Day 6

Product and Quotient Rules

A Warm Up to make you think!

1. Find f'(x) given

$$a.\ \ f(x) = \frac{1}{x} \qquad\qquad b.\ \ f(x) = x^2 - c^2$$

2. A tangent line to f(x) is y + 7 = 4(x - 1). What is f'(x) and at which point?

3. Find the derivative of: **(exponents should be positive and whole!)**

a. $\dfrac{x^4}{8} + \dfrac{2}{x^2} - \dfrac{5x}{x^4}$

b. $\sqrt{x} + \dfrac{3}{\sqrt[3]{x^4}} - x$

A Warm Up to make you think!

1. Find f'(x) given

$$a.\ f(x)=\frac{1}{x} \quad f'(x)=\frac{-1}{x^2} \qquad b.\ f(x)=x^2-c^2 \quad \frac{dy}{dx}=2x$$

2. A tangent line to f(x) is y+7=4(x-1). What is f'(x) and at which point?

$$f'(x)=4\ at\ (1,-7)$$

3. Find the derivative of: **(exponents should be positive!)**

a. $$\frac{x^4}{8}+\frac{2}{x^2}-\frac{5x}{x^4}$$

$$\frac{x^3}{2}-\frac{4}{x^3}+\frac{15}{x^4}$$

b. $$\sqrt{x}+\frac{3}{\sqrt[3]{x^4}}-x$$

$$\frac{1}{2\sqrt{x}}-\frac{4}{x^{\frac{7}{3}}}-1$$

Homework Questions?

Notes: Product Rule & Quotient Rule

The Product Rule

- The product of two differentiable functions ***f*** and ***g*** is itself differentiable. The derivative of ***fg*** is the first function times the derivative of the second, plus the second function times the derivative of the first.

$$\frac{d}{dx}\left[f(x)g(x)\right] = f'(x)g(x) + f(x)g'(x)$$

Deriv of 1st * 2nd =

Deriv of 1st • 2nd + 1st • Deriv of 2nd

Find each derivative.

$$\frac{d}{dx}\left[f(x)g(x)\right] = f'(x)g(x) + f(x)g'(x)$$

1. $h(x) = (3 - 2x^2)(5 + 4x)$

2. $f(x) = x^3(2x^2 - 3x)$

Find each derivative.

$$\frac{d}{dx}[f(x)g(x)] = f'(x)g(x) + f(x)g'(x)$$

1. $h(x) = (3 - 2x^2)(5 + 4x)$

$$h'(x) = -24x^2 - 20x + 12$$

2. $f(x) = x^3(2x^2 - 3x)$

$$f'(x) = 10x^4 - 12x^3$$

Find each derivative.

$$\frac{d}{dx}\left[f(x)g(x)\right] = f'(x)g(x) + f(x)g'(x)$$

3. $g(x) = \sqrt{x^3}\left(-8x^{-4}\right)$

4. $y = -\frac{x}{3}\left(\frac{7}{x^2}\right)$

Find each derivative.

$$\frac{d}{dx}\left[f(x)g(x)\right] = f'(x)g(x) + f(x)g'(x)$$

3. $g(x) = \sqrt{x^3}\left(-8x^{-4}\right)$

$$g'(x) = \frac{20}{\sqrt{x^7}}$$

4. $y = -\frac{x}{3}\left(\frac{7}{x^2}\right)$

$$y' = \frac{7}{3x^2}$$

Now let's find tangent lines!

- When x = 2, find the slope of the tangent line to the function: $y = x^3(7 - 4x)$

Now let's find tangent lines!

- When x = 2, find the slope of the tangent line to the function: $y = x^3(7-4x)$

$$y' = (x^3)(-4) + (3x^2)(7-4x)$$

$$y' = -4x^3 + 21x^2 - 12x^3$$

$$y' = -16x^3 + 21x^2$$

$$y'(2) = -16(2)^3 + 21(2)^2$$

$$\boxed{y'(2) = -44}$$

You Try!! Find the equation of a tangent line when x = 4

$$f(x) = \frac{4x}{7}(\sqrt{x^3})$$

Find the equation of a tangent line when x=4

$$f(x)=\frac{4x}{7}(\sqrt{x^3})$$

$$f'(x)=\frac{4x}{7}(\frac{3}{2}x^{\frac{1}{2}})+\frac{4}{7}(\sqrt{x^3})$$

$$f(4)=\frac{4(4)}{7}(\sqrt{4^3})$$

$$f'(4)=\frac{4(4)}{7}(\frac{3}{2}(4)^{\frac{1}{2}})+\frac{4}{7}(\sqrt{(4)^3})$$

$$=\frac{128}{7}$$

$$f'(4)=\frac{16}{7}(3)+\frac{32}{7}=\frac{80}{7}$$

$$y-\frac{128}{7}=\frac{80}{7}(x-4)$$

$$\boxed{y=\frac{80}{7}x-\frac{192}{7}}$$

Final Answer

The Quotient Rule

- The derivative of ***f/g*** is given by the denominator times the derivative of the numerator minus the numerator times the derivative of the denominator, all divided by the square of the denominator.

$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right]=\frac{g(x)f'(x)-f(x)g'(x)}{\left[g(x)\right]^2},\ g(x)\neq 0$$

Deriv of Hi/Lo = (Lo • dHi – Hi • dLo) / (Lo•Lo)

Deriv of Top/Bot = (Bot • Top' – Top • Bot') / $(\text{Bot})^2$

Find each derivative.

$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right]=\frac{g(x)f'(x)-f(x)g'(x)}{\left[g(x)\right]^2},g(x)\neq 0$$

Deriv of $\frac{\text{Hi}}{\text{Lo}}$ = $\frac{\text{Lo • dHi – Hi • dLo}}{\text{Lo•Lo}}$

6. $f(x)=\dfrac{3-(\frac{1}{x})}{x+5}$

Find each derivative.

$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right]=\frac{g(x)f'(x)-f(x)g'(x)}{\left[g(x)\right]^2}, g(x)\neq 0$$

Deriv of $\frac{\text{Hi}}{\text{Lo}}$ = $\frac{\text{Lo} \bullet \text{dHi} - \text{Hi} \bullet \text{dLo}}{\text{Lo}\bullet\text{Lo}}$

5. $y=\dfrac{5x-2}{x^2+1}$

Find each derivative. ANSWERS

$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x)f'(x) - f(x)g'(x)}{\left[g(x)\right]^2}, g(x) \neq 0$$

5. $y = \dfrac{5x-2}{x^2+1}$

$$y' = \frac{-5x^2+4x+5}{x^4+2x^2+1}$$

6. $f(x) = \dfrac{3-(\frac{1}{x})}{x+5}$

$$f'(x) = \frac{-3x^2+2x+5}{x^4+10x^3+25x^2}$$

Find each derivative.

$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x)f'(x) - f(x)g'(x)}{\left[g(x)\right]^2}, g(x) \neq 0$$

Deriv of $\frac{\text{Hi}}{\text{Lo}}$ = $\frac{\text{Lo} \bullet \text{dHi} - \text{Hi} \bullet \text{dLo}}{\text{Lo} \bullet \text{Lo}}$

7. $y = \dfrac{x^2 + 3x}{6}$

Find each derivative.

$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right]=\frac{g(x)f'(x)-f(x)g'(x)}{\left[g(x)\right]^2}, g(x)\neq 0$$

Deriv of $\frac{\text{Hi}}{\text{Lo}}$ = $\frac{\text{Lo} \bullet \text{dHi} - \text{Hi} \bullet \text{dLo}}{\text{Lo}\bullet\text{Lo}}$

8. $g(x)=\dfrac{x^3+\sqrt[4]{x}}{6-x}$

Find each derivative. ANSWERS

$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right]=\frac{g(x)f'(x)-f(x)g'(x)}{\left[g(x)\right]^2}, g(x)\neq 0$$

7. $y=\dfrac{x^2+3x}{6}$

$$y'=\frac{2x+3}{6}$$

8. $g(x)=\dfrac{x^3+\sqrt[4]{x}}{6-x}$

$$g'(x)=\frac{-8x^{15/4}+72x^{11/4}+3x+6}{4(x^2-12x+36)x^{3/4}}$$

Find the equation of a tangent line to h(x) when x= -3

$$h(x) = \frac{x^2 - 4}{3x + 7}$$

Find the equation of a tangent line to h(x) when x= -3

$$h(x)=\frac{x^2-4}{3x+7}$$

$$h'(x)=\frac{(3x+7)(2x)-(x^2-4)(3)}{(3x+7)^2}$$

$$h'(-3)=\frac{(3(-3)+7)(2(-3))-((-3)^2-4)(3)}{(3(-3)+7)^2}$$

$$h'(-3)=\frac{12-15}{4}=\frac{-3}{4}$$

$$h(-3)$$

$$=\frac{(-3)^2-4}{3(-3)+7}$$

$$=\frac{5}{-2} \quad =-\frac{5}{2}$$

$$y+\frac{5}{2}=-\frac{3}{4}(x+3)$$

$$y=-\frac{3}{4}x-\frac{19}{4}$$

Final Answer

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Finish for HW!