KEY

Monday

Day 6

Product and Quotient Rules

A Warm Up to make you think!

1. Find f'(x) given

$a.\ f(x)=\frac{1}{x}$ $\qquad b.\ f(x)=x^2-c^2$

2. A tangent line to f(x) is y + 7 = 4(x - 1). What is f'(x) and at which point?

3. Find the derivative of: **(exponents should be positive!)**

a. $\frac{x^4}{8}+\frac{2}{x^2}-\frac{5x}{x^4}$ $\qquad$ b. $\sqrt{x}+\frac{3}{\sqrt[3]{x^4}}-x$

A Warm Up to make you think!

1. Find f'(x) given $f'(x) = -x^{-2}$ $f'(x) =$

a. $f(x) = \frac{1}{x} = x^{-1}$ $\boxed{\frac{-1}{x^2}}$ b. $f(x) = x^2 - c^2$ $\boxed{2x}$

2. A tangent line to f(x) is y + 7 = 4(x - 1). What is f'(x) and at which point?

$\boxed{f'(x) = 4} = m$ $\boxed{\text{at } (1, -7)}$

3. Find the derivative of: **(exponents should be positive!)**

a. $\frac{x^4}{8} + \frac{2}{x^2} - \frac{5x}{x^4}$ b. $\sqrt{x} + \frac{3}{\sqrt[3]{x^4}} - x$

$f(x) = \frac{x^4}{8} + 2x^{-2} - 5x^{-3}$

$\boxed{f'(x) = \frac{x^3}{2} - \frac{4}{x^3} + \frac{15}{x^4}}$

$f(x) = x^{1/2} + 3x^{-4/3} - x$

$f'(x) = \frac{1}{2}x^{-1/2} - 4x^{-7/3} - 1$

$\boxed{f'(x) = \frac{1}{2\sqrt{x}} - \frac{4}{\sqrt[3]{x^7}} - 1}$

A Warm Up to make you think!

1. Find f'(x) given

a. $f(x) = \frac{1}{x}$ $f'(x) = \frac{-1}{x^2}$ b. $f(x) = x^2 - c^2$ $\frac{dy}{dx} = 2x$

2. A tangent line to f(x) is y+7=4(x-1). What is f'(x) and at which point?

$f'(x) = 4 \ at \ (1, -7)$

3. Find the derivative of: **(exponents should be positive!)**

a. $\frac{x^4}{8} + \frac{2}{x^2} - \frac{5x}{x^4}$ b. $\sqrt{x} + \frac{3}{\sqrt[3]{x^4}} - x$

$\frac{x^3}{2} - \frac{4}{x^3} + \frac{15}{x^4}$ $\frac{1}{2\sqrt{x}} - \frac{4}{x^{\frac{7}{3}}} - 1$

Notes: Product Rule & Quotient Rule

The Product Rule

- The product of two differentiable functions ***f*** and ***g*** is itself differentiable. The derivative of ***fg*** is the first function times the derivative of the second, plus the second function times the derivative of the first.

$$\frac{d}{dx}[f(x)g(x)] = f(x)g'(x) + g(x)f'(x)$$

Deriv of 1st * 2nd =

1st * Deriv of 2nd + 2nd * Deriv of 1st

Find each derivative.

$$\frac{d}{dx}[f(x)g(x)] = f(x)g'(x) + g(x)f'(x)$$

1. $h(x) = \underset{f(x)}{(3-2x^2)}\underset{g(x)}{(5+4x)}$

$h'(x) = (3-2x^2)(4) + (5+4x)(-4x)$

$h'(x) = 12 - 8x^2 - 20x - 16x^2$ $\boxed{-24x^2 - 20x + 12}$

2. $f(x) = x^3(2x^2 - 3x)$

$f'(x) = x^3(4x-3) + (2x^2-3x)(3x^2)$

$f'(x) = 4x^4 - 3x^3 + 6x^4 - 9x^3$

$\boxed{f'(x) = 10x^4 - 12x^3}$

Find each derivative.

$$\frac{d}{dx}[f(x)g(x)] = f(x)g'(x) + g(x)f'(x)$$

1. $h(x) = (3-2x^2)(5+4x)$

2. $f(x) = x^3(2x^2 - 3x)$

Find each derivative.

$$\frac{d}{dx}[f(x)g(x)] = f(x)g'(x) + g(x)f'(x)$$

3. $g(x) = \sqrt{x^3}(-8x^{-4})$

$= x^{3/2}(-8x^{-4})$

$g'(x) = (x^{3/2})(32x^{-5}) + (-8x^{-4})(\frac{3}{2}x^{1/2})$

$-10/2$ $\quad$ $-8/2$

$32x^{-7/2} - 12x^{-7/2} = 20x^{-7/2}$

$\boxed{\frac{20}{\sqrt{x^7}}}$

4. $y = -\frac{x}{3}\left(\frac{7}{x^2}\right)$

$7x^{-2}$

$= \frac{-x}{3}(7x^{-2})$

$= \left(\frac{-x}{3}\right)(-14x^{-3}) + (7x^{-2})\left(-\frac{1}{3}\right)$

$= \frac{14}{3}x^{-2} - \frac{7}{3}x^{-2} = \frac{7}{3}x^{-2}$

$\boxed{\frac{7}{3x^2}}$

Find each derivative.

$$\frac{d}{dx}[f(x)g(x)] = f(x)g'(x) + g(x)f'(x)$$

3. $g(x) = \sqrt{x^3}(-8x^{-4})$

4. $y = -\frac{x}{3}\left(\frac{7}{x^2}\right)$

Now let's find tangent lines!

- When x = 2, find the slope of the tangent line to the function: $y = x^3(7-4x)$

$f'(x) = (x^3)(-4) + (7-4x)(3x^2)$

$-4x^3 + 21x^2 - 12x^3$

$f'(x) = -16x^3 + 21x^2$

$f'(2) = -16(2)^3 + 21(2)^2$

$\boxed{-44}$

Now let's find tangent lines!

- When x = 2, find the slope of the tangent line to the function: $y = x^3(7-4x)$

$y' = (x^3)(-4) + (3x^2)(7-4x)$

$y' = -4x^3 + 21x^2 - 12x^3$

$y' = -16x^3 + 21x^2$

$y'(2) = -16(2)^3 + 21(2)^2$

$\boxed{y'(2) = -44}$

Find the equation of a tangent line when x = 4

$$f(x)=\frac{4x}{7}(\sqrt{x^3})$$

$x^{3/2}$

$$f'(x)=\left(\frac{4x}{7}\right)\left(\frac{3}{2}x^{1/2}\right)+\left(\sqrt{x^3}\right)\left(\frac{4}{7}\right)$$

$$=\frac{6}{7}x^{3/2}+\frac{4}{7}x^{3/2}=\frac{10}{7}x^{3/2}$$

$$f'(4)=\frac{10}{7}(4)^{3/2}=80/7=m$$

$$f(4)=\frac{4(4)}{7}(\sqrt{4^3})=128/7\ y_1$$

$$y-128/7=80/7(x-4)$$

$$y-\frac{128}{7}=\frac{80}{7}x-\frac{320}{7}$$

$$\boxed{y=\frac{80}{7}x-\frac{192}{7}}$$

Find the equation of a tangent line when x=4

$$f(x)=\frac{4x}{7}\cdot x^{3/2}(\sqrt{x^3})$$

$$f'(x)=\frac{4x}{7}\left(\frac{3}{2}x^{\frac{1}{2}}\right)+\frac{4}{7}\cdot x^{3/2}(\sqrt{x^3})=\frac{10}{7}x^{3/2}$$

$$f'(x)=\frac{10}{7}x^{3/2}$$

$$f'(4)=\frac{4(4)}{7}\left(\frac{3}{2}(4)^{\frac{1}{2}}\right)+\frac{4}{7}\left(\sqrt{(4)^3}\right)$$

$$f'(4)=\frac{16}{7}(3)+\frac{32}{7}=\frac{80}{7}$$

$$f(4)=\frac{4(4)}{7}(\sqrt{4^3})=\frac{128}{7}$$

$$y-\frac{128}{7}=\frac{80}{7}(x-4)$$

$$\boxed{y=\frac{80}{7}x-\frac{192}{7}}$$

Final Answer

The Quotient Rule

- The derivative of ***f/g*** is given by the denominator times the derivative of the numerator minus the numerator times the derivative of the denominator, all divided by the square of the denominator.

$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right]=\frac{g(x)f'(x)-f(x)g'(x)}{[g(x)]^2},g(x)\neq 0$$

$$\frac{\text{bot}\cdot\text{top}'-\text{top}\cdot\text{bot}'}{(\text{bot})^2}$$

Find each derivative.

$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right]=\frac{g(x)f'(x)-f(x)g'(x)}{[g(x)]^2},g(x)\neq 0$$

5. $y=\dfrac{5x-2}{x^2+1}=\dfrac{(x^2+1)(5)-(5x-2)(2x)}{(x^2+1)^2}=\dfrac{5x^2+5-10x^2+4x}{(x^2+1)^2}$

$(x^2+1)(x^2+1)$

$$\boxed{\frac{-5x^2+4x+5}{x^4+2x^2+1}}$$

$3-x^{-1}$

6. $f(x)=\dfrac{3-(\frac{1}{x})}{x+5}=\dfrac{(x+5)(\frac{1}{x^2})-(3-x^{-1})(1)}{(x+5)^2}=\dfrac{\left(\frac{1}{x}+\frac{5}{x^2}-3+x^{-1}\right)x^2}{x^2(x^2+10x+25)}$

$$\frac{x+5-3x^2+x}{x^2(x^2+10x+25)}$$

$$\boxed{\frac{-3x^2+2x+5}{x^4+10x^3+25x^2}}$$

Find each derivative.

$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right]=\frac{g(x)f'(x)-f(x)g'(x)}{[g(x)]^2}, g(x)\neq 0$$

5. $y=\dfrac{5x-2}{x^2+1}$

6. $f(x)=\dfrac{3-(\frac{1}{x})}{x+5}$

Find each derivative.

$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right]=\frac{g(x)f'(x)-f(x)g'(x)}{[g(x)]^2}, g(x)\neq 0$$

7. $y=\dfrac{x^2+3x}{6} \Rightarrow \dfrac{6(2x+3)-(x^2+3x)(0)}{6^2} = \boxed{\dfrac{2x+3}{6}}$

8. $g(x)=\dfrac{x^3+\sqrt[4]{x}}{6-x}$ ($x^{1/4}$) $\Rightarrow \dfrac{(6-x)(3x^2+\frac{1}{4}x^{-3/4})-(x^3+x^{1/4})(-1)}{(6-x)^2}$

$$\frac{18x^2+\frac{3}{2}x^{-3/4}-3x^3-\frac{1}{4}x^{1/4}+x^3+x^{1/4}}{x^2-12x+36} =$$

$$\boxed{\frac{-2x^3+18x^2+\frac{3}{4}x^{1/4}+\frac{3}{2}x^{-3/4}}{x^2-12x+36}} = \boxed{\frac{-8x^{15/4}+72x^{11/4}+3x+6}{4(x^2-12x+36)x^{3/4}}}$$

Find each derivative.

$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right]=\frac{g(x)f'(x)-f(x)g'(x)}{[g(x)]^2},g(x)\neq 0$$

$$7.\ y=\frac{x^2+3x}{6}$$

$$8.\ g(x)=\frac{x^3+\sqrt[4]{x}}{6-x}$$

Find the equation of a tangent line to h(x) when x= -3

$$h(x)=\frac{x^2-4}{3x+7}$$

$$h(-3)=\frac{(-3)^2-4}{3(-3)+7}=\frac{5}{-2}$$

$(-3, -5/2)$

$$h'(x)=\frac{(3x+7)(2x)-(x^2-4)(3)}{(3x+7)^2}$$

$$=\frac{6x^2+14x-3x^2+12}{9x^2+42x+49}=\frac{3x^2+14x+12}{9x^2+42x+49}$$

$$h'(-3)=\frac{3(-3)^2+14(-3)+12}{9(-3)^2+42(-3)+49}=\frac{-3}{4}$$

$$y+5/2=-3/4(x+3)$$

$$y=-3/4x-9/4-\frac{10}{4}$$

$$\boxed{y=\frac{-3}{4}x-\frac{19}{4}}$$

Find the equation of a tangent line to h(x) when x= -3

$$h(x) = \frac{x^2 - 4}{3x + 7}$$

$$h'(x) = \frac{(3x+7)(2x) - (x^2 - 4)(3)}{(3x+7)^2}$$

$$h'(-3) = \frac{(3(-3)+7)(2(-3)) - ((-3)^2 - 4)(3)}{(3(-3)+7)^2}$$

$$h'(-3) = \frac{12 - 15}{4} = \frac{-3}{4}$$

$$h(-3)$$

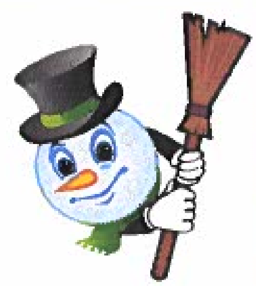
$$= \frac{(-3)^2 - 4}{3(-3)+7}$$

$$= \frac{5}{-2} = -\frac{5}{2}$$

$$y + \frac{5}{2} = -\frac{3}{4}(x+3)$$

$$\boxed{y = -\frac{3}{4}x - \frac{19}{4}}$$

Final Answer



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Finish for HW!