UNIT 1 DAY6 RULES OF PROBABILITY

7.2-7.3

WARM UP DAY 6

1) What is the sample space for events: B = drawing a black card and C = drawing a Club? Are Events B & C mutually Exclusive? What is the probability of B and C?
2) On Friday and Monday WCPSS had three options for school schedules: Open, Closed, Delayed. Create the sample space for both days using a tree diagram and then write the sample space.
3) List the events D where school is delayed at least one of the days.

Riddle of the Day: The ages of a father and son add up to 66. The father's age is the son's age reversed. How old could they be?

Warm-Up Continued ->

WARM UP DAY 6 PART 2

4. A card is drawn at random from a fair standard deck of cards. Find each of the following:

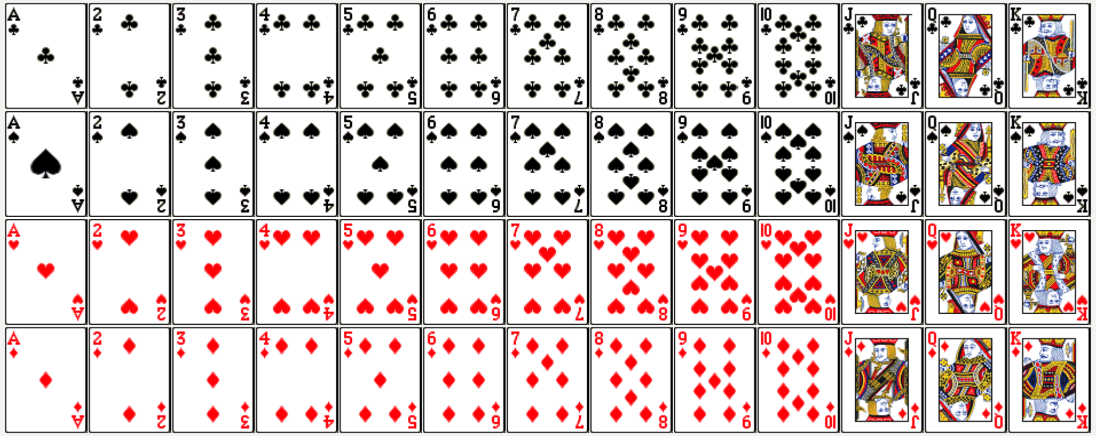
**Please include fraction AND decimal answers!

P(heart) _______________

P(black card) _______________

P(2 or jack) _______________

P(not a heart) _______________



WARM UP ANSWERS

1) What is the sample space for events: B = drawing a black card and C=drawing a Club? Are Events B & C mutually Exclusive? What is the probability of B and C?

   S = {all spades Ace-King ; and all clubs Ace-King} ;
   No these are not Mutually Exclusive because all clubs are black.
   Probability of B and C is 0.

2) On Friday and Monday WCPSS had three options for school schedules: Open, Closed, Delayed. Create the sample space for both days using a tree diagram and then write the sample space.

   S = {OO, OD, OC, CC, CO, CD, DD, DO, DC}

3) List the events D where school is delayed at least one of the days.

   {DD}, {DO}, {OD}, {CD}, {DC}

WARM UP DAY 6 PART 2

4. A card is drawn at random from a fair standard deck of cards. Find each of the following:

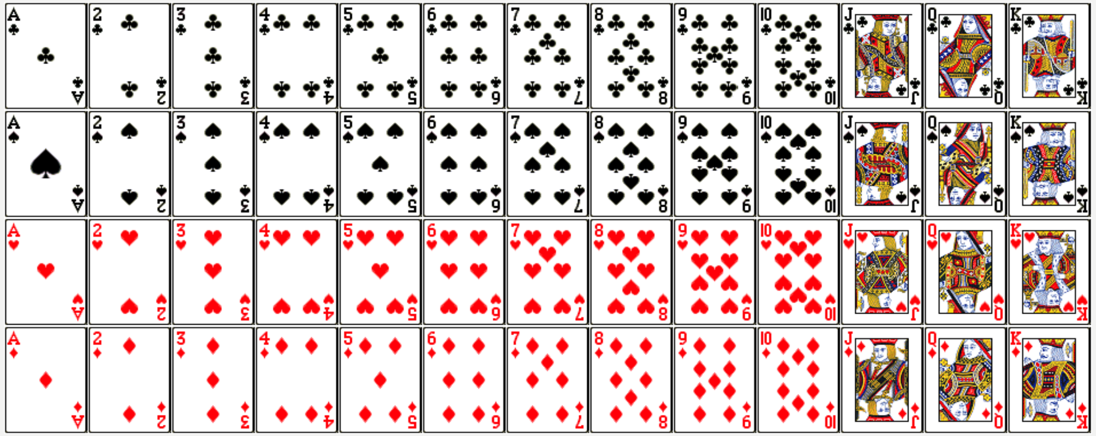
P(heart) $\frac{13}{52} = \frac{1}{4} = 0.25$

P(black card) $\frac{26}{52} = \frac{1}{2} = 0.5$

P(2 or jack) $\frac{8}{52} = \frac{2}{13} = 0.154$

P(not a heart) $\frac{39}{52} = \frac{3}{4} = 0.75$

**Please include fraction AND decimal answers!



RIDDLE OF THE DAY

The ages of a father and son add up to 66.
The father's age is the son's age reversed.
How old could they be?

There are three possible solutions for this: the father-son duo could be 51 and 15 years old, 42 and 24 years old or 60 and 06 years old.

HW QUESTIONS?

THEORETICAL VS EMPIRICAL PROBABILITIES

PROBABILITY DISTRIBUTIONS

DEFINITION OF PROBABILITY

- **Probability** describes the chance that an uncertain event will occur.

- Probability is always a number between 0 and 1.
  It is often given as a % between 0 and 100.

- Notation for probability:
  P(E) means probability of event E occurring.

Theoretical Probability of an event is the number of ways that the event can occur, divided by the total number of outcomes. It is finding the probability of events that come from a sample space of known equally likely outcomes.

Theoretical Probability Formula

$$P(E) = \frac{n(E)}{n(S)} = \frac{\#\ of\ outcomes\ in\ E}{total\ \#\ of\ outcomes\ in\ S}$$

$P(E)$ = probability that an event, E, will occur.
$n(E)$ = number of equally likely outcomes of E.
$n(S)$ = number of equally likely outcomes of sample space S.

Cite: http://www.regentsprep.org/regents/math/algebra/apr5/theoprop.htm

EQUALLY LIKELY

- If an experiment's sample space, $S = \{s_1, s_2, s_3, \ldots\}$, has equally likely outcomes, then we assume the probability of each event $\{s_1\}$, $\{s_2\}$, $\{s_3\}$… are 1/n, n = number of events.

Example :

A single 6-sided die is rolled. What is the probability of each outcome?

$P(1) = \frac{1}{6}$

$P(4) = \frac{1}{6}$

$P(\text{even}) = \frac{3}{6} = \frac{1}{2}$

$P(\text{odd}) = \frac{3}{6} = \frac{1}{2}$

Another Example of Equally Likely Outcomes was this part of the warm-up ☺

4. A card is drawn at random from a fair standard deck of cards. Find each of the following:

P(heart) $\frac{13}{52} = \frac{1}{4} = 0.25$

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P(2 or jack) $\frac{8}{52} = \frac{2}{13} = 0.154$

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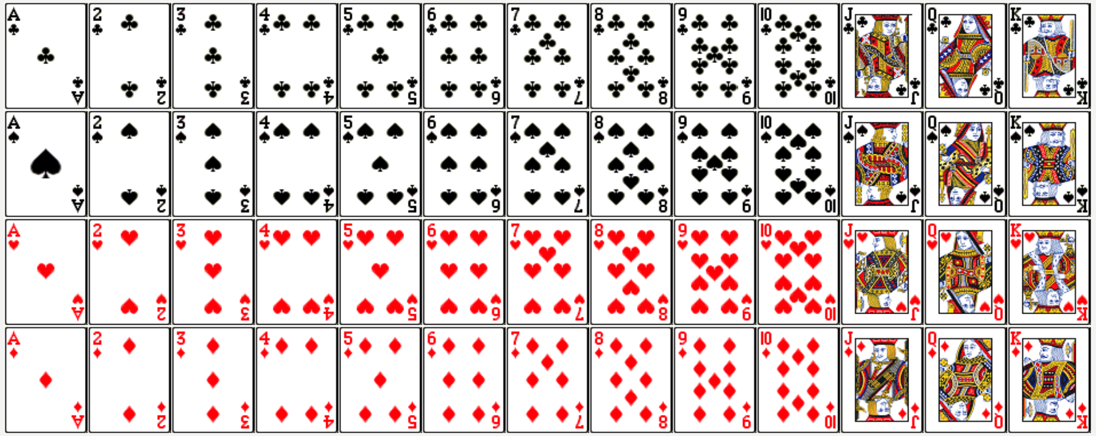
**Please include fraction AND decimal answers!



LET'S EXPERIMENT!

- If we were to actually flip a coin 10 times, would we get heads 50% of the time? After 20 times? 50 times? 1000 times?

- If our relative frequency of a particular event E, flipping a coin, approaches the P(E) after many, many trials, then P(E) is called Empirical probability of E.

EMPIRICAL PROBABILITY

- I need a volunteer to flip a coin 10 times!
- Sample Space S = {H, T}

Trial	1	2	3	4	5	6	7	8	9	10
Outcome										

- Relative frequency of Heads: _____/10 = _____

  Tails: _____/10 = _____

Was P(heads) = 50%??????

Empirical Probability of an event is an "estimate" that the event will happen based on how often the event occurs after collecting data or running an experiment (in a large number of trials). It is based specifically on direct observations or experiences.

(Also known as Experimental Probability)

Empirical Probability Formula

$$P(E) = \frac{\#\ of\ times\ event\ E\ occurs}{total\ \#\ of\ observed\ occurrences}$$

$P(E)$ = probability that an event, E, will occur.
top = number of ways the specific event occurs.
bottom = number of ways the experiment could occur.

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PROBABILITY DISTRIBUTION

- An experiment with a finite sample space of simple events can create a probability distribution.
- Each event in the experiment is assigned a probability and collectively they make the probability distribution for that sample space.
- Usually probability distributions are displayed using tables.
- All probability distributions should total 1 (because they display100% of the outcomes).

GRADES EXAMPLE

- A high school transcript consists of letters grades. If our sample space S = {A, B, C, D, F,} then we have events E: {A}, {B}, {C}, {D}, {F}.
- Find each empirical probability.

Event	A	B	C	D	F	Total
Frequency	6	7	3	1	0	17
P(E)	0.35	0.41	0.18	0.06	0	1

- Collectively this makes an empirical probability distribution.

You Try! A pair of fair dice is cast. Answer using fractions and decimals.

- a. Calculate the probability that the sum of the two numbers of the dice is less than 6.
- b. Calculate the probability that the sum of the two numbers on the dice is more than 7.
- c. Find the probability distribution for the following experiment:

Event	**Probability**
At least 1 die shows 2	
Both show the same number	
Both show different numbers and neither is a 2	

You Try! Answers….. A pair of fair dice is cast. Answer using fractions and decimals.

- a. Calculate the probability that the sum of the two numbers of the dice is less than 6.

$$\frac{10}{36} = \frac{5}{18} = 0.278$$

- b. Calculate the probability that the sum of the two numbers on the dice is more than 7.

Remember this. We did it the other day…

$$S \begin{Bmatrix} (1,1) & (2,1) & (3,1) & (4,1) & (5,1) & (6,1) \\ (1,2) & (2,2) & (3,2) & (4,2) & (5,2) & (6,2) \\ (1,3) & (2,3) & (3,3) & (4,3) & (5,3) & (6,3) \\ (1,4) & (2,4) & (3,4) & (4,4) & (5,4) & (6,4) \\ (1,5) & (2,5) & (3,5) & (4,5) & (5,5) & (6,5) \\ (1,6) & (2,6) & (3,6) & (4,6) & (5,6) & (6,6) \end{Bmatrix}$$

$$\frac{15}{36} = \frac{5}{12} = 0.417$$

You Try! Answers…..

c. Find the probability distribution for the following experiment:

- Suppose you roll a pair of dice:

**Please include fraction AND decimal answers!

Event	Probability
At least 1 die shows 2	$\frac{11}{36} = 0.306$
Both show the same number	$\frac{6}{36} = \frac{1}{6} = 0.167$
Both show different numbers and neither is a 2	$\frac{20}{36} = \frac{5}{9} = 0.556$

$$S \begin{Bmatrix} (1,1) & (2,1) & (3,1) & (4,1) & (5,1) & (6,1) \\ (1,2) & (2,2) & (3,2) & (4,2) & (5,2) & (6,2) \\ (1,3) & (2,3) & (3,3) & (4,3) & (5,3) & (6,3) \\ (1,4) & (2,4) & (3,4) & (4,4) & (5,4) & (6,4) \\ (1,5) & (2,5) & (3,5) & (4,5) & (5,5) & (6,5) \\ (1,6) & (2,6) & (3,6) & (4,6) & (5,6) & (6,6) \end{Bmatrix}$$

ADDING EVENTS

WHAT HAPPENS WHEN WE COMBINE EVENTS?

- The union of two (or more) events E and F in a probability distribution would just be adding the individual event probabilities together…

$$P(E \text{ or } F) = P(E \cup F) = P(E) + P(F)$$

This is used for **Mutually Exclusive Events**

Ex 1: A card is drawn from a well-shuffled deck of 52 playing cards.

- What is the probability that it is a Jack or King?

$$\frac{4}{52}+\frac{4}{52}=\frac{8}{52}=0.154$$

Ex 2: A die is rolled.

- What is the probability of rolling a 6 or an odd number?

$$\frac{1}{6}+\frac{3}{6}=\frac{4}{6}=0.667$$

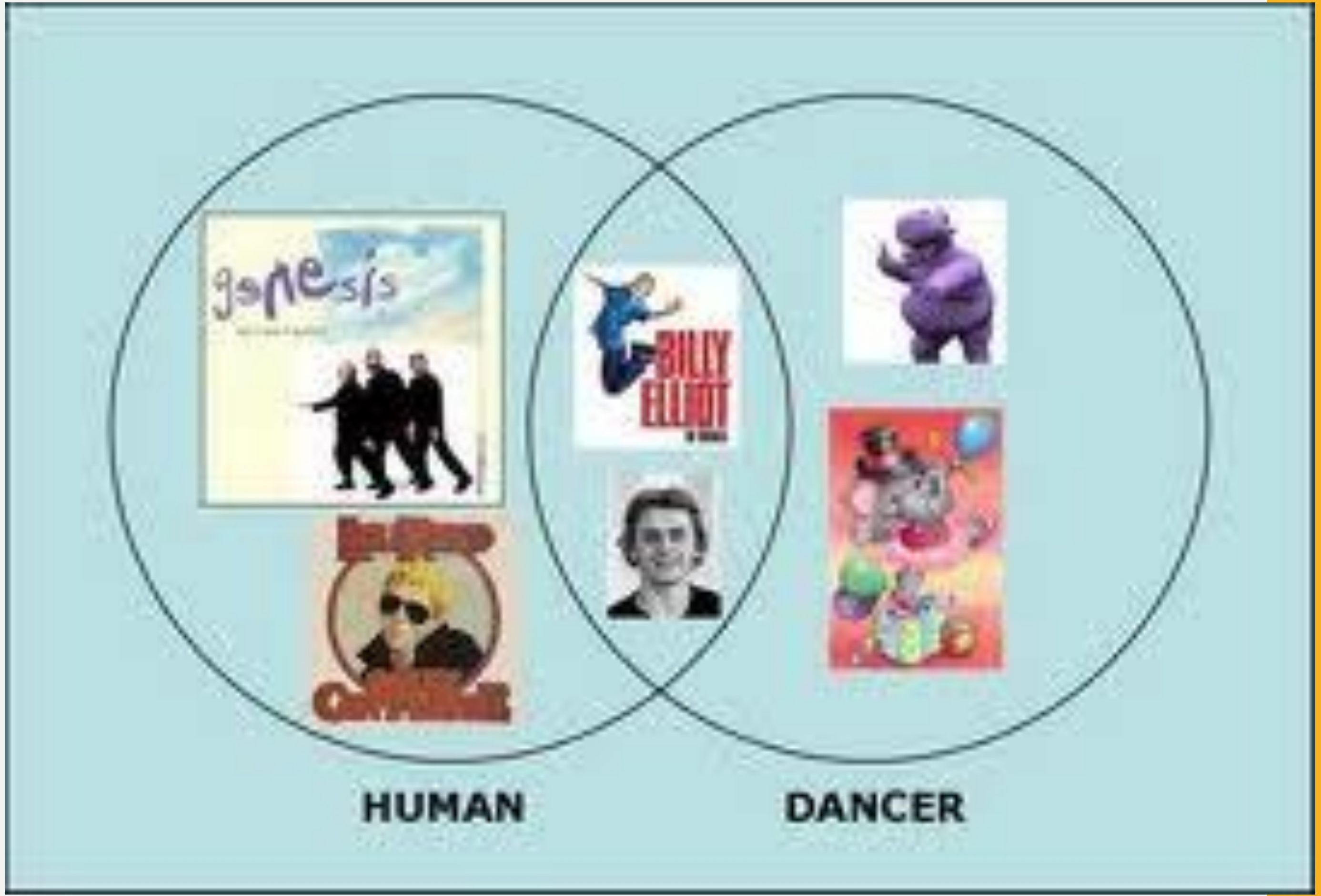
PROBABILITY OF MUTUALLY EXCLUSIVE EVENTS

*If A and B are mutually exclusive (no overlap), then

P(A and B) = __0__ .

- What if E and F are NOT mutually exclusive?

We call them
Mutually Inclusive!



Mutually Inclusive Events: Two events that **can** occur at the same time.

❖Combining not mutually exclusive events!

If **E** and **F** are **mutually <u>inclusive</u> events**, then

$$P(E \text{ or } F) =$$

$$P(E \cup F) = P(E) + P(F) - P(E \cap F)$$

What does this remind you of from earlier?

EXAMPLES

1. What is the probability of choosing a card from a deck of cards that is a club or a ten?

P(choosing a club or a ten)
= P(club) + P(ten) – P(10 and club)
= 13/52 + 4/52 – 1/52
= 16/52
= 4/13 or 0.308

The probability of choosing a club or a ten is 4/13 or 30.8%

2. What is the probability of choosing a number from 1 to 10 that is less than 5 or odd?

P(<5 or odd)

<5 = {1,2,3,4}   odd = {1,3,5,7,9}

= P(<5) + P(odd) – P(<5 and odd)
= 4/10 + 5/10 – 2/10
= 7/10

The probability of choosing a number less than 5 or an odd number is 7/10 or 70%.

You Try!!!

A bag contains 26 tiles with a letter on each, one tile for each letter of the alphabet. What is the probability of reaching into the bag and …

3) randomly choosing a tile with one of the first 10 letters of the alphabet on it or randomly choosing a tile with a vowel on it?

4) randomly choosing a tile with one of the last 5 letters of the alphabet on it or randomly choosing a tile with a vowel on it?

You Try Answers…

3. A bag contains 26 tiles with a letter on each, one tile for each letter of the alphabet. What is the probability of reaching into the bag and randomly choosing a tile with one of the first 10 letters of the alphabet on it or randomly choosing a tile with a vowel on it?

P(one of the first 10 letters or vowel)

= P(one of the 1st 10 letters) + P(vowel) – P(1st 10 and vowel)
= 10/26 + 5/26 – 3/26
= 12/26 or 6/13

The probability of choosing either one of the first 10 letters or a vowel is 6/13 or 46.2%

You Try Answers…

4. A bag contains 26 tiles with a letter on each, one tile for each letter of the alphabet. What is the probability of reaching into the bag and randomly choosing a tile with one of the last 5 letters of the alphabet on it or randomly choosing a tile with a vowel on it?

P(one of the last 5 letters or vowel)

= P(one of last 5 letters) + P(vowel) – P(last 5 and vowel)
= 5/26 + 5/26 – 0
= 10/26 or 5/13

The probability of choosing either one of the first 10 letters or a vowel is 5/13 or 38.5%

❖Rules of Complements

- If **E** is an event of an experiment and **E**^{c} denotes the complement of **E**, then

$$P(E^c) = 1 - P(E)$$

Example:

If 3 prizes for every 1000 raffle tickets,
P(not win) = 1 – P(win) = 1 – 3/1000 = 997/1000

- **Example:**

  The quality-control department of Vista Vision has determined from records that 3% of TV sets sold experience video problems, 1% experience audio problems, and 0.1% experience both video as well as audio problems before the expiration of the 90-day warranty.

- What is the probability that a Vista Vision TV will not experience video or audio problems before the warranty expires?

$$P(\text{not video or audio}) = 1 - P(\text{video or audio})$$

$$P(\text{video or audio}) = P(\text{video}) + P(\text{audio}) - P(\text{video} \cap \text{audio})$$

$$= 0.03 + 0.01 - 0.001$$

$$= 0.039$$

$$P(E^c) = 1 - 0.039 = 0.961$$

96.1%

- Let **E** and **F** be two events of an experiment with sample space **S**. Suppose P(E)= 0.2, P(F)= 0.1, and $P(E \cap F)$= 0.05. Compute:

$$P(E \cup F) = P(E) + P(F) - P(E \cap F)$$
$$= .2 + .1 - .05$$
$$= .25$$

(DeMorgan's Law)

$$P(E^c \cap F^c) = P[(E \cup F)^c]$$
$$= 1 - P(E \cup F)$$
$$= 1 - .25$$
$$= .75$$

$$P(E^c \cap F) = P(F) - P(E \cap F)$$
$$= .1 - .05$$
$$= .05$$

You Try!! Let **E** and **F** be two *mutually exclusive* events and suppose P(E) = 0.1 and P(F) = 0.6. Compute:

$P(E \cap F)$ = 0
(mutually exclusive)

$P(E \cup F)$ = 0.7

$P(E^c)$ = 0.9

$P(E^c \cap F^c)$ (DeMorgan's Law)

$$= P[(E \cup F)^c]$$
$$= 1 - P(E \cup F)$$
$$= 1 - 0.7$$
$$= 0.3$$

$P(E^c \cup F^c)$

Remember, $P(E^c \cup F^c) =$

$$P(E^c) + P(F^c) - P(E^c \cap F^c)$$
$$P(E^c) = 1 - P(E) = 1 - 0.1 = 0.9$$
$$P(F^c) = 1 - P(F) = 1 - 0.6 = 0.4$$
$$P(E^c \cap F^c) = 0.3$$
$$= 0.9 + 0.4 - 0.3 = 1$$

HW
PACKET P.9-10