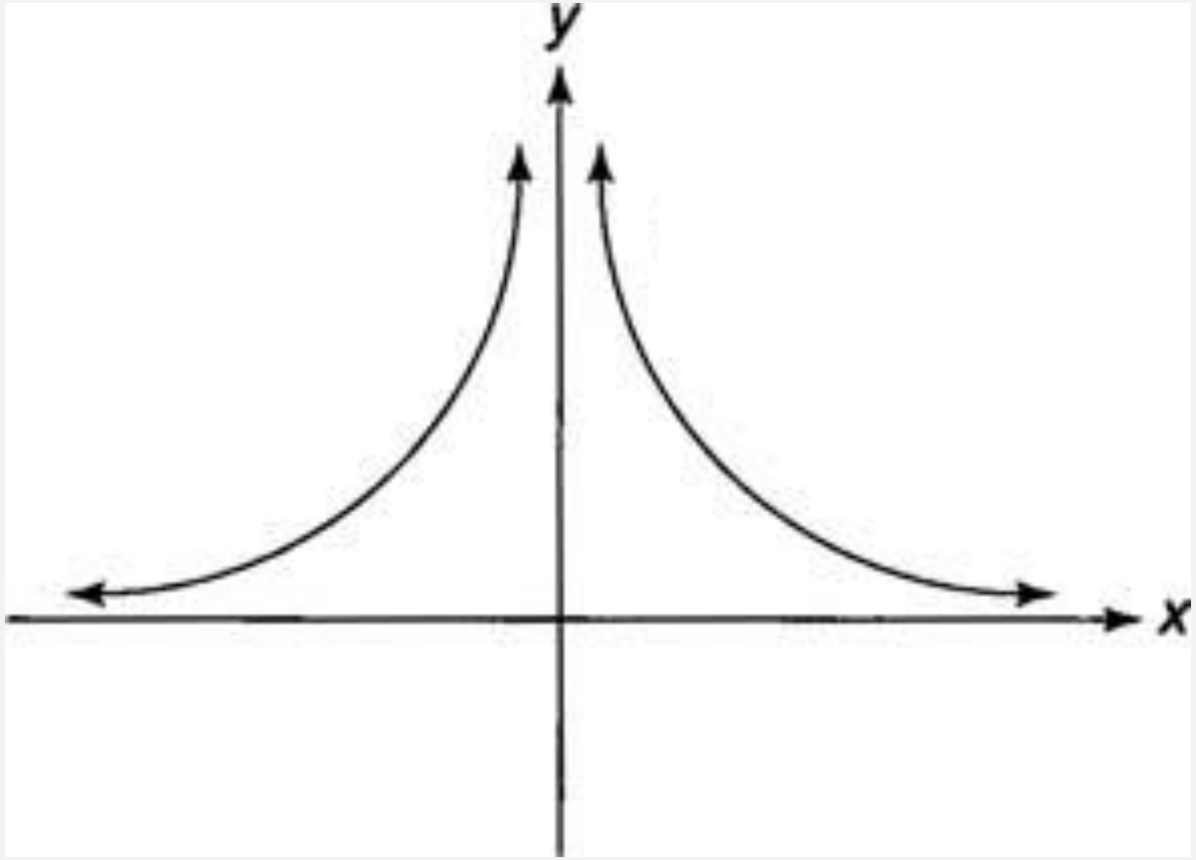




Even though the two curves are on different sides of x = 0, since **both curves approach** the **same y-value**, as x approaches 0, we have a limit. ☺

WHAT ABOUT THIS… ANSWERS

$$\lim_{x \to 0} f(x) = ? \quad \infty$$



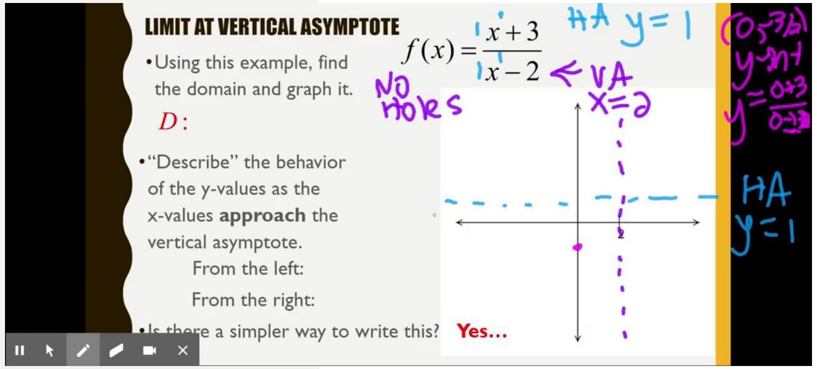
Even though the two curves are on different sides of x = 0, since **both curves approach** the **same y-value**, as x approaches 0, we have a limit. ☺

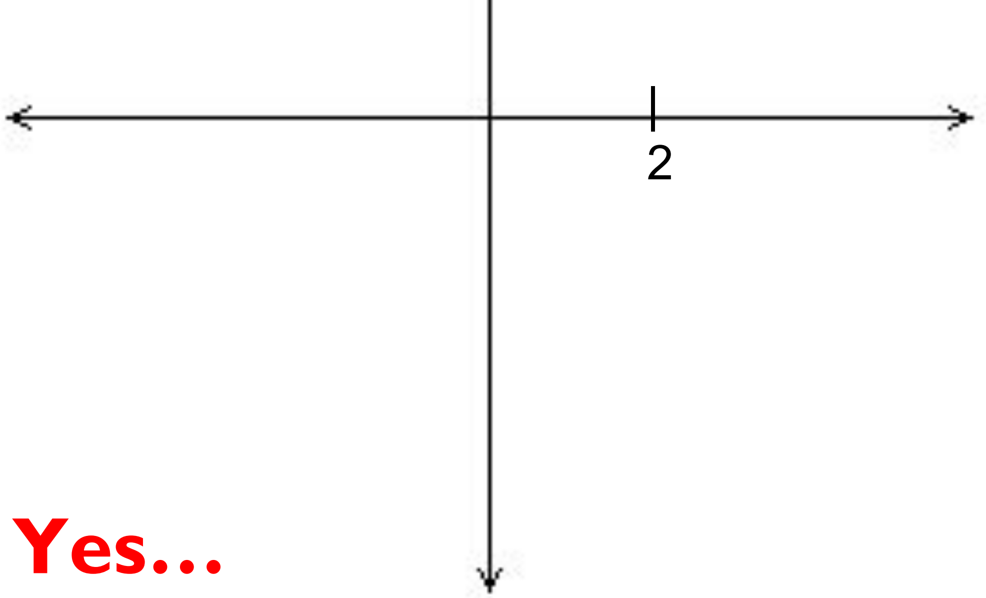
LIMIT AT VERTICAL ASYMPTOTE

$$f(x) = \frac{x+3}{x-2}$$

- Using this example, find the domain and graph it.

$D:$





- "Describe" the behavior of the y-values as the x-values **approach** the vertical asymptote.

    From the left:

    From the right:

- Is there a simpler way to write this? **Yes...**

LIMIT AT VERTICAL ASYMPTOTE
ANSWERS

$$f(x) = \frac{x+3}{x-2}$$

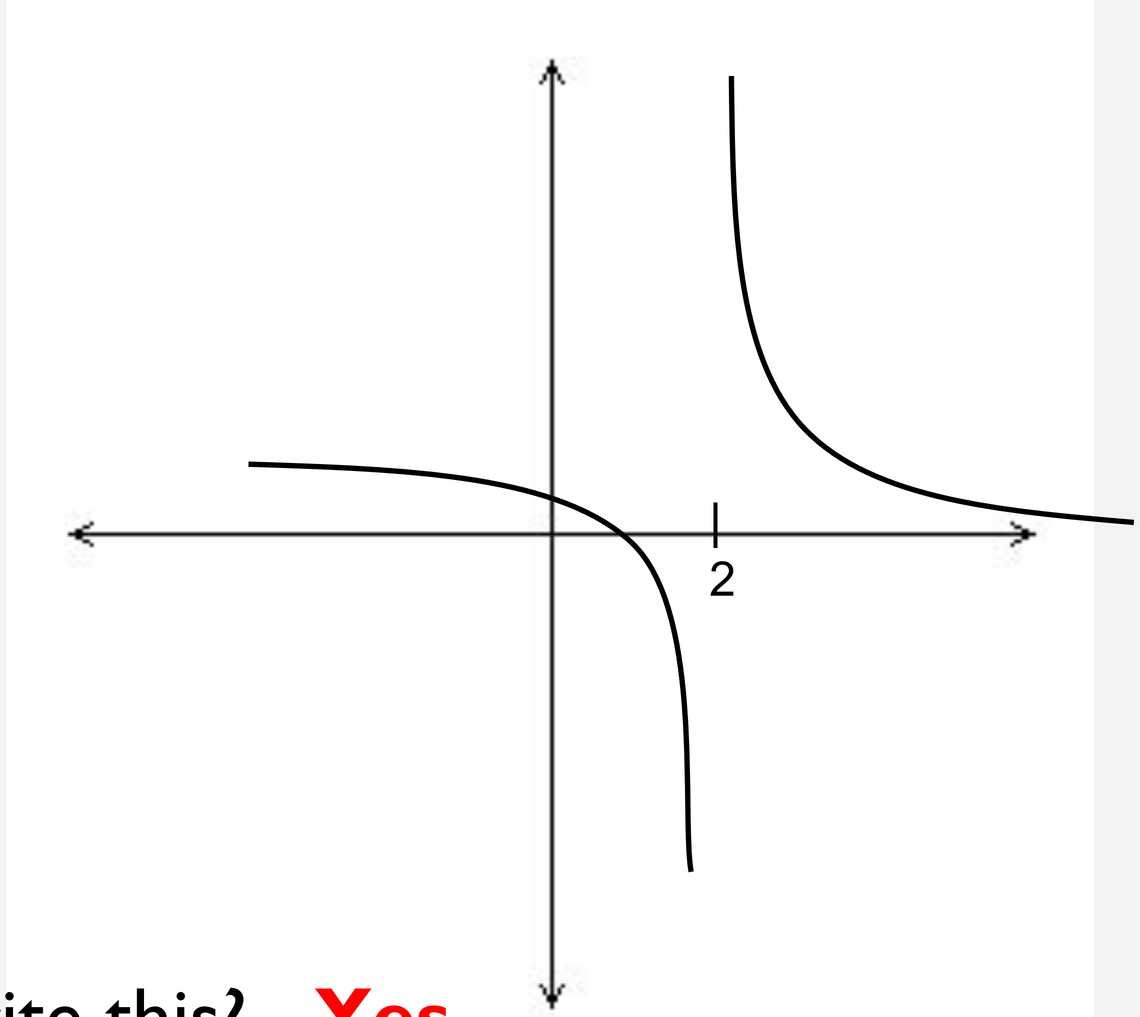
- Using this example, find the domain and graph it.

$$D : (-\infty, 2) \cup (2, \infty)$$

- "Describe" the behavior of the y-values as the x-values **approach** the vertical asymptote.

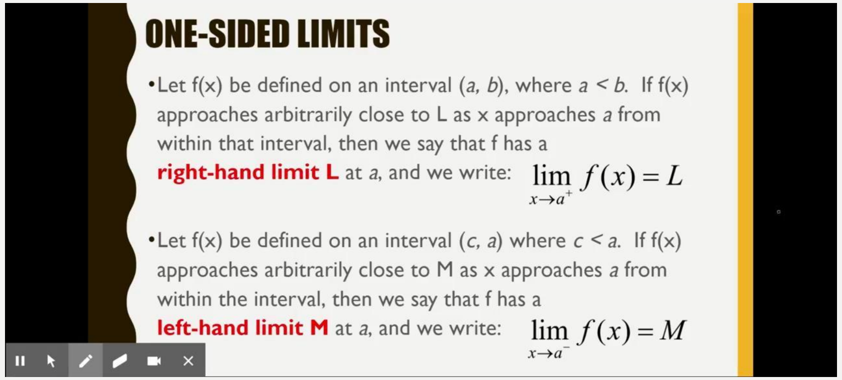
From the left: $-\infty$

From the right: ∞



Is there a simpler way to write this? **Yes…**

ONE-SIDED LIMITS



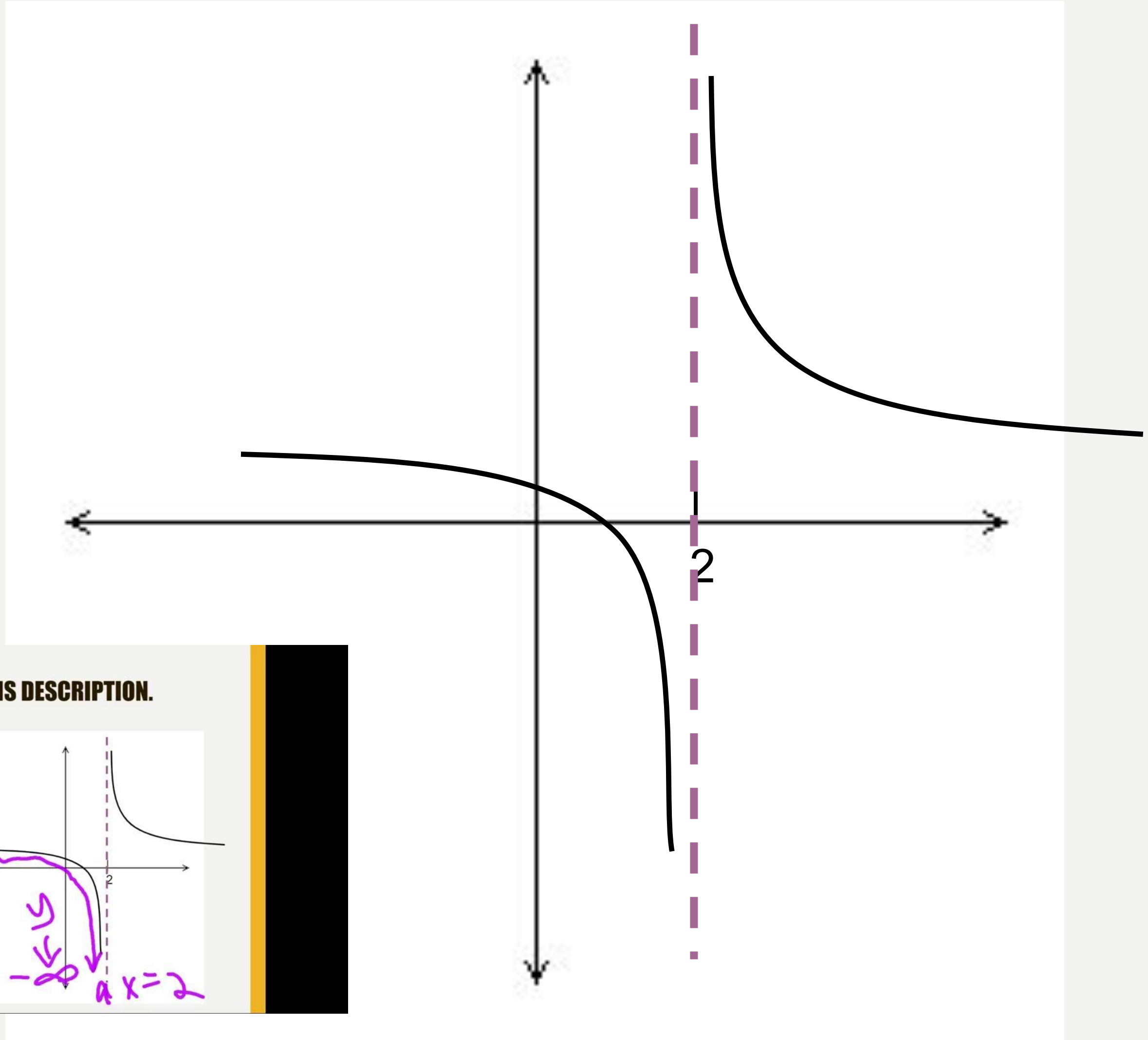
- Let f(x) be defined on an interval (*a*, *b*), where *a* < *b*. If f(x) approaches arbitrarily close to L as x approaches *a* from within that interval, then we say that f has a **right-hand limit L** at *a*, and we write: $\lim_{x\to a^+} f(x) = L$

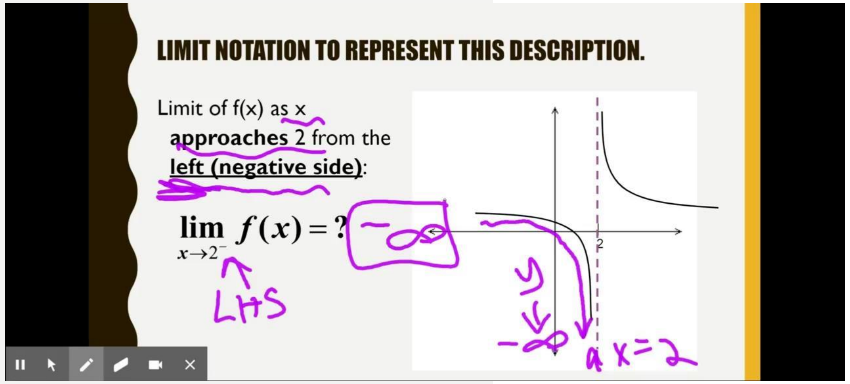
- Let f(x) be defined on an interval (*c*, *a*) where *c* < *a*. If f(x) approaches arbitrarily close to M as x approaches *a* from within the interval, then we say that f has a **left-hand limit M** at *a*, and we write: $\lim_{x\to a^-} f(x) = M$

LIMIT NOTATION TO REPRESENT THIS DESCRIPTION.

Limit of f(x) as x **approaches** 2 from the **left (negative side)**:

$$\lim_{x \to 2^-} f(x) = ?$$



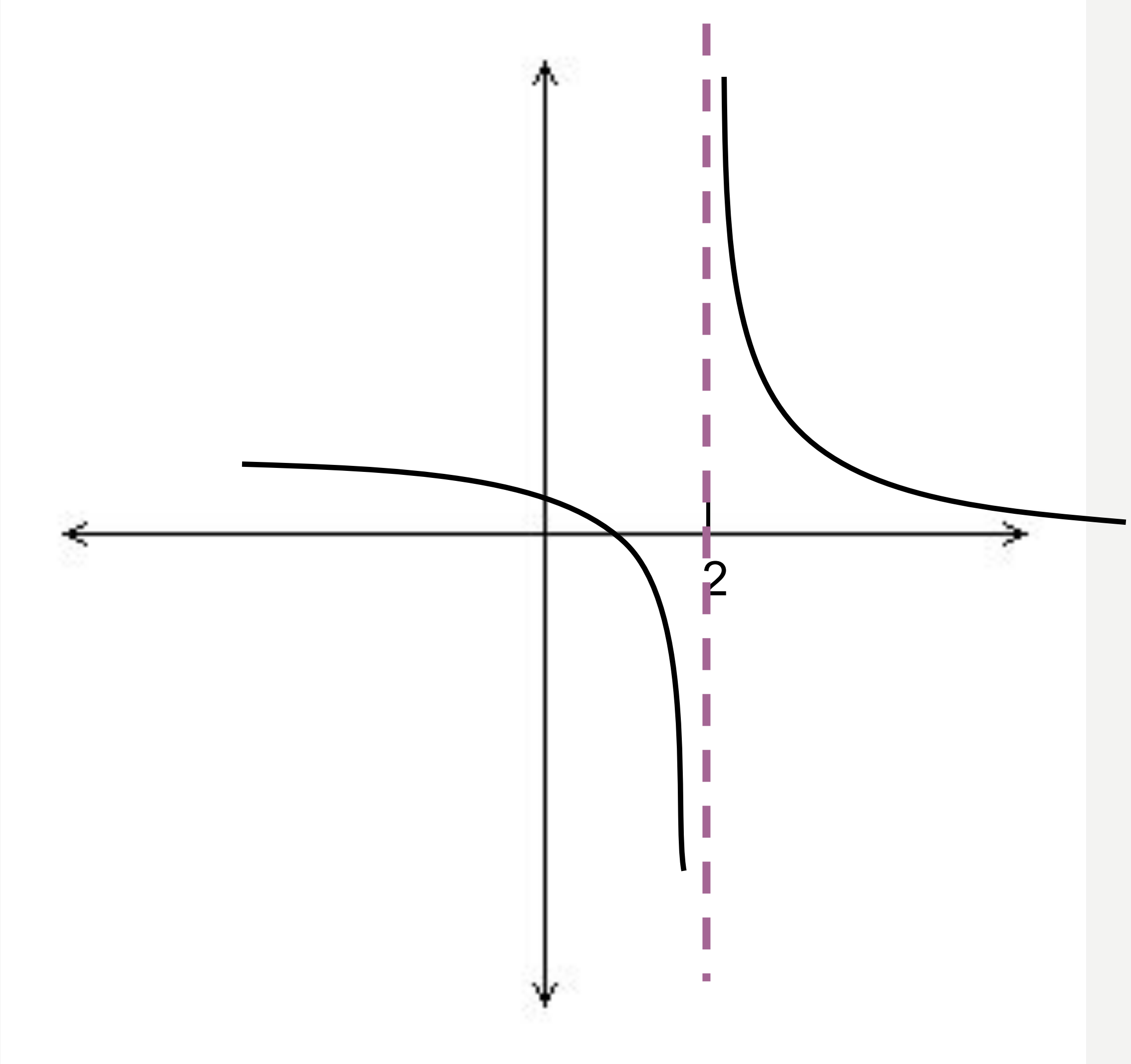


LIMIT NOTATION TO REPRESENT THIS DESCRIPTION. ANSWER

Limit of f(x) as x **approaches** 2 from the **left (negative side)**:

$$\lim_{x \to 2^-} f(x) = ?$$

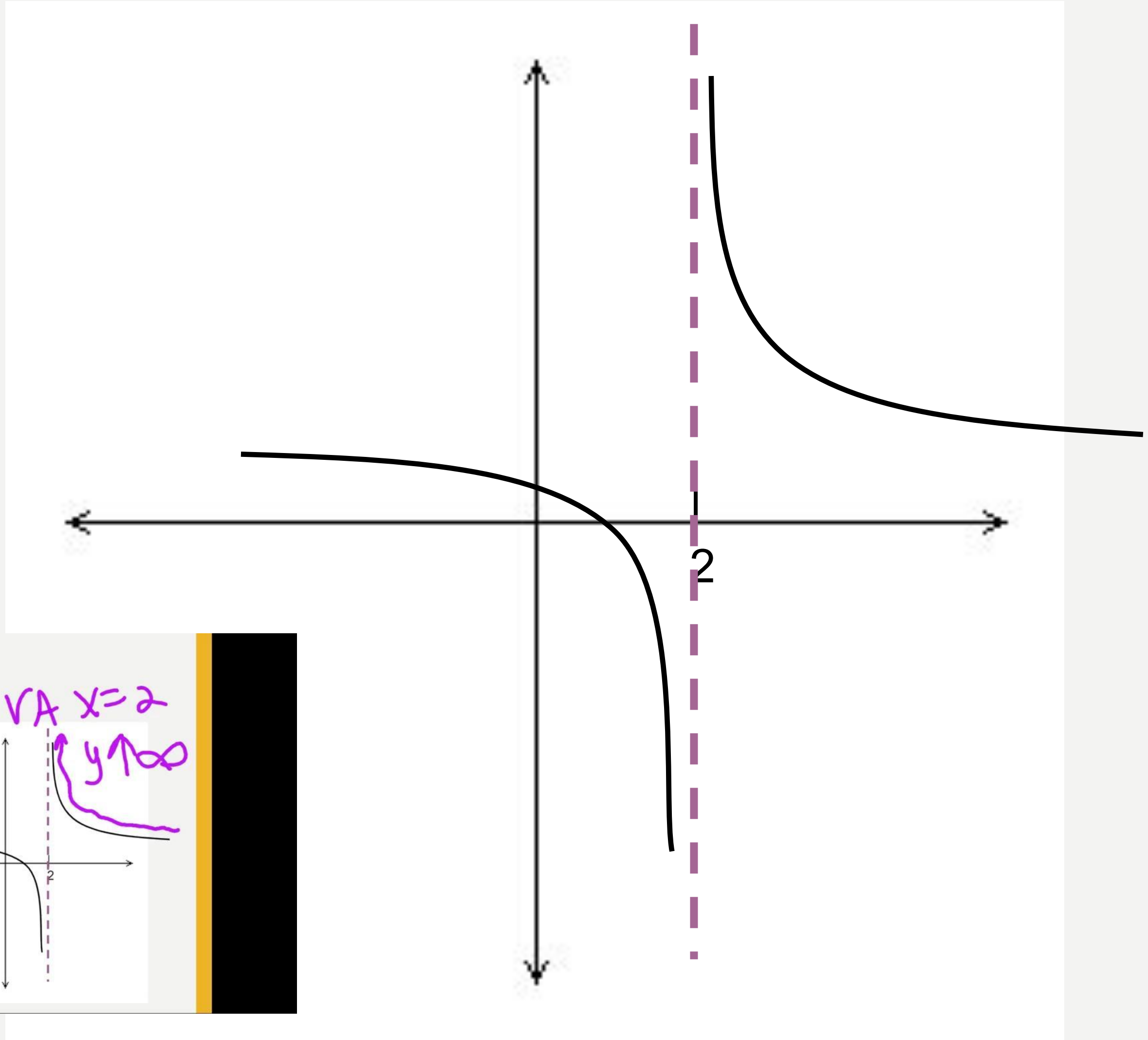
$$\lim_{x \to 2^-} f(x) = -\infty$$

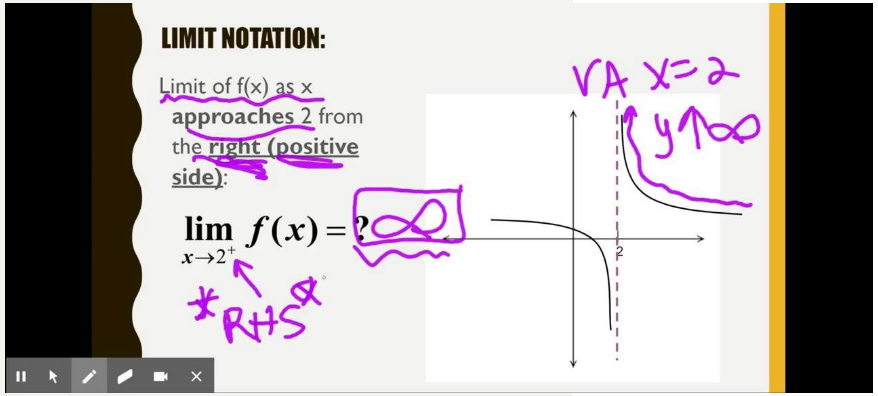


LIMIT NOTATION:

Limit of f(x) as x **approaches** 2 from the **right (positive side)**:

$$\lim_{x \to 2^{+}} f(x) = ?$$



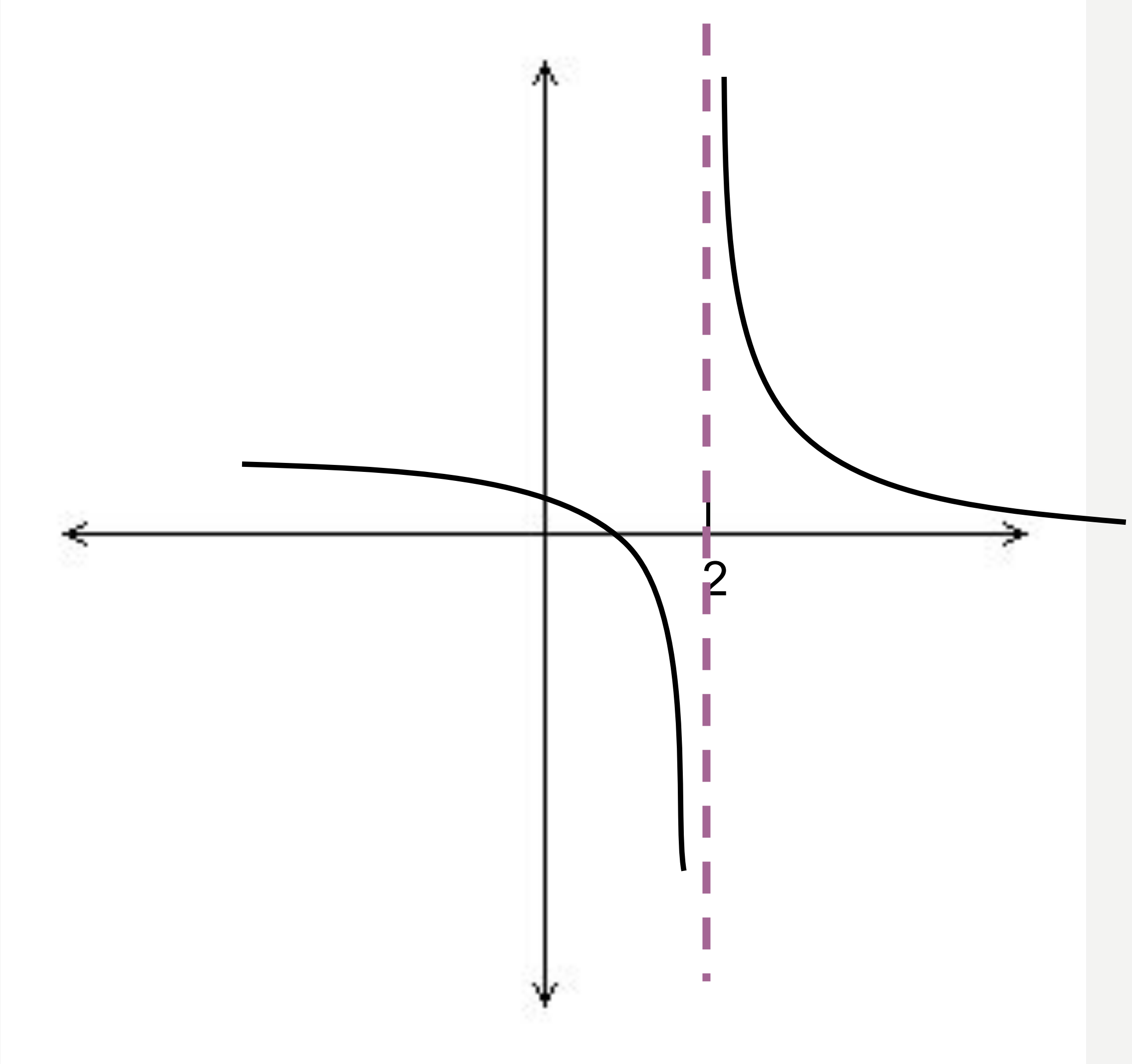


LIMIT NOTATION: ANSWER

Limit of f(x) as x **approaches** 2 from the **right (positive side)**:

$$\lim_{x \to 2^+} f(x) = ?$$

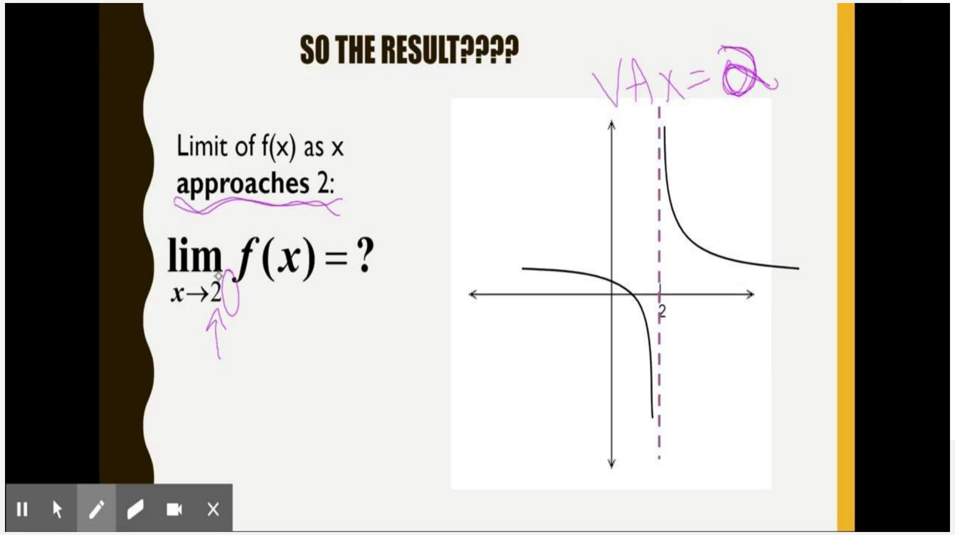
$$\lim_{x \to 2^+} f(x) = \infty$$

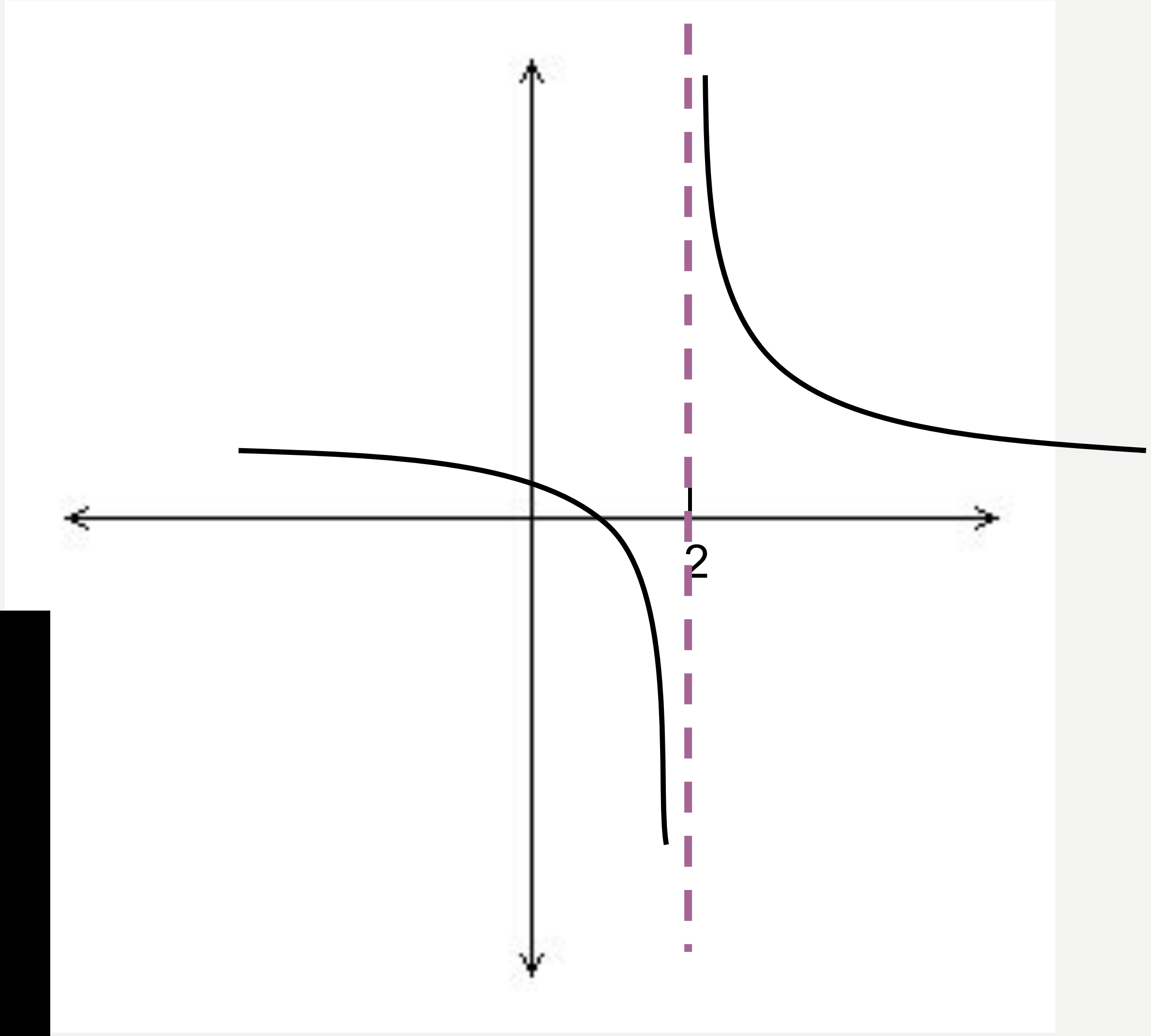


SO THE RESULT????

Limit of f(x) as x **approaches** 2:

$$\lim_{x \to 2} f(x) = ?$$

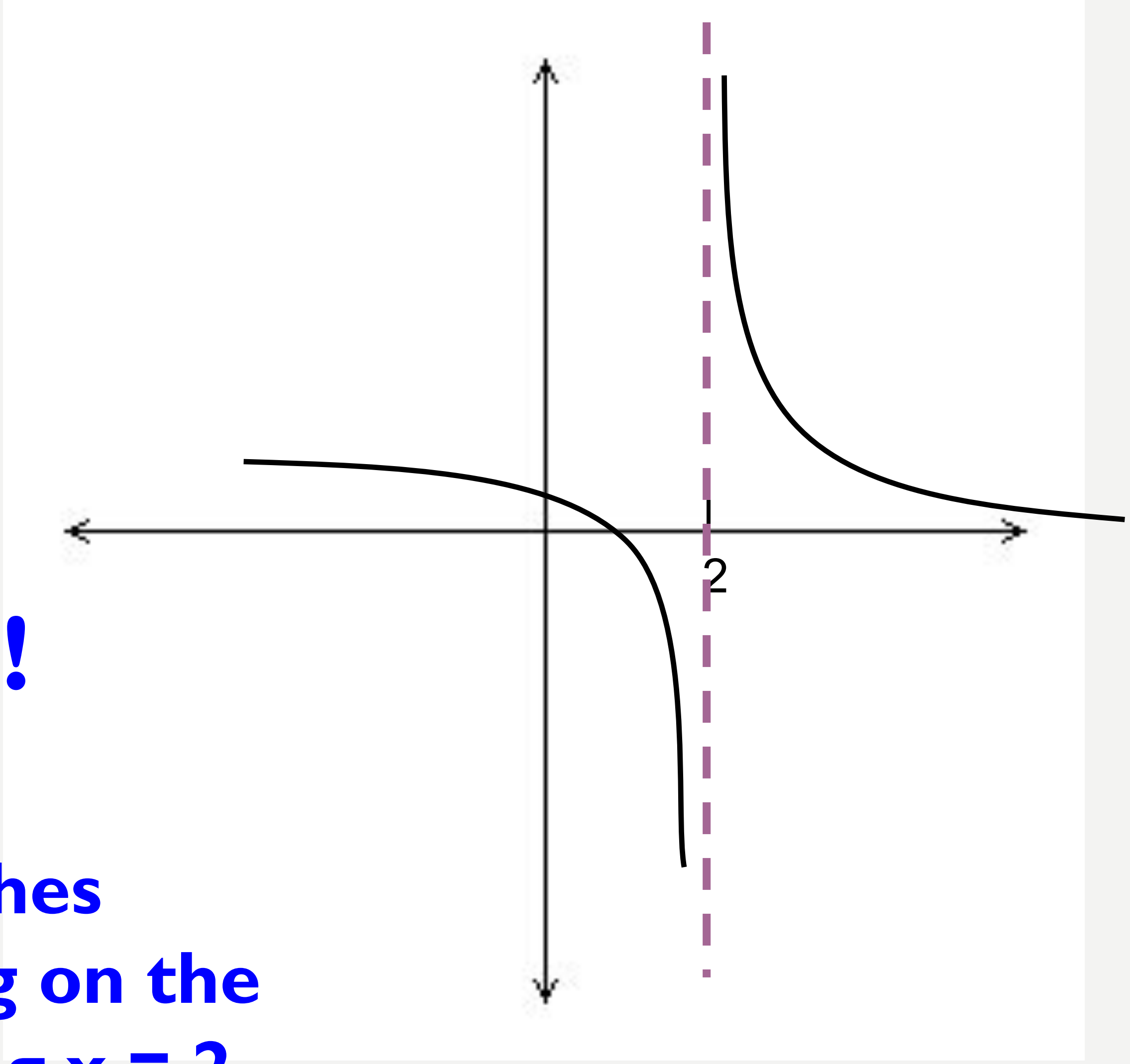




SO THE RESULT???? ANSWER

Limit of f(x) as x approaches 2:

$$\lim_{x \to 2} f(x) = ?$$



DOES NOT EXIST!

(a.k.a. "DNE")

Because the graph approaches different y-values depending on the direction you're approaching x = 2

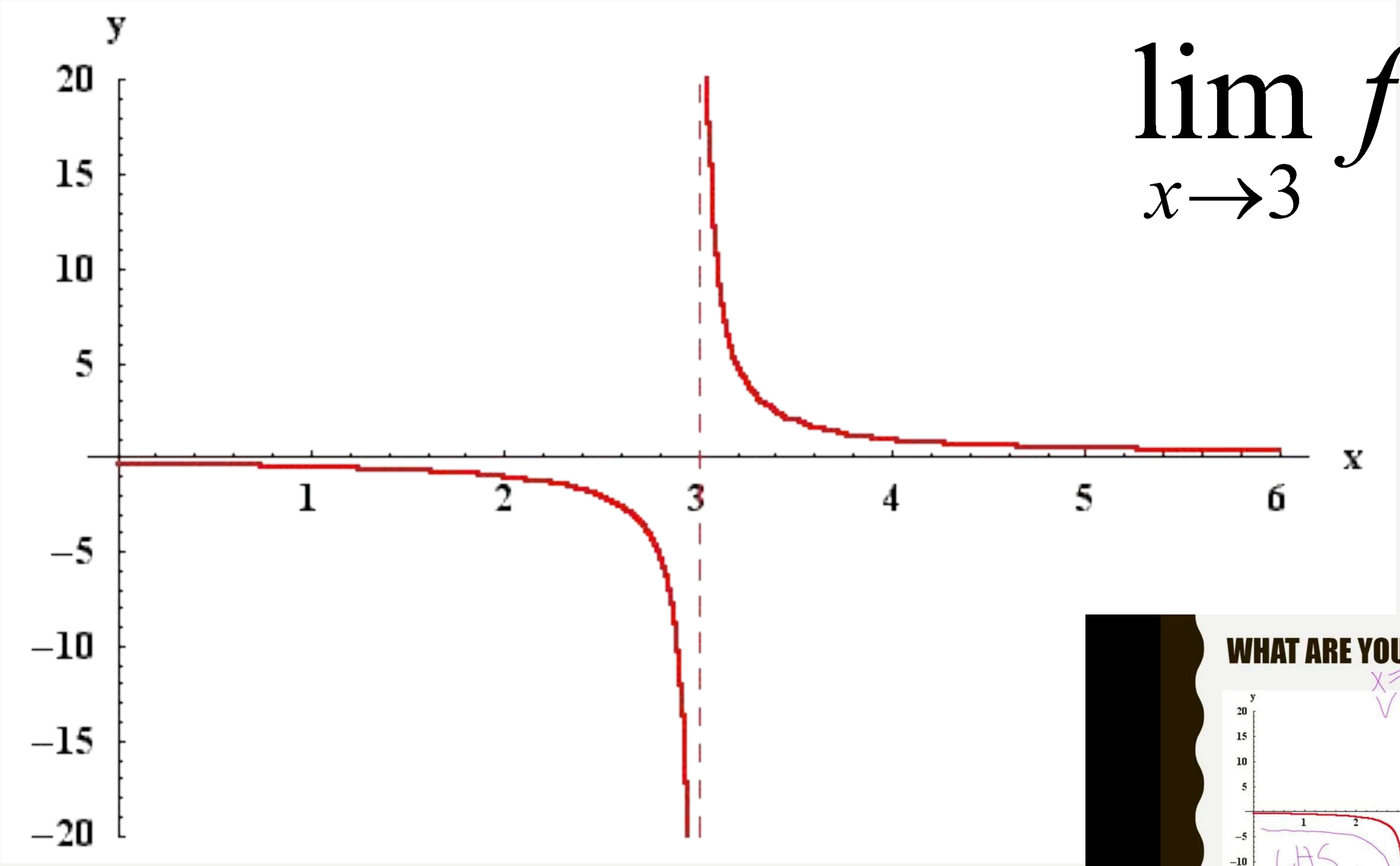
SO, A LIMIT DOES NOT EXIST (DNE) IF:

- f(x) approaches a different value from the right side than from the left.
- f(x) oscillates between two fixed values as x approaches c.

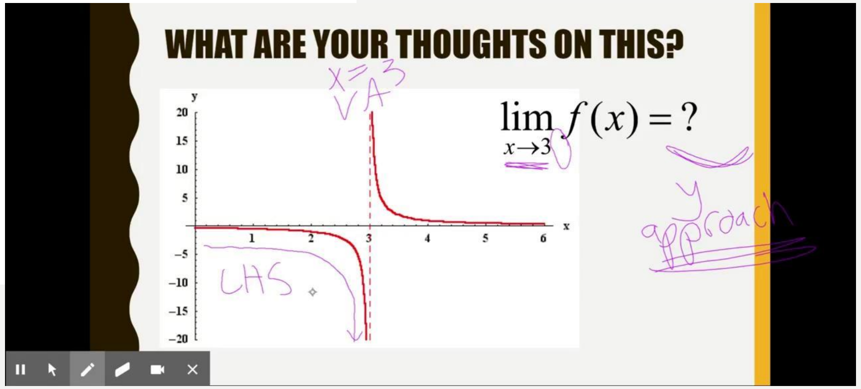
- Ex: Graph y = sin(x). Find limit as x approaches infinity.

DOES NOT EXIST!

WHAT ARE YOUR THOUGHTS ON THIS?

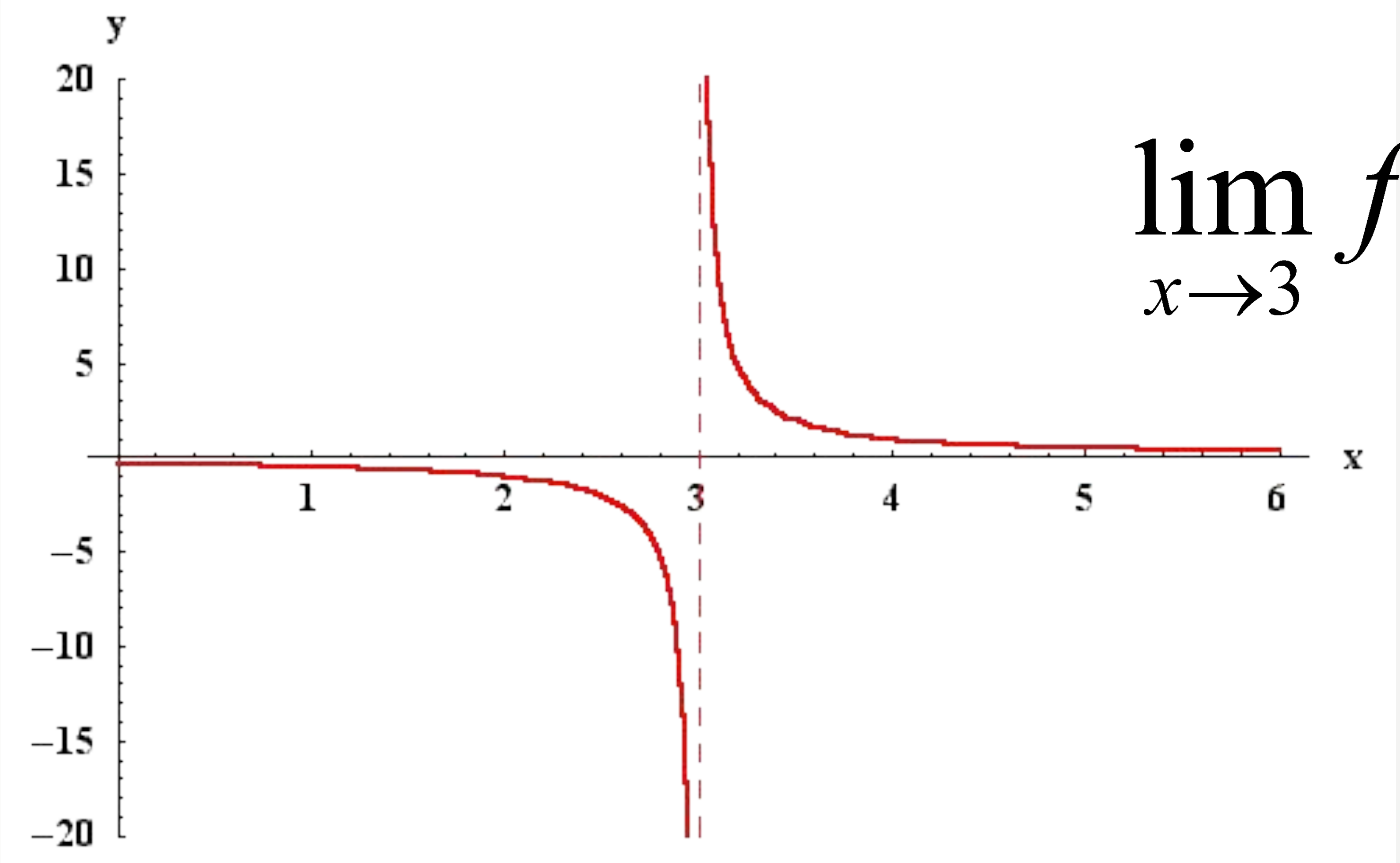


$$\lim_{x \to 3} f(x) = ?$$



WHAT ARE YOUR THOUGHTS ON THIS?

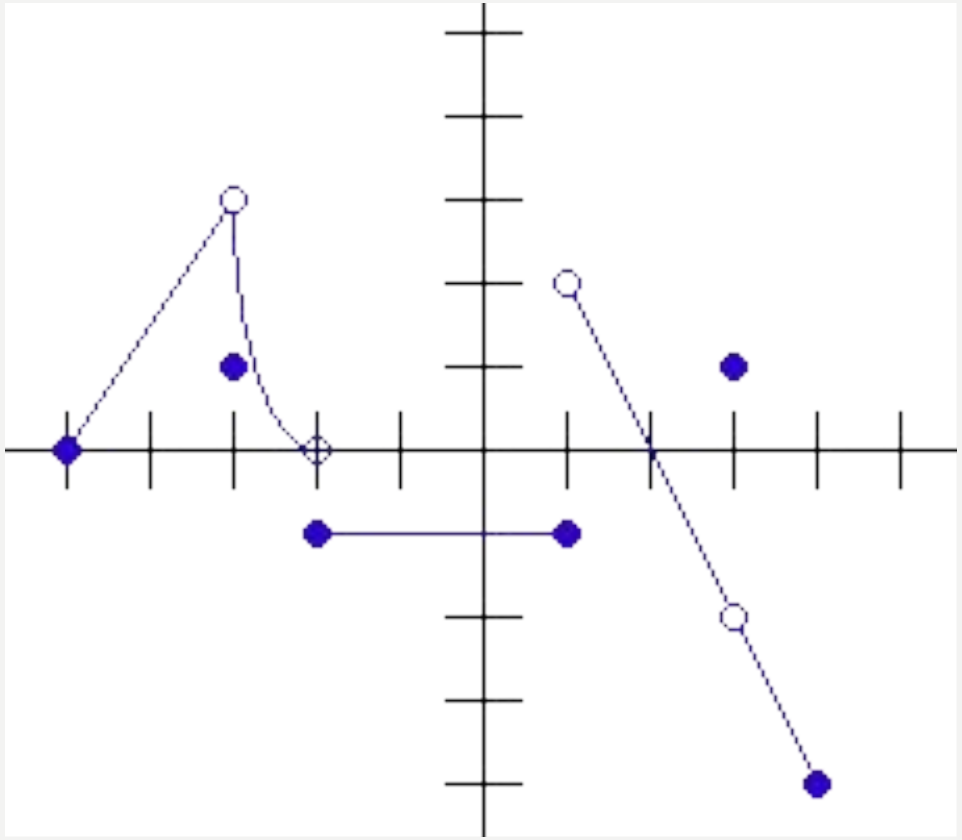
ANSWER



$$\lim_{x \to 3} f(x) = ?$$

DNE

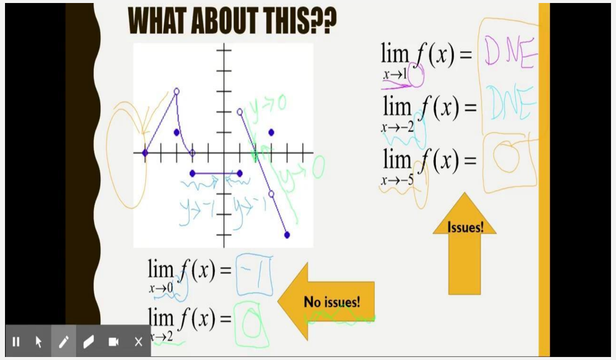
WHAT ABOUT THIS??



$$\lim_{x \to 1} f(x) =$$

$$\lim_{x \to -2} f(x) =$$

$$\lim_{x \to -5} f(x) =$$



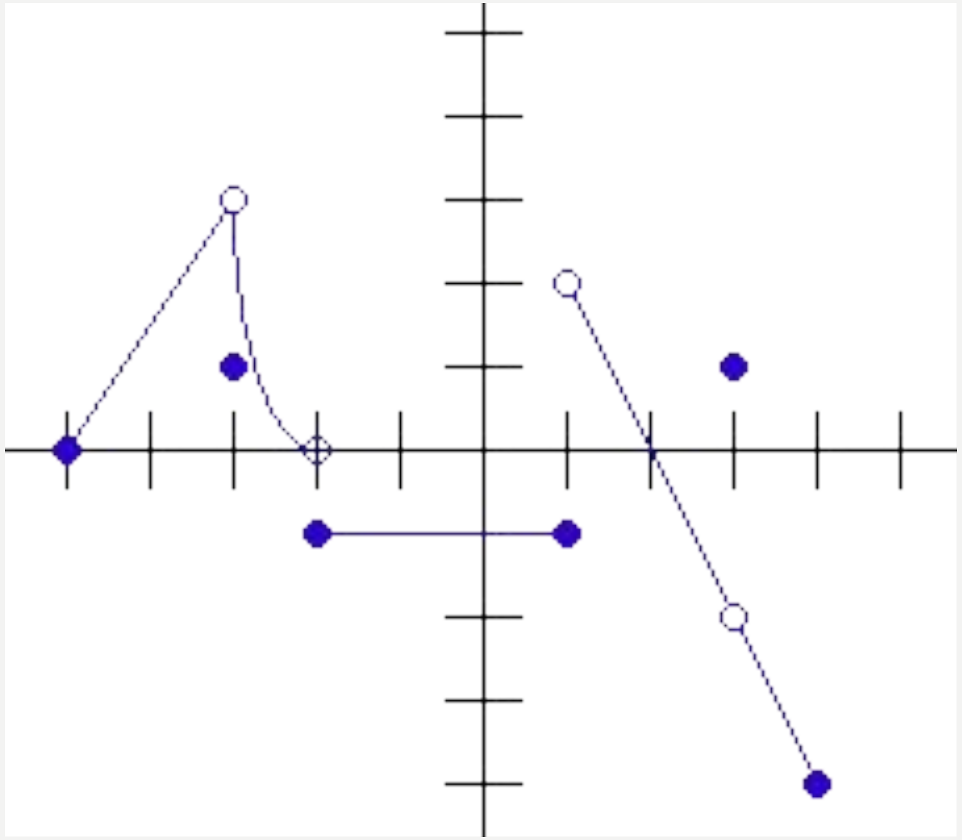
Issues!

$$\lim_{x \to 0} f(x) =$$

$$\lim_{x \to 2} f(x) =$$

No issues!

WHAT ABOUT THIS??



$$\lim_{x\to 0} f(x) = \textbf{-1}$$

$$\lim_{x\to 2} f(x) = \textbf{0}$$

No issues!

ANSWERS

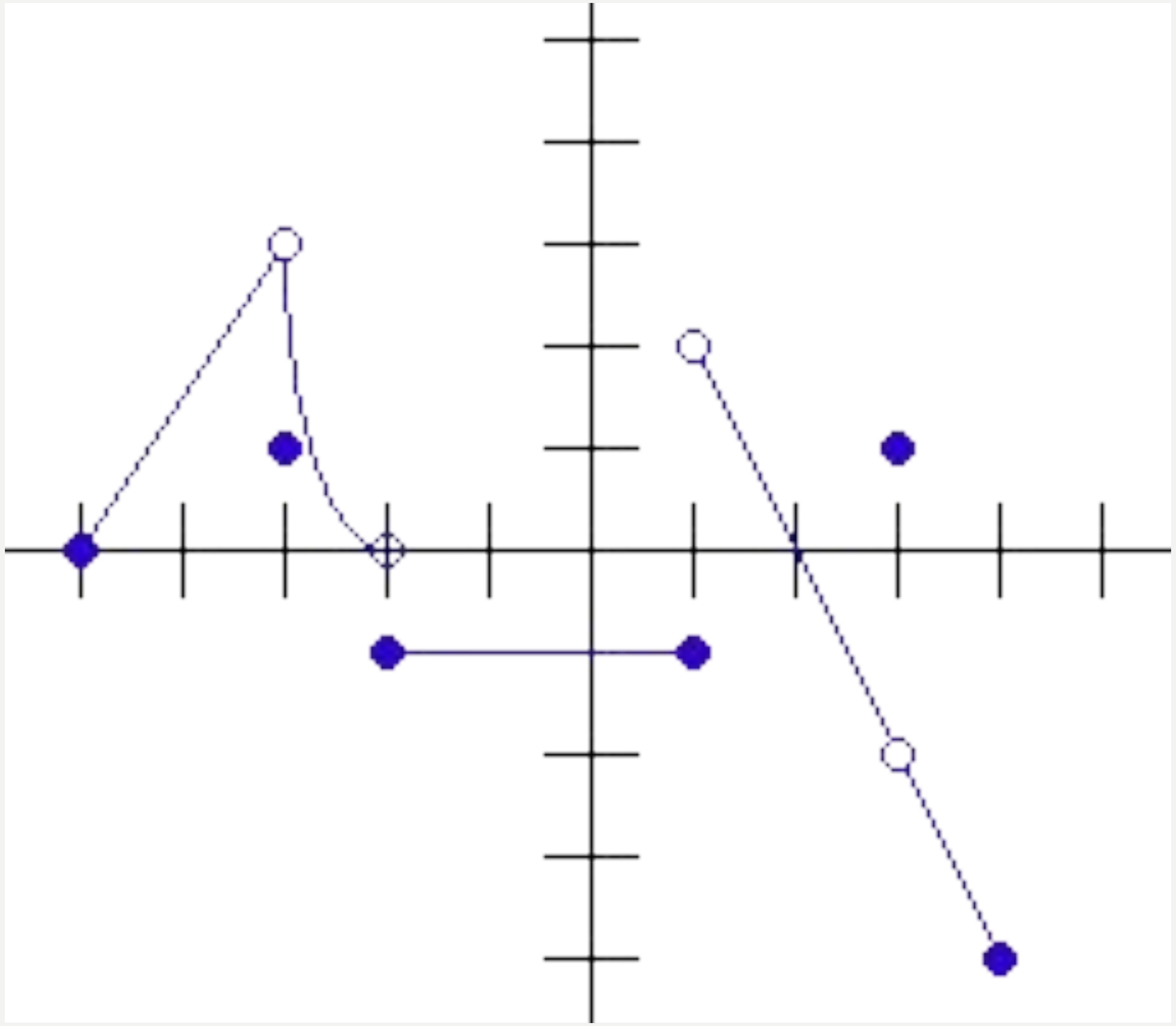
$$\lim_{x\to 1} f(x) = \textbf{DNE}$$

$$\lim_{x\to -2} f(x) = \textbf{DNE}$$

$$\lim_{x\to -5} f(x) = \textbf{0}$$

Issues!

YOU TRY! EVALUATE. ***



$$\lim_{x \to -2^-} f(x) =$$

$$\lim_{x \to -2^+} f(x) =$$

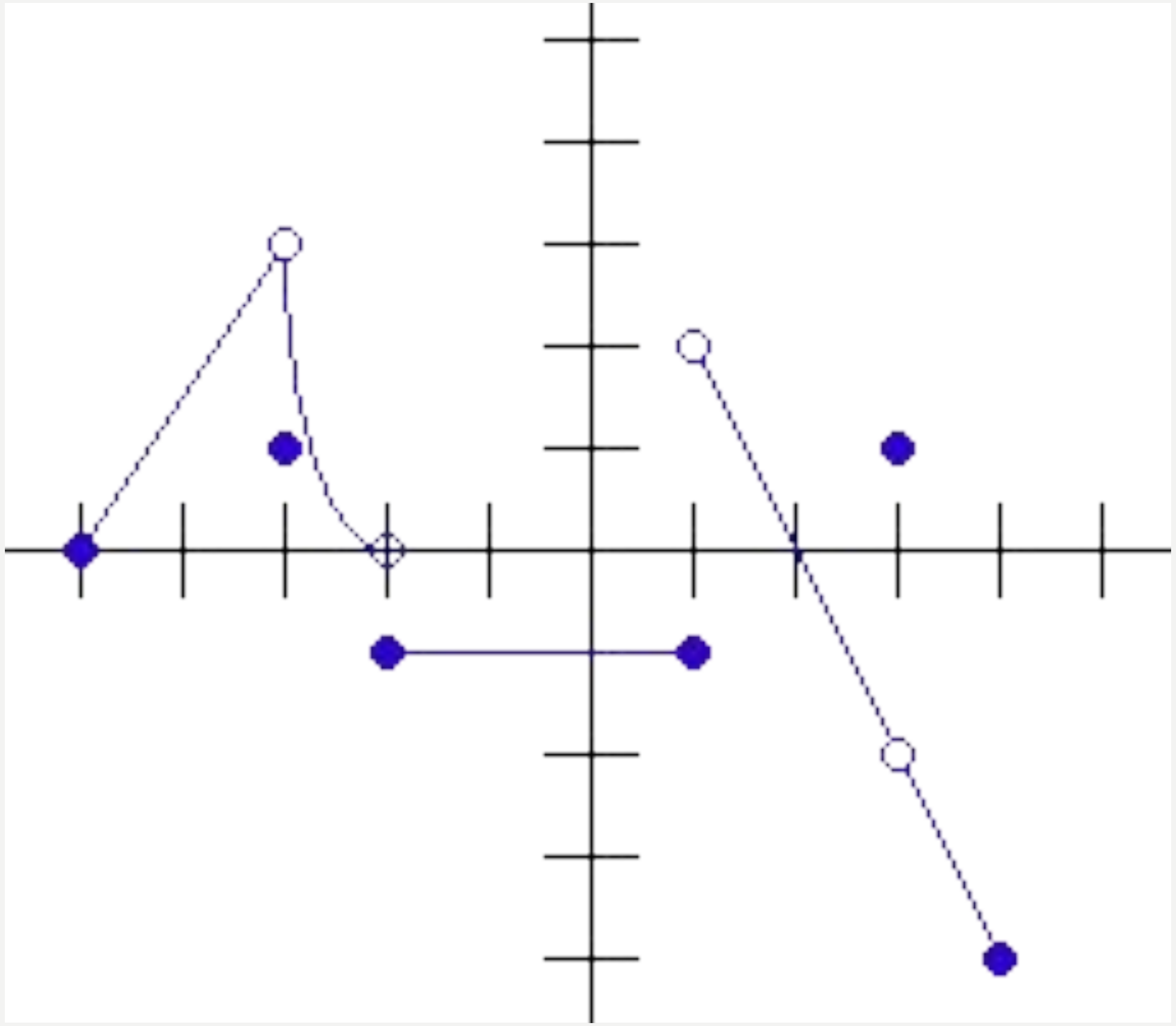
$$\lim_{x \to 1^-} f(x) =$$

$$\lim_{x \to 1^+} f(x) =$$

YOU TRY! EVALUATE.

***

ANSWERS

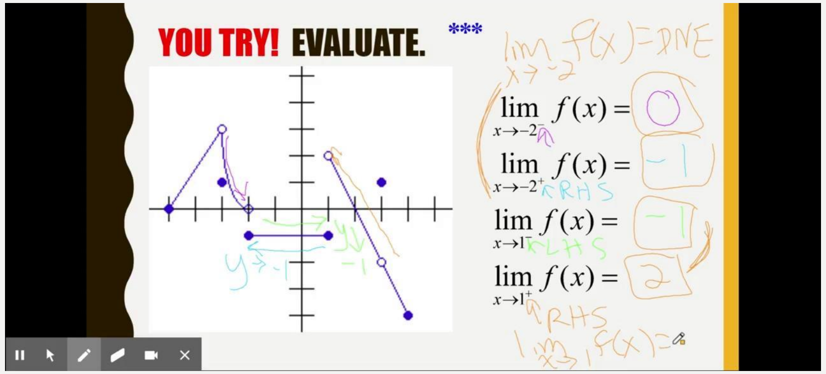


$$\lim_{x \to -2^-} f(x) = 0$$

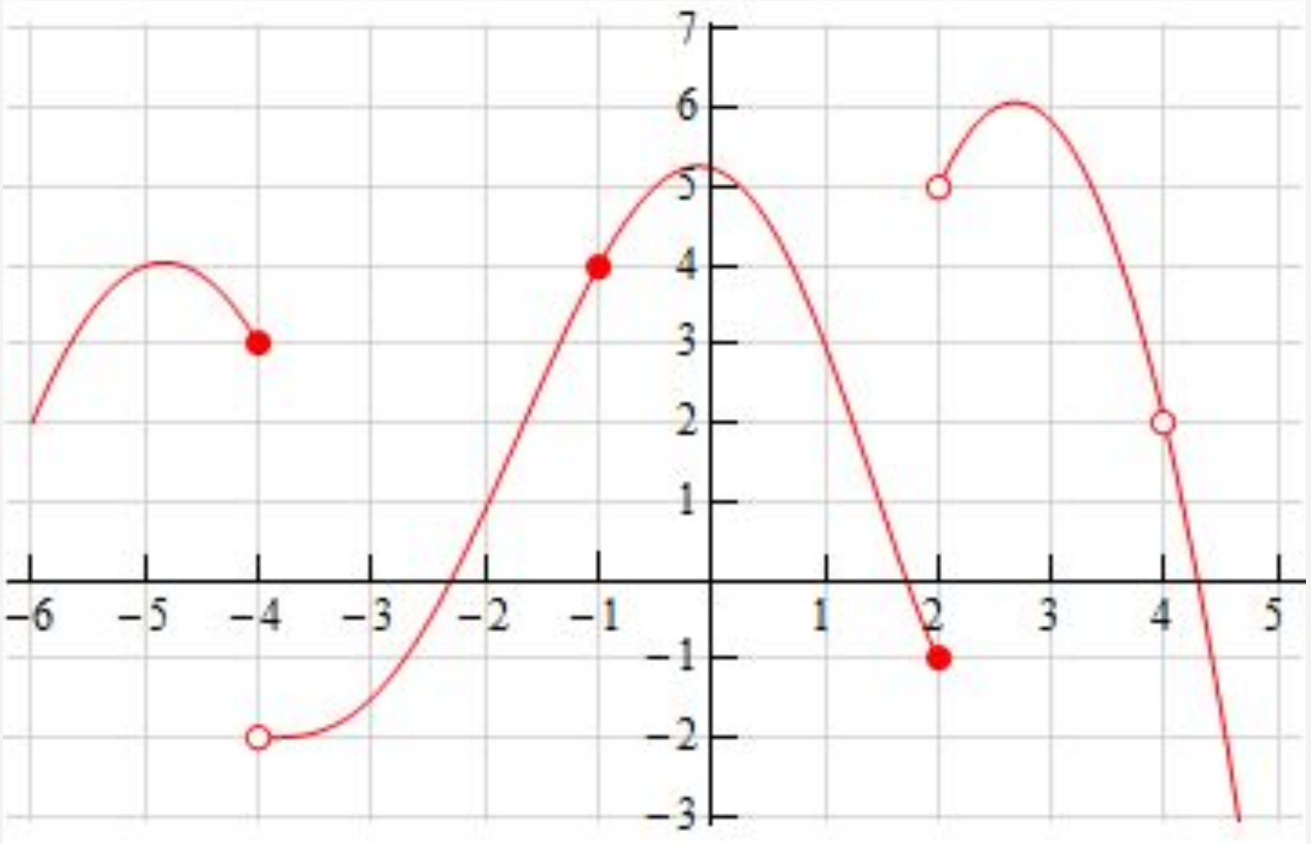
$$\lim_{x \to -2^+} f(x) = -1$$

$$\lim_{x \to 1^-} f(x) = -1$$

$$\lim_{x \to 1^+} f(x) = 2$$



REVIEW



$$\lim_{x\to -4} f(x) =$$

$$\lim_{x\to -4^-} f(x) =$$

$$\lim_{x\to -1} f(x) =$$

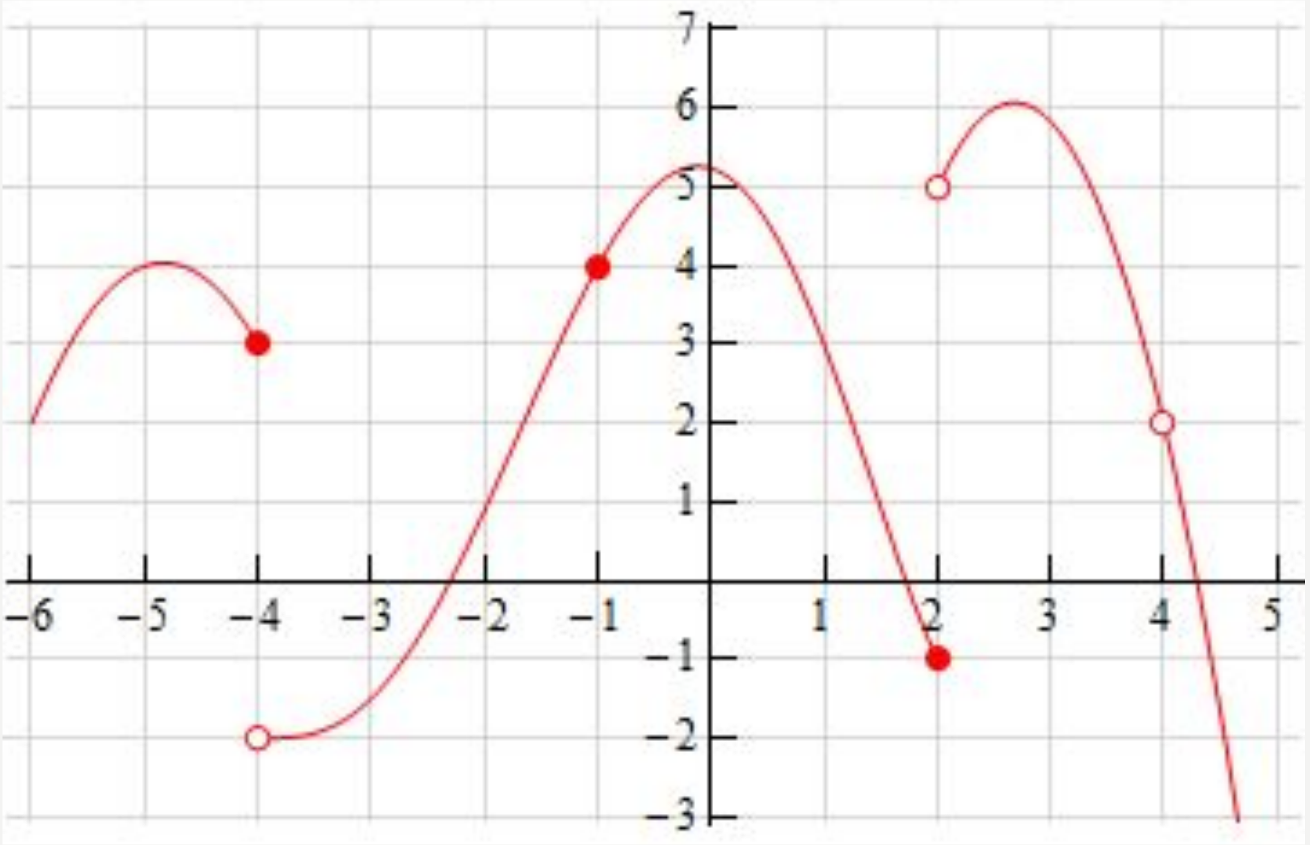
$$\lim_{x\to 4} f(x) =$$

$$f(4) =$$

$$f(2) =$$

REVIEW

ANSWERS



$$\lim_{x\to -4} f(x) = \text{DNE}$$

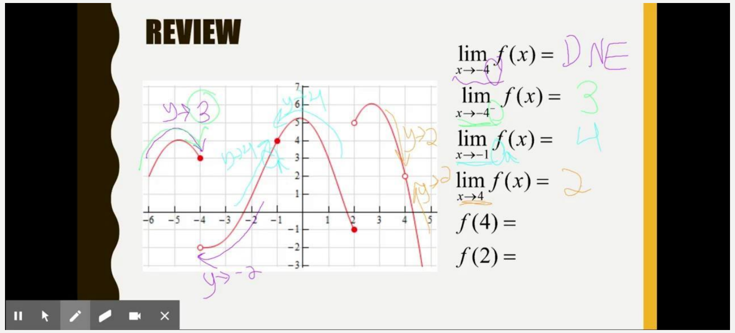
$$\lim_{x\to -4^-} f(x) = 3$$

$$\lim_{x\to -1} f(x) = 4$$

$$\lim_{x\to 4} f(x) = 2$$

$$f(4) = \text{DNE}$$

$$f(2) = -1$$



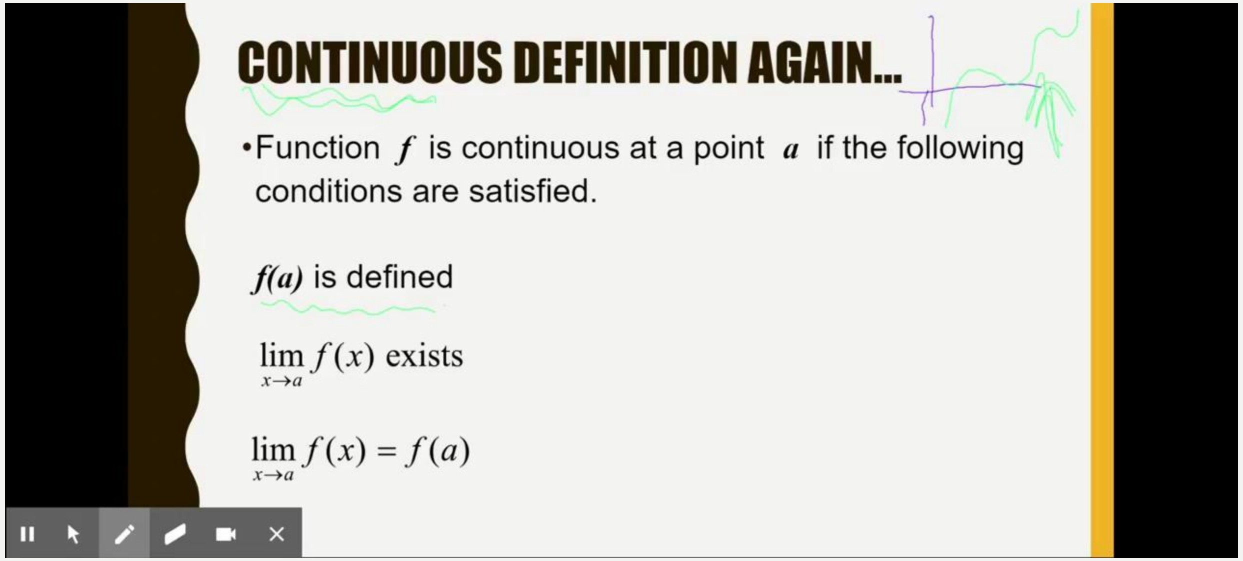
CONTINUOUS DEFINITION AGAIN…

- Function f is continuous at a point a if the following conditions are satisfied.

$f(a)$ is defined

$\lim_{x \to a} f(x)$ exists

$\lim_{x \to a} f(x) = f(a)$



PRACTICE

- Find the values requested for $g(x) = \dfrac{3x^2 + 16x - 12}{x^2 + 2x - 24}$

D:

R:

Removable Disc:

NonRemovable Disc:

Horizontal Asymptote:

Increasing:

Decreasing:

End Behavior, written as limits

The following limits

$\lim_{x \to -6} g(x) =$ $\lim_{x \to 4} g(x) =$ $\lim_{x \to 4^-} g(x) =$ $\lim_{x \to 4^+} g(x) =$

PRACTICE ANSWERS

$$g(x)=\frac{3x^2+16x-12}{x^2+2x-24}$$

- Find the values requested for

D: (-∞, -6) U (-6, 4) U (4,∞)

R: (-∞, 2) U (2, 3) U (3,∞)

Removable Disc: Hole at (-6, 2)

NonRemovable Disc: VA at x = 4

Horizontal Asymptote: y = 3

Increasing: None

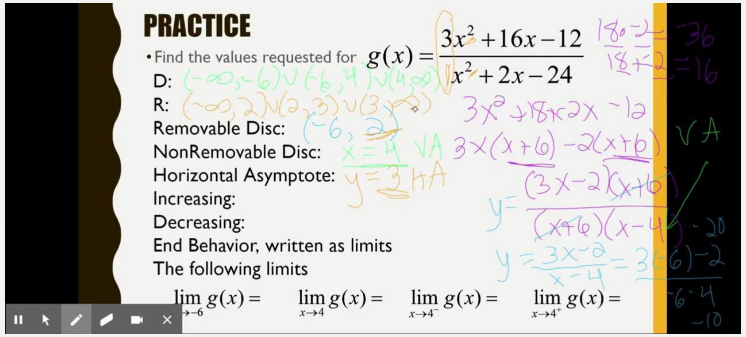
Decreasing: (-∞, -6) U (-6, 4) U (4,∞)

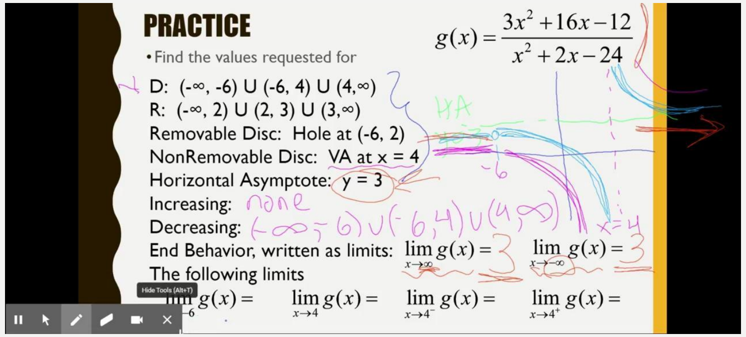
End Behavior, written as limits.

$$\lim_{x\to\infty} g(x)=3 \qquad \lim_{x\to-\infty} g(x)=3$$

The following limits

$$\lim_{x\to-6} g(x)=2 \qquad \lim_{x\to4} g(x)=DNE \qquad \lim_{x\to4^-} g(x)=-\infty \qquad \lim_{x\to4^+} g(x)=\infty$$





LIMIT PRACTICE HANDOUT

THIS EXTRA HANDOUT IS ON THE WEBSITE, IF YOU'D LIKE MORE PRACTICE 😊

HW DAY 7:

PACKET P.9 AND HANDOUT INTRO TO LIMITS (ON WEBSITE)