Unit 5 Day 7

Chain Rule

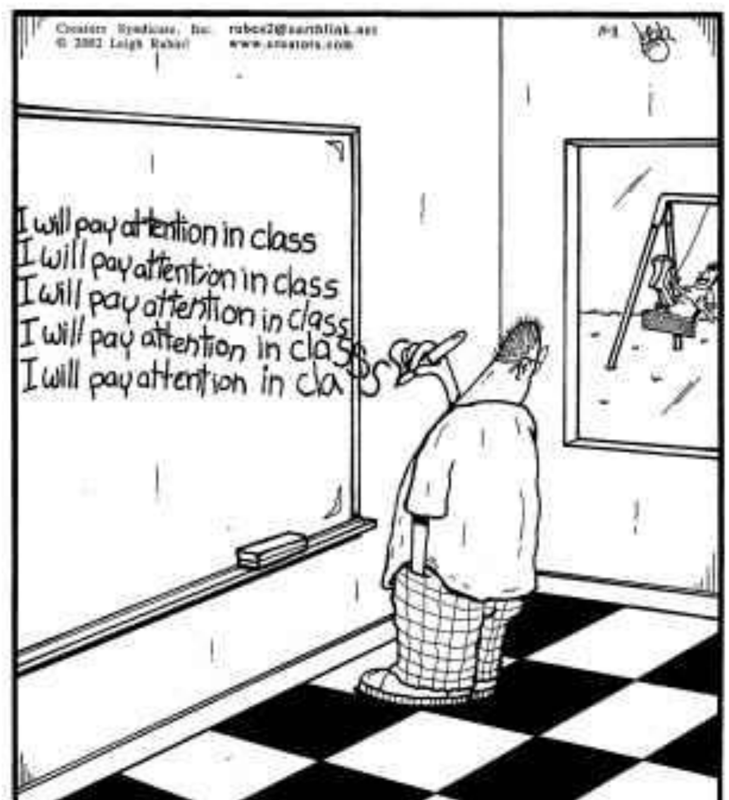
Warm Up ~

Find g(h(x)) and simplify.

1. $g(x)=\dfrac{2x-1}{3x^2+2}$; *and* $h(x)=x-5$

Given h(x) = f(g(x)) below, identify f(x) and g(x).

2. $h(x)=\sqrt{x^2-7}$



Find an equation of a tangent line to the given function.

3. $g(x)=\dfrac{2x-1}{3x^2+2}; x=-2$

Warm Up ~ ANSWERS

Find g(h(x)) and simplify.

1. $g(x)=\dfrac{2x-1}{3x^2+2};\ \ and\ \ h(x)=x-5$

$$g(h(x))=g(x-5)$$

$$=\frac{2(x-5)-1}{3(x-5)^2+2}=\frac{2x-10-1}{3(x^2-10x+25)+2}$$

$$=\frac{2x-11}{3x^2-30x+77}$$

Given h(x) = f(g(x)) below, identify f(x) and g(x).

2. $h(x)=\sqrt{x^2-7}$ $\quad f(x)=\sqrt{x}\quad g(x)=x^2-7$

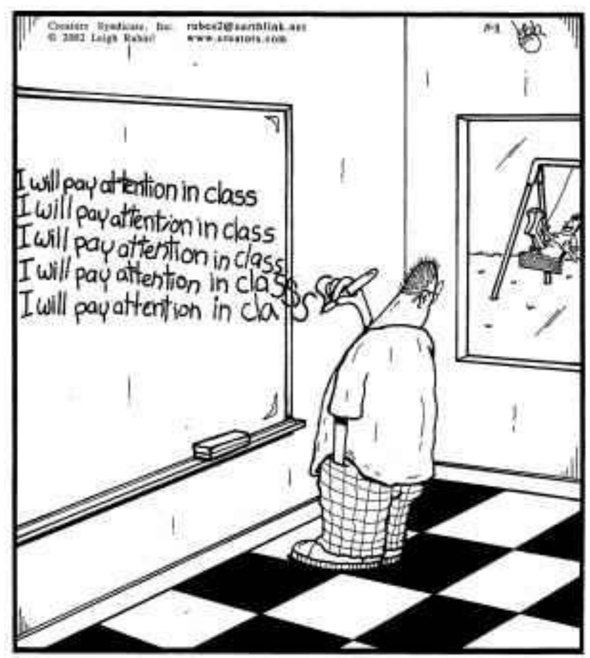
Warm Up ~ ANSWERS

Find an equation of a tangent line to the given function.

3. $g(x) = \dfrac{2x-1}{3x^2+2}; x = -2$

$$y + \frac{5}{14} = -\frac{8}{49}(x+2)$$

$$y = -\frac{8}{49}x - \frac{67}{98}$$



HW Questions?

Notes: Chain Rule

Comparing Prior Skills to New Skills

Prior Skill

$\sqrt{x}\cdot(x^2+x)$ is a combination of two functions

Find $\dfrac{d}{dx}$ using Product Rule

New Skill

$\sqrt{(x^2+x)}$ is a composition of two functions

Find $\dfrac{d}{dx}$ using Chain Rule

The Chain Rule

Remember the composition of two functions?

$$f \circ g = f(g(x))$$

The chain rule is used to find the derivative of the composition of two functions.

$$\frac{d}{dx}\left[f(g(x))\right] = f'(g(x))g'(x)$$

Chain Rule

$$\frac{d}{dx}\left[f(g(x))\right] = f'(g(x))g'(x)$$

$$y = (2x^3 + x + 7)^5$$

Derivative of outside function evaluated at inside function

Derivative of inside function

Chain Rule

$$\frac{d}{dx}[f(g(x))] = f'(g(x))g'(x)$$

$$y = (2x^3 + x + 7)^5$$

$$\frac{dy}{dx} = \left(5(2x^3 + x + 7)^4\right) \bullet \left(6x^2 + 1\right)$$

Derivative of outside function evaluated at inside function

Derivative of inside function

$$\frac{d}{dx}[f(g(x))] = f'(g(x))g'(x)$$

Ex: Find y' for $y = (x^2 + 1)^3$

$f(x) = x^3$

$g(x) = x^2 + 1$

Ex: Find y' for $y = (3x - 2x^2)^3$

$f(x) = x^3$

$g(x) = 3x - 2x^2$

$$\frac{d}{dx}\left[f(g(x))\right] = f'(g(x))g'(x)$$

Ex: Find y' for $y = (x^2 + 1)^3$

$f(x) = x^3$

$g(x) = x^2 + 1$

$$y' = 3(x^2 + 1)^2(2x) = 6x(x^2 + 1)^2$$

Ex: Find y' for $y = (3x - 2x^2)^3$

$f(x) = x^3$

$g(x) = 3x - 2x^2$

$$y' = 3(3x - 2x^2)^2(3 - 4x)$$

$$\frac{d}{dx}\left[f(g(x))\right] = f'(g(x))g'(x)$$

Examples: Find the derivative of the following.

Ex. $y = \sqrt{x+1}$

Rewrite as $(x+1)^{\frac{1}{2}}$

You Try!

Ex. $y = \sqrt{9x+1}$

Rewrite as $(9x+1)^{\frac{1}{2}}$

Examples ANSWERS:

$$\frac{d}{dx}\left[f(g(x))\right] = f'(g(x))g'(x)$$

Find the derivative of the following.

Ex. $y = \sqrt{x+1}$

Rewrite as $(x+1)^{\frac{1}{2}}$

$$y' = \frac{1}{2\sqrt{x+1}}$$

You Try!

Ex. $y = \sqrt{9x+1}$

Rewrite as $(9x+1)^{\frac{1}{2}}$

$$y' = \frac{9}{2\sqrt{9x+1}}$$

$$\frac{d}{dx}\left[f(g(x))\right] = f'(g(x))g'(x)$$

Ex:

Find f'(x) for $f(x) = \sqrt[3]{(x^2+2)^2}$

$$=$$

$$\frac{d}{dx}\left[f(g(x))\right] = f'(g(x))g'(x)$$

Ex:

Find f'(x) for $f(x) = \sqrt[3]{(x^2+2)^2}$

$$= \left(x^2+2\right)^{2/3}$$

$$f'(x) = \frac{2}{3}\left(x^2+2\right)^{-1/3}(2x) = \frac{4x}{3\sqrt[3]{x^2+2}}$$

Differentiate

$$g(t) = \frac{-7}{(2t-3)^2}$$ rewritten as

Differentiate ANSWERS

$$g(t) = \frac{-7}{(2t-3)^2}$$ rewritten as $= -7(2t-3)^{-2}$

$g'(t) = 14(2t-3)^{-3}(2)$ $$= \frac{28}{(2t-3)^3}$$

Differentiate- This needs the product and chain rule!

$h(x) = x^2\sqrt{1-x^2}$ rewritten as

Remember the Product Rule??

$$\frac{d}{dx}\left[f(x)g(x)\right] = g(x)f'(x) + f(x)g'(x)$$

Differentiate- This needs the product and chain rule!

$$h(x) = x^2\sqrt{1-x^2}$$ rewritten as $$x^2\left(1-x^2\right)^{1/2}$$

Remember the Product Rule??

$$\frac{d}{dx}\left[f(x)g(x)\right] = f(x)g'(x) + g(x)f'(x)$$

$$f(x) = x^2 \qquad g(x) = \left(1-x^2\right)^{\frac{1}{2}}$$

$$h'(x) = x^2\left(\frac{1}{2}\right)\left(1-x^2\right)^{-1/2}(-2x) + \left(1-x^2\right)^{1/2}(2x)$$

$$h'(x) = -x^3\left(1-x^2\right)^{-1/2} + 2x\left(1-x^2\right)^{1/2}$$

Find the derivative of the following.

1. $y = \sqrt{-x^4 - 1}(-x - 2)$

ANSWER: Find the derivative of the following.

1. $y = \sqrt{-x^4 - 1}(-x - 2)$

$$y' = -\sqrt{-x^4 - 1} + \frac{2x^4 + 4x^3}{\sqrt{-x^4 - 1}}$$

You Try!

Find the derivative of the following.

2. $y = (3x - 1)(-3x^2 - 4)^{-3}$

You Try!

ANSWER:

Find the derivative of the following.

2. $y = (3x - 1)(-3x^2 - 4)^{-3}$

Example Problems (GETTING your ANSWER to MATCH a GIVEN ANSWER)—Find the derivative of the following.

1. $y = x^3(2x - 5)^4$

Example Problems (GETTING your ANSWER to MATCH a GIVEN ANSWER)—Find the derivative of the following.

$$1. y = x^3(2x-5)^4$$

$$\frac{dy}{dx} = (2x-5)^3(14x^3-15x^2)$$

Differentiate

$$f(x) = \frac{x}{\sqrt[3]{x^2 + 4}}$$ rewritten as

Quotient Rule $$\frac{\text{Bot} * \text{Top}' - \text{Top} * \text{Bot}'}{(\text{Bot})^2}$$

Differentiate

$$f(x) = \frac{x}{\sqrt[3]{x^2+4}}$$ rewritten as $$= \frac{x}{\left(x^2+4\right)^{1/3}}$$

Quotient Rule $\dfrac{\text{Bot * Top' – Top * Bot'}}{(\text{Bot})^2}$

$$f'(x) = \frac{(x^2+4)^{1/3}(1) - x(1/3)(x^2+4)^{-2/3}(2x)}{(x^2+4)^{2/3}}$$

$$f'(x) = \frac{(x^2+4)^{1/3} - (2x^2/3)(x^2+4)^{-2/3}}{(x^2+4)^{2/3}}$$

Differentiate

$$y = \left(\frac{3x - 1}{x^2 + 3} \right)^2$$

Differentiate

$$y=\left(\frac{3x-1}{x^2+3}\right)^2$$

$$y'=2\left(\frac{3x-1}{x^2+3}\right)^1\left(\frac{(x^2+3)(3)-(3x-1)(2x)}{(x^2+3)^2}\right)$$

$$=\frac{2(3x-1)\left(3x^2+9-6x^2+2x\right)}{(x^2+3)^3}$$

$$=\frac{2(3x-1)\left(-3x^2+2x+9\right)}{(x^2+3)^3}$$

Extra Warm-Up #2 on next slide

Warm Up ~ ANSWERS

Find an equation of a tangent line to the given function.

1. $f(x) = (x+2)(4x^2 + 3x - 1);\ (-3, -26)$

$$y + 26 = 47(x+3)$$

$$y = 47x + 115$$

2. $g(x) = \dfrac{2x-1}{3x^2+2};\ x = -2$

$$y + \frac{5}{14} = -\frac{8}{49}(x+2)$$

$$y = -\frac{8}{49}x - \frac{67}{98}$$