Day 6 ~ Increasing/Decreasing, and Extrema

A Graphical Approach

Warm Up ~ Day 6

1) $$f(x)=\frac{x-4}{x^2-7x+12}$$

Find the a) domain b) x & y intercepts

c) range d) discontinuities

e) end behavior using limit notation

2) $$g(x)=\sqrt{3x^2+7x-6}$$

Find the a) domain b) x & y intercepts

c) range

Warm Up ~ Day 6 ANSWERS

1) $f(x) = \dfrac{x-4}{x^2-7x+12}$

Find the

a) domain $D: (-\infty, 3) \cup (3,4) \cup (4, \infty)$

b) x & y intercepts $x-\text{int}: none$ $y-\text{int}: (0, -1/3)$

c) range $R: (-\infty, 0) \cup (0, 1) \cup (1, \infty)$

d) discontinuities Hole (Removable Disc.) at (4,1)

V.A. (NonRemovable Disc.) at x=3

e) end behavior using limit notation

$\lim_{x \to \infty} f(x) = 0$ $\lim_{x \to -\infty} f(x) = 0$

Warm Up ~ Day 6 ANSWERS

2) $g(x) = \sqrt{3x^2 + 7x - 6}$

Find the

a) domain $Domain: \ (-\infty, -3] \cup [\frac{2}{3}, \infty)$

b) x & y intercepts $x - \text{int} : (-3, 0) \ and \ (2/3, 0)$

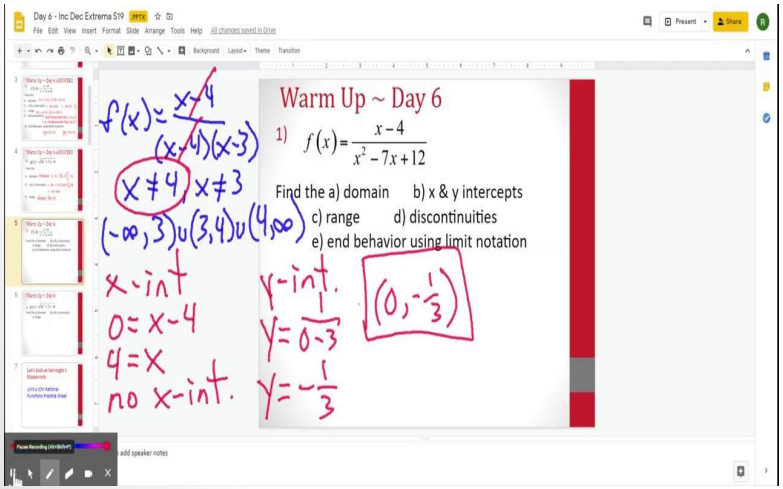
$y - \text{int} : none$

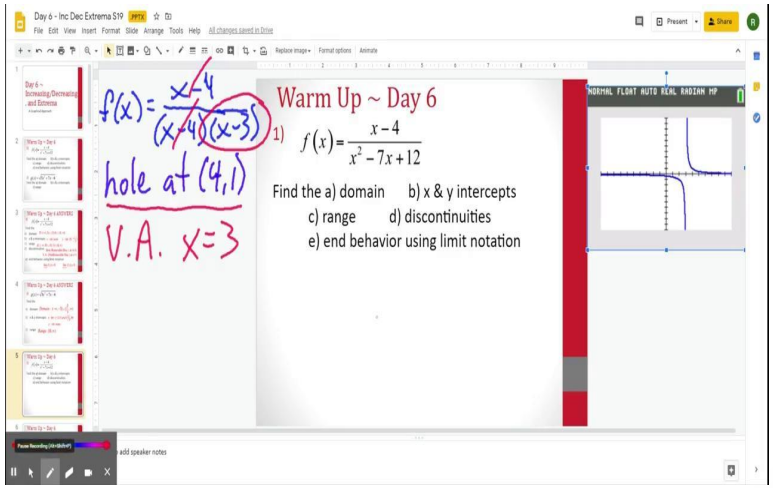
c) range $Range : [0, \infty)$

Warm Up ~ Day 6

1) $$f(x)=\frac{x-4}{x^2-7x+12}$$

Find the a) domain b) x & y intercepts
c) range d) discontinuities
e) end behavior using limit notation





Warm Up ~ Day 6

2) $g(x) = \sqrt{3x^2 + 7x - 6}$

Find the a) domain b) x & y intercepts
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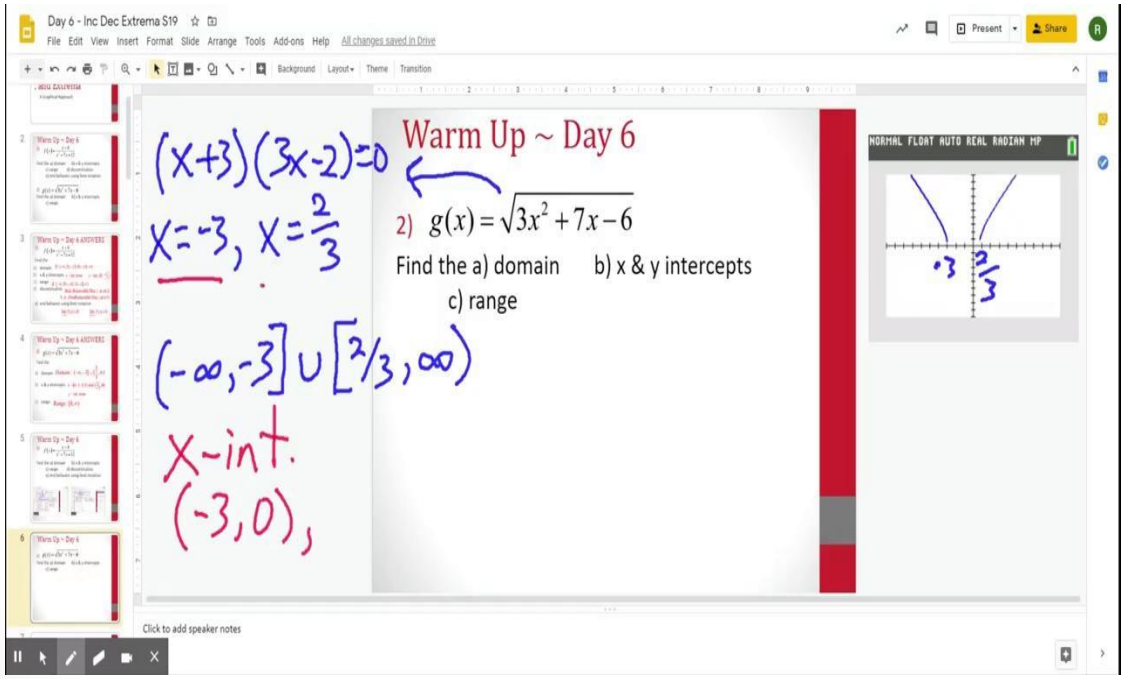


Announcements

Tonight's Homework

Packet p. 7-8

Notes Day 6

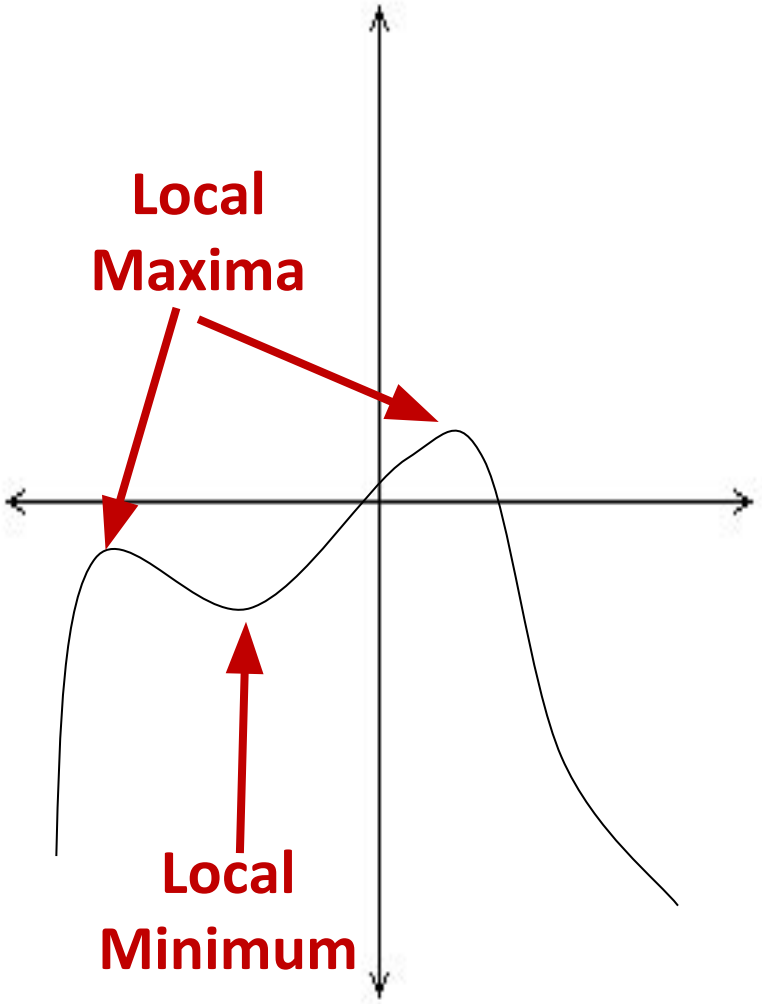
Part 1 – Increasing, Decreasing, Max and Min

Local/Relative Extrema

- Local Maximum – the maximum **y-value** of a function on some small interval

- Local Minimum – the minimum **y-value** of a function on some small interval

- Local Extrema are also called “relative” extrema



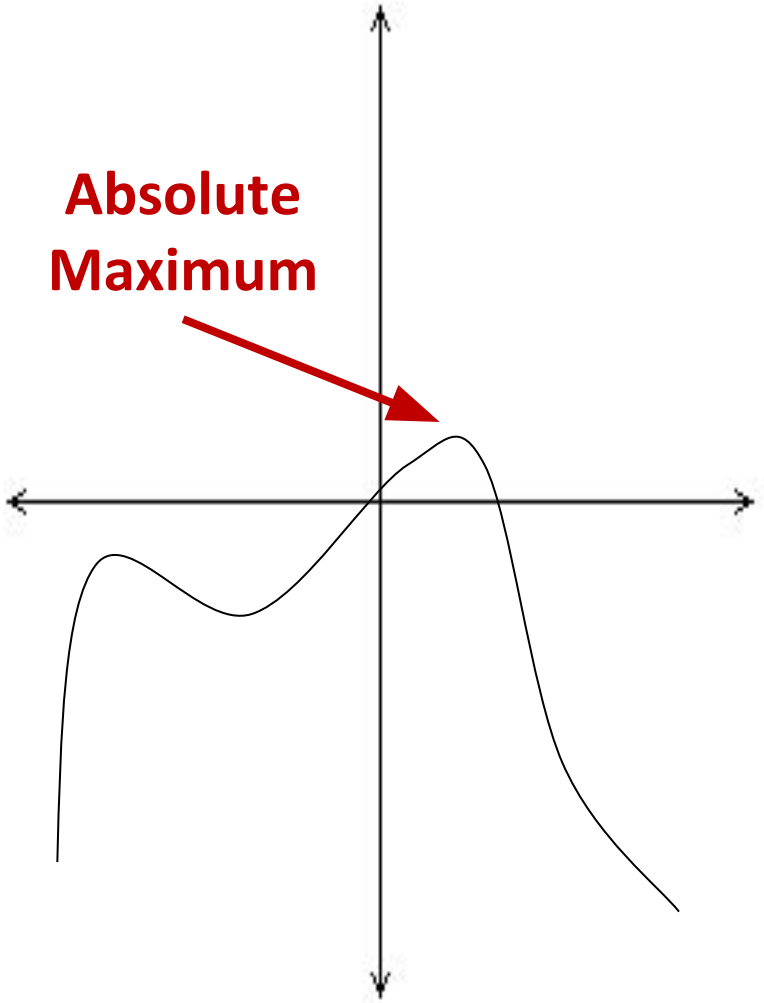
Absolute Extrem

- **Absolute** extrema are the maximum and minimum of **all** range values.

- Report all extrema (local or absolute) as:

  Max/Min *y*-value occurs at *x*-value

- Occurs when y-values change from increasing to decreasing

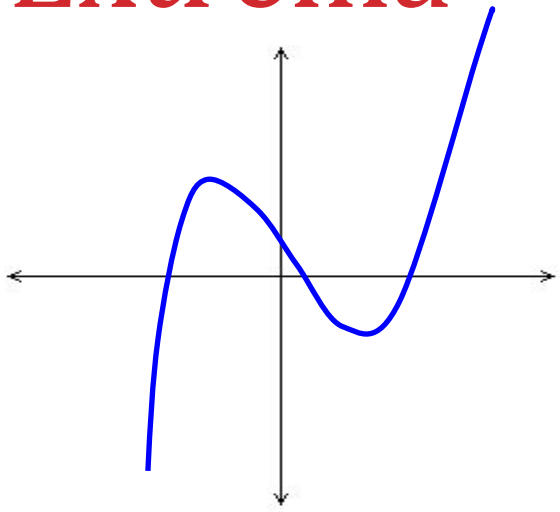


Absolute Max Example:

Maximum 3 occurs at x = 4

Local & Absolute Extrema

- What are the local extrema of: $g(x) = x^3 - 4x + 1$



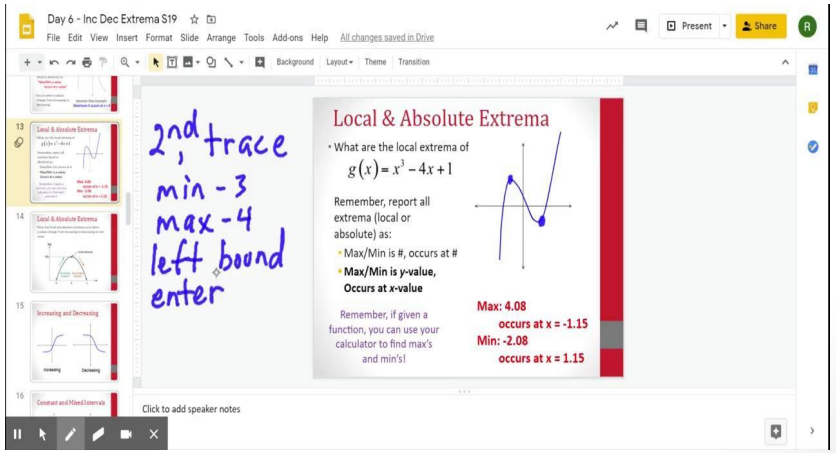
Remember, report all extrema (local or absolute) as:

Max: 4.08 occurs at x = -1.15

Min: -2.08 occurs at x = 1.15

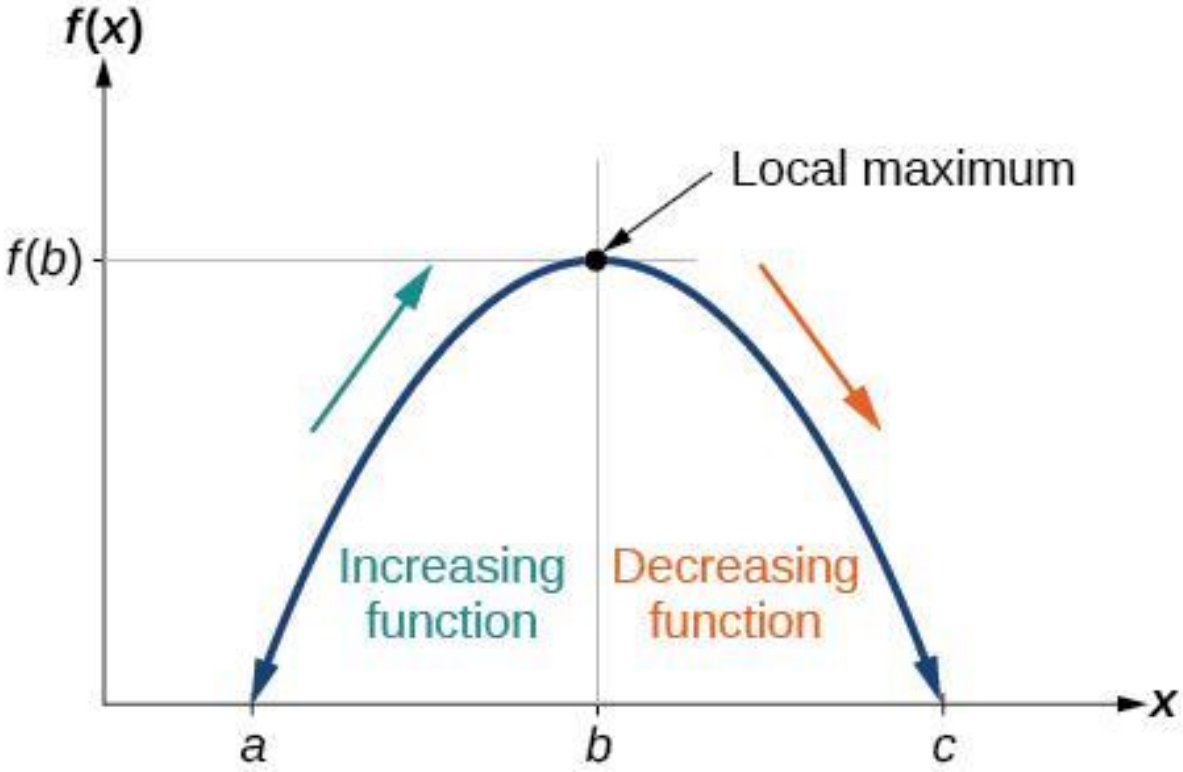
- Max/Min is #, occurs at #
- **Max/Min is *y*-value, Occurs at *x*-value**

Remember, if given a function, you can use your calculator to find max's and min's!

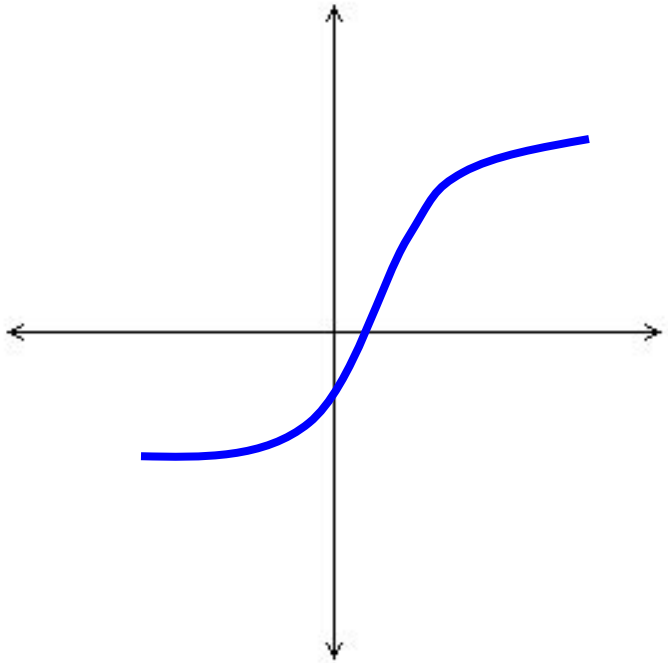


Local & Absolute Extrema

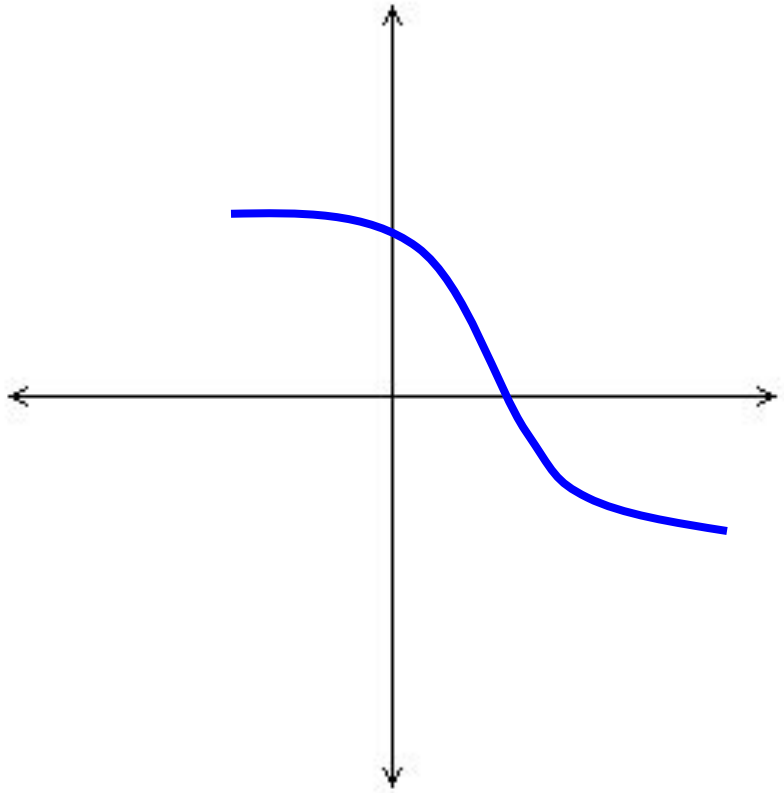
- Note that local and absolute extrema occur when y-values change from increasing to decreasing or vice versa



Increasing and Decreasing

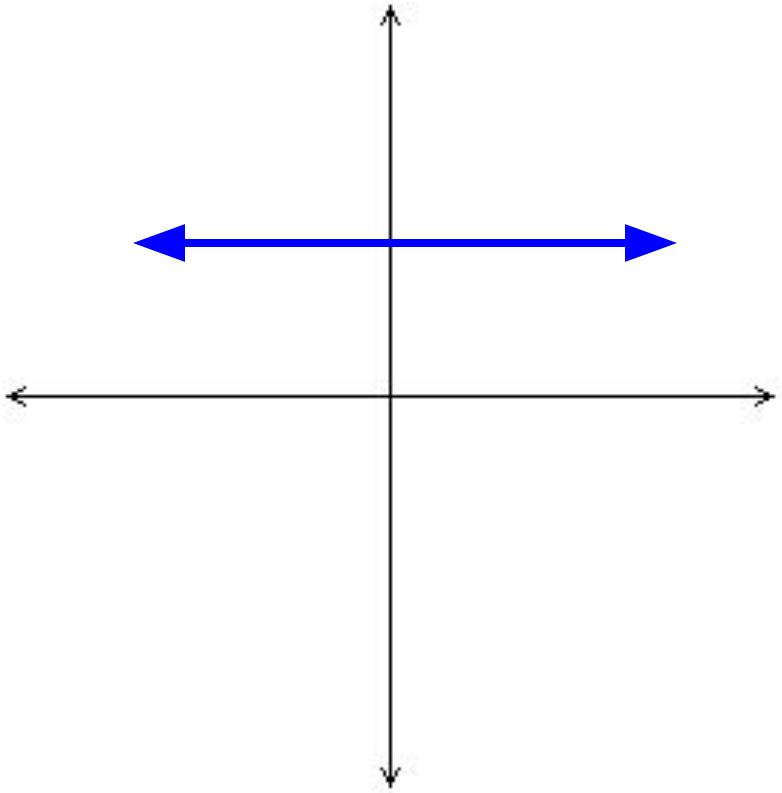


Increasing

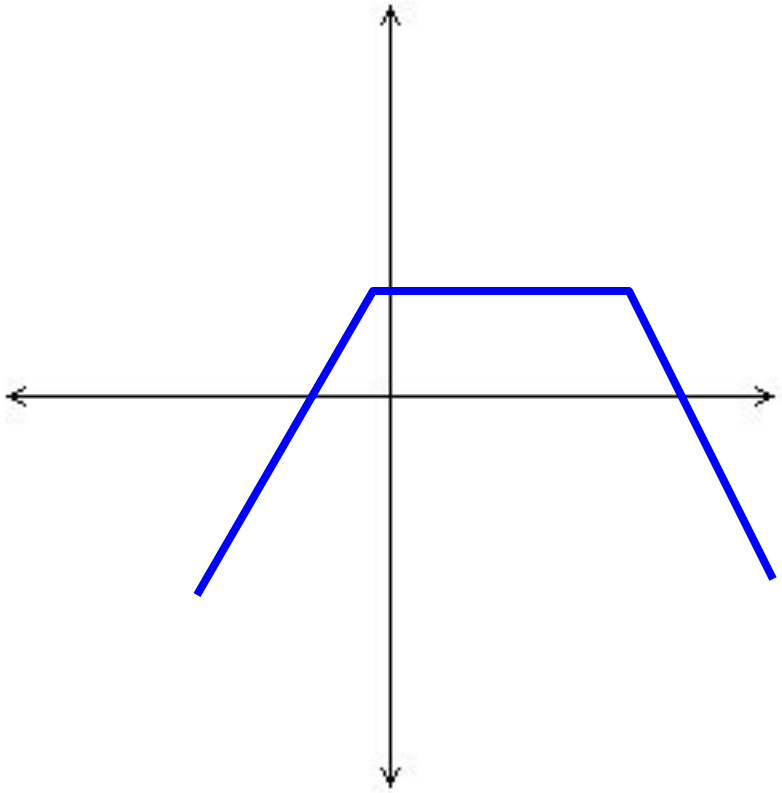


Decreasing

Constant and Mixed Intervals



Constant



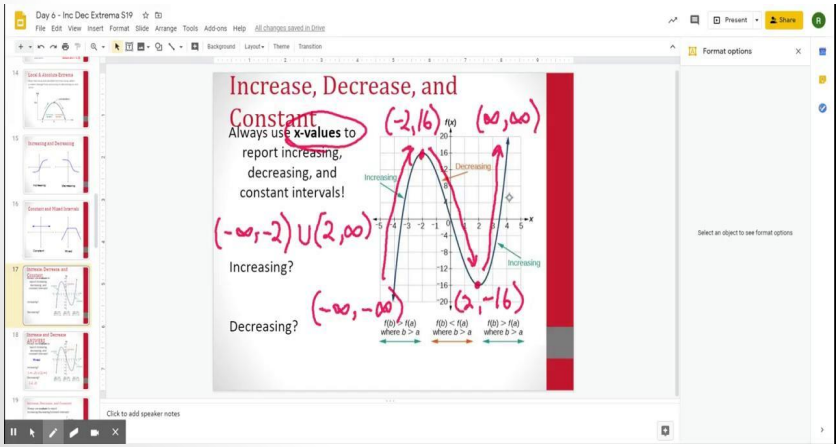
Mixed

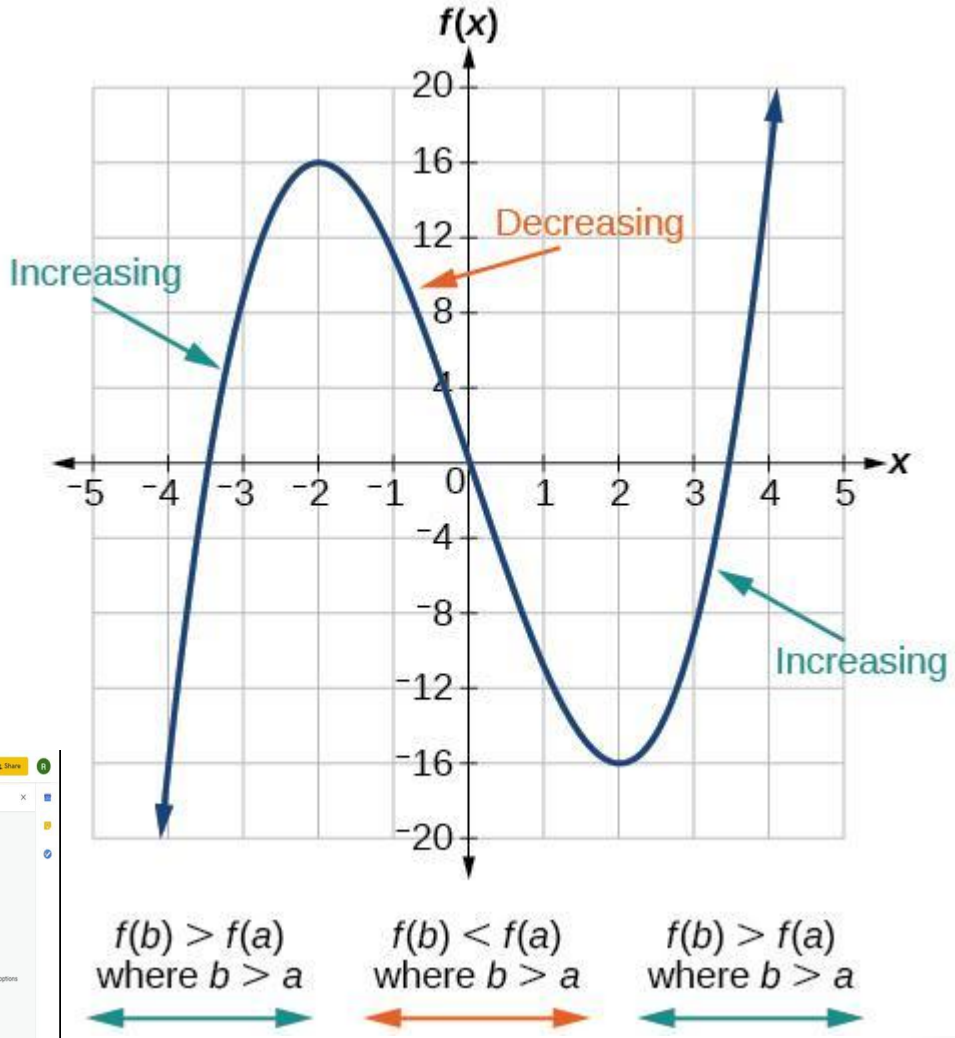
Increase, Decrease, and Constant

Always use **x-values** to report increasing, decreasing, and constant intervals!

Increasing?

Decreasing?





Increase and Decrease ANSWERS

Always use **x-values** to report increasing, decreasing, and constant intervals!

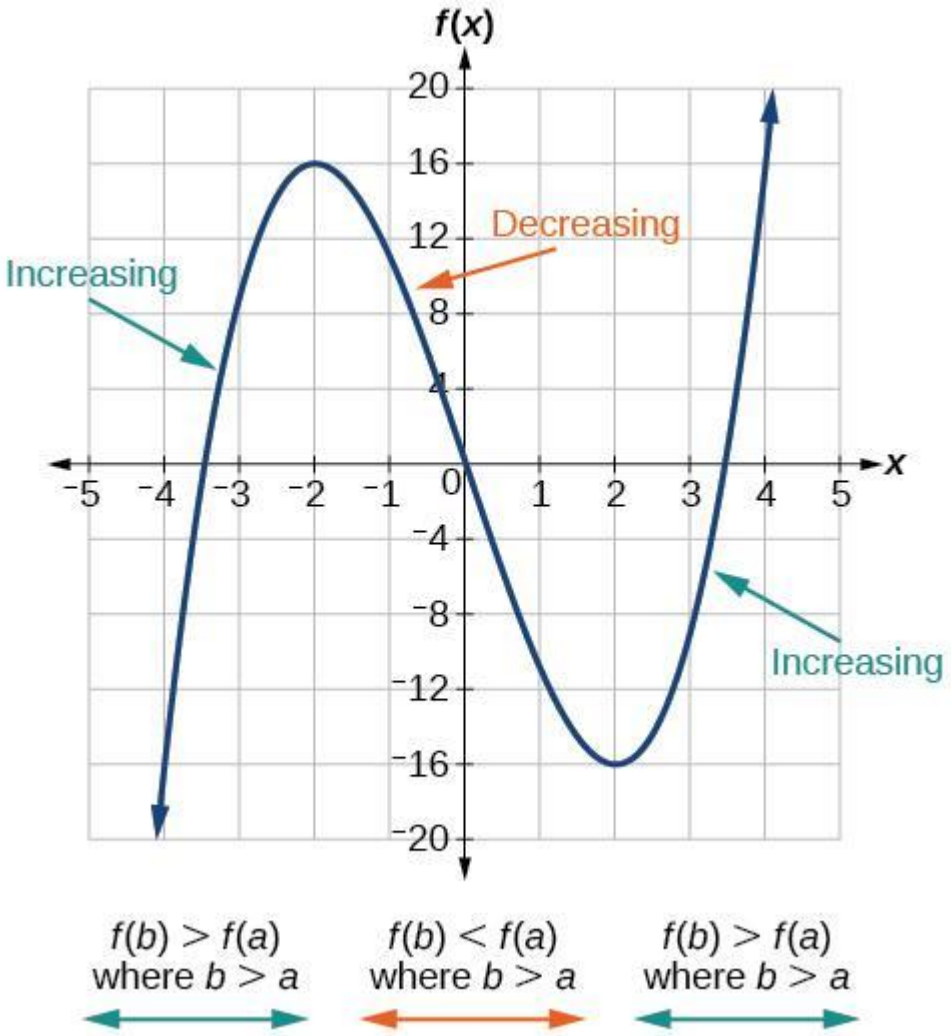
Mixed

Increasing?

$(-\infty,-2) \cup (2,\infty)$

Decreasing?

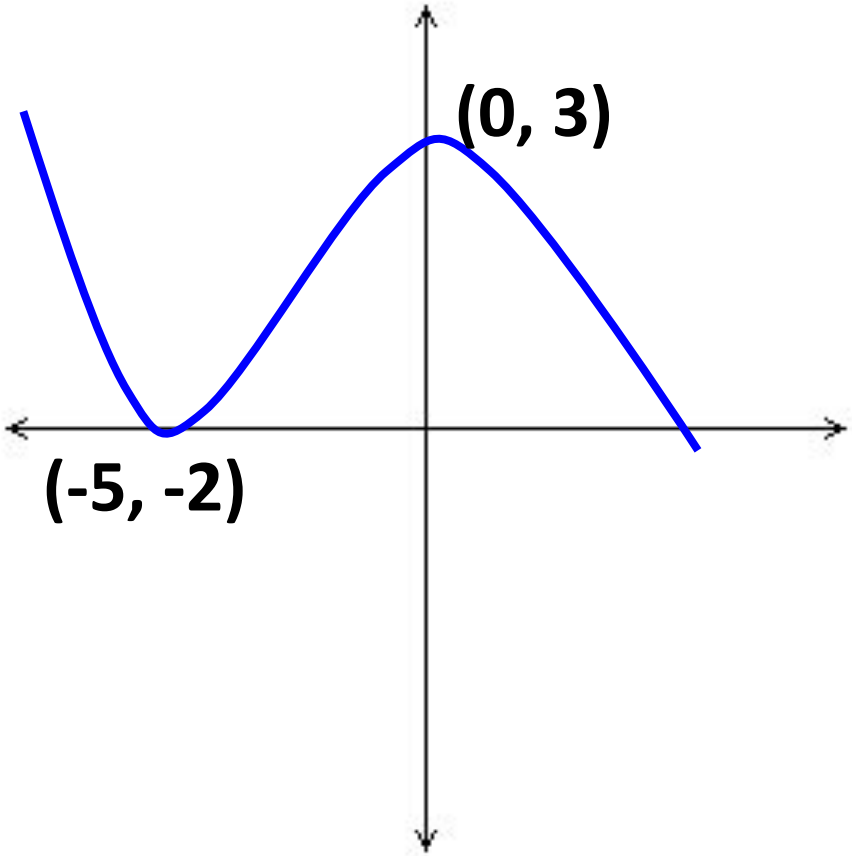
$(-2, 2)$



Increase, Decrease, and Constant

Always use **x-values** to report increasing/decreasing/constant intervals!

Mixed



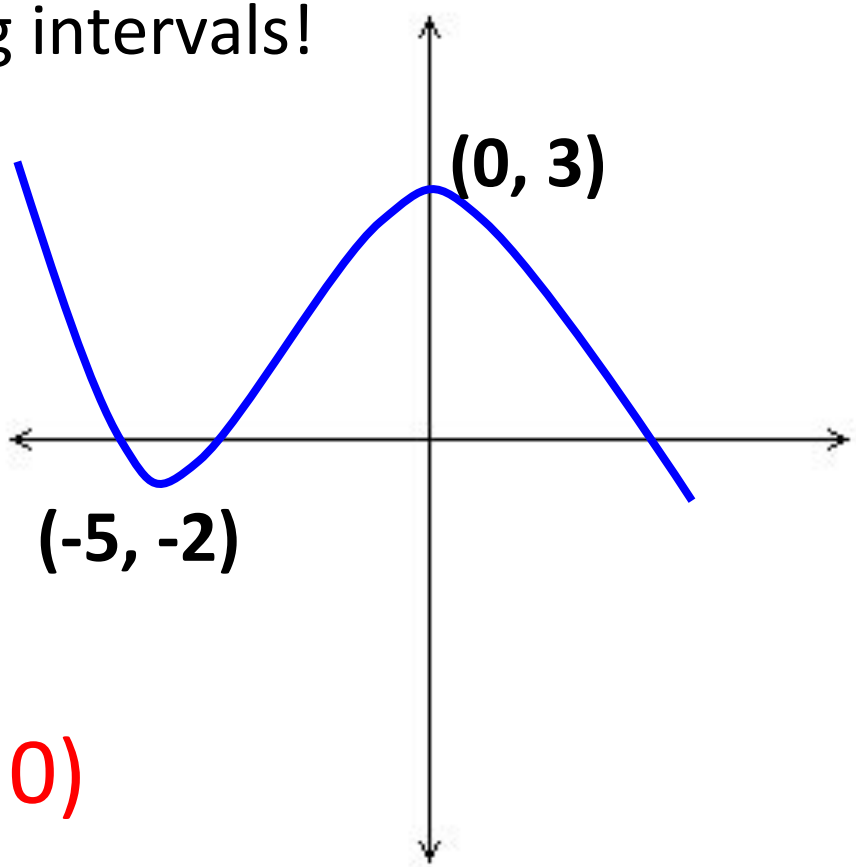
Increasing?

Decreasing?

Increase and Decrease ANSWERS

Always use **x-values** to report increasing/decreasing intervals!

Mixed



Increasing? (-5 , 0)

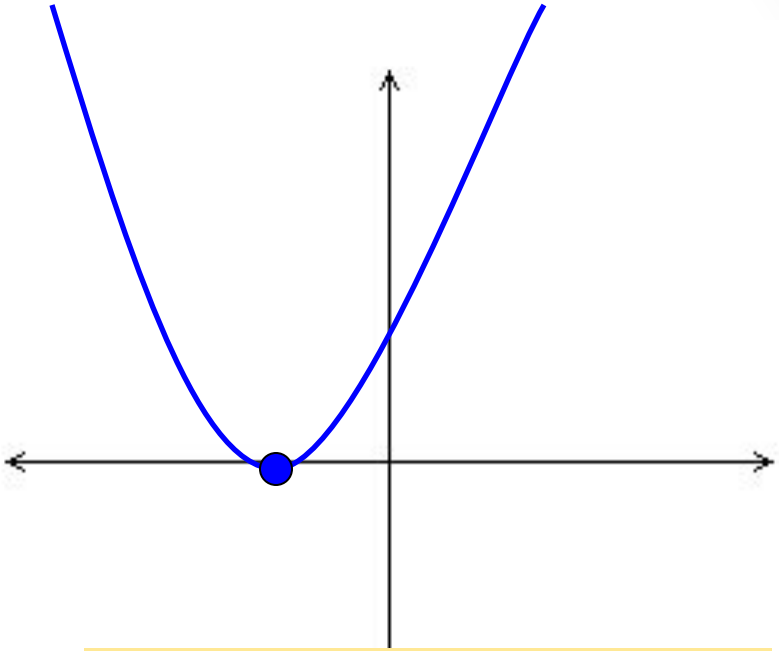
Decreasing? (-∞,-5) U (0,∞)

Increasing/Decreasing (Algebraic Models)

- Ex:

$$f(x)=(x+2)^2$$

- What is the interval when this function is:
  - Increasing
  - Decreasing

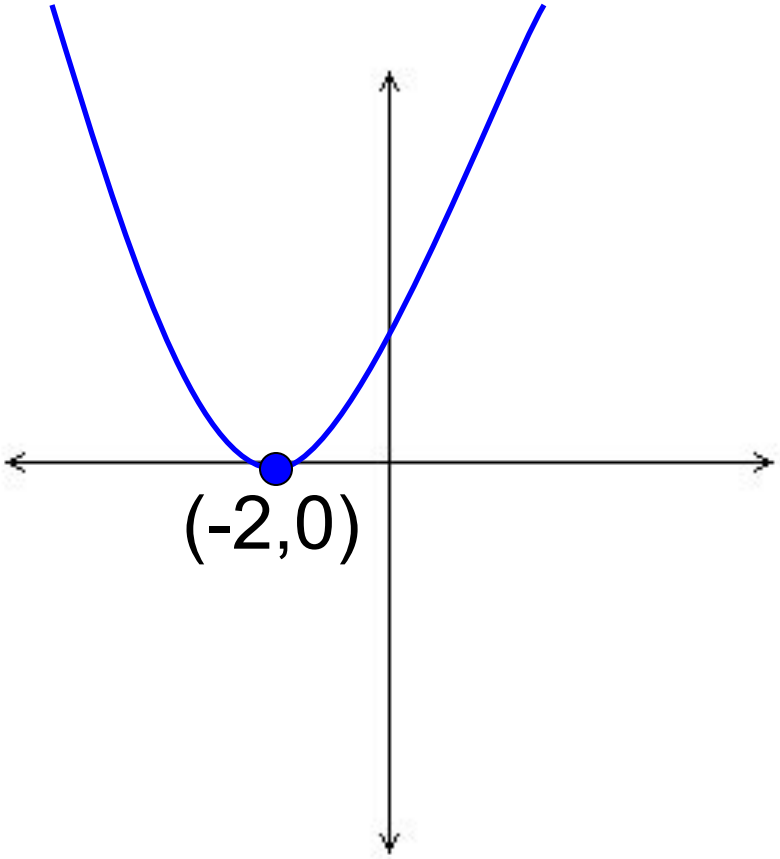


HINT: 1st Get any Max and/or Min Values in Calculator OR with Algebra!

Increasing/Decreasing (Algebraic Models) ANSWERS

- Ex:

$$f(x)=(x+2)^2$$



- What is the interval when this function is:
  - Increasing $(-2,\infty)$
  - Decreasing $(-\infty,-2)$

Always use x values!

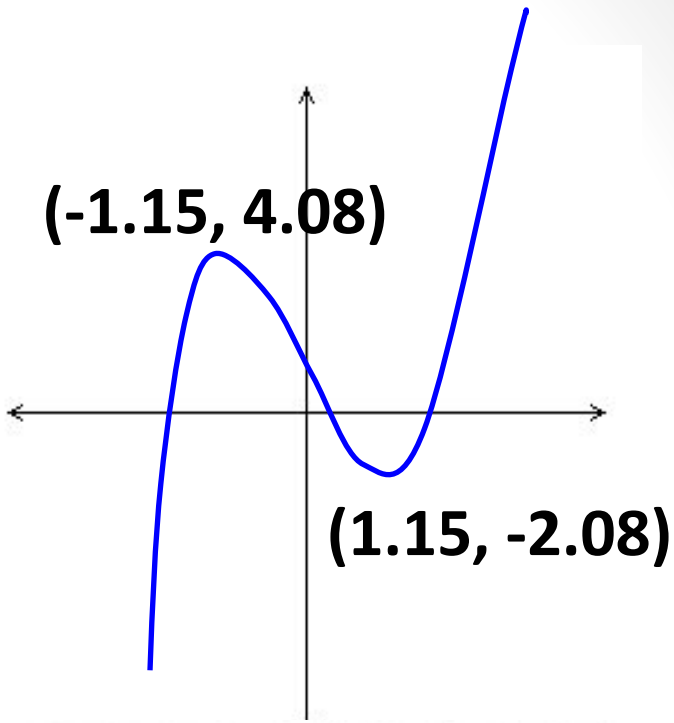
You Try!

1) Back to this example…

$$g(x) = x^3 - 4x + 1$$

- When is the function increasing?

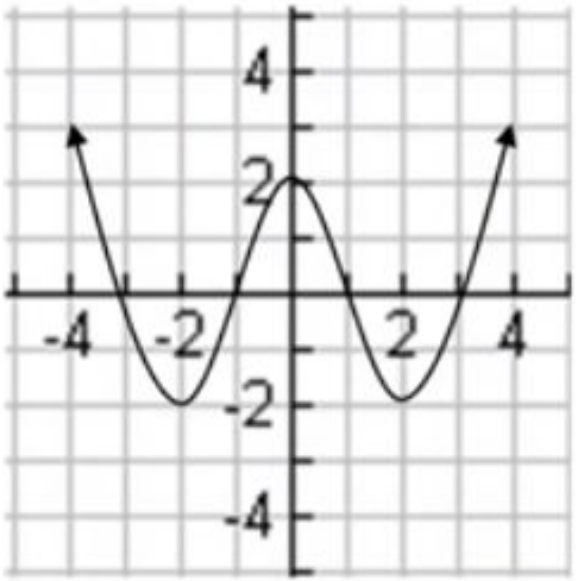
- When is the function decreasing?



2) Using the graph shown to the right,

- When is the function increasing?

- When is the function decreasing?



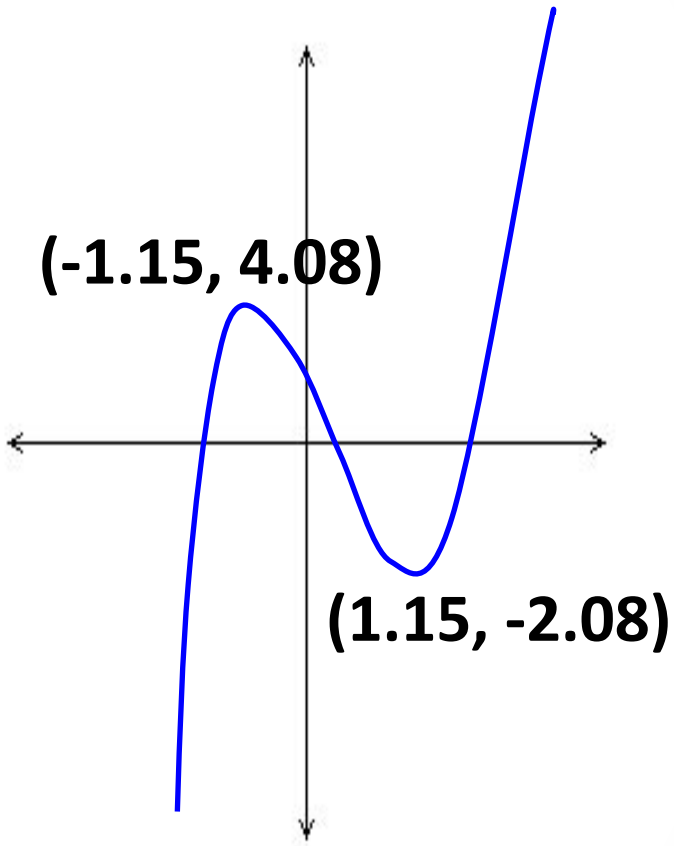
You Try!…ANSWERS

1) We found the extrema:

Max: 4.08 occurs at x = -1.1

Min: -2.08 occurs at x = 1.1

$$g(x) = x^3 - 4x + 1$$



- When is the function increasing?

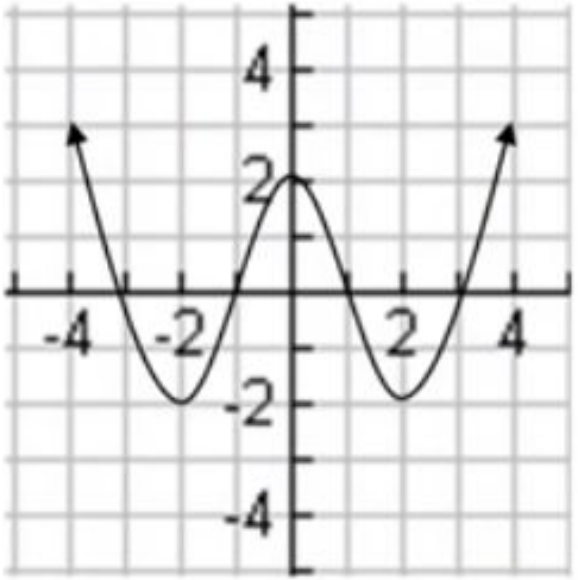
$(-\infty,-1.15) \cup (1.15,\infty)$

- When is the function decreasing?

$(-1.15,1.15)$

You Try!

2) Using the graph shown to the right,



First, find the extrema

Max: 2 occurs at x = 0

Min: -2 occurs at x = -2 and 2

- When is the function increasing?

$(-2,0) \cup (2,\infty)$

- When is the function decreasing?

$(-\infty,-2) \cup (0,2)$

Student Practice

- Find the intervals increasing/decreasing.
- State the domain and range.

- A) $g(x) = \dfrac{x^2}{x^2 - 1}$

Increasing:

Decreasing:

Domain:

Range:

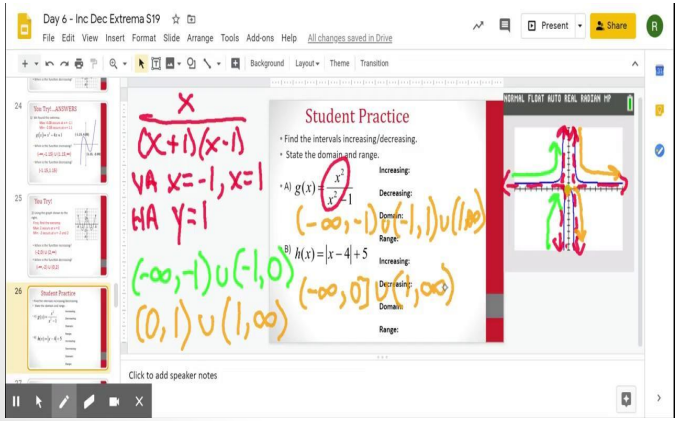
- B) $h(x) = |x - 4| + 5$

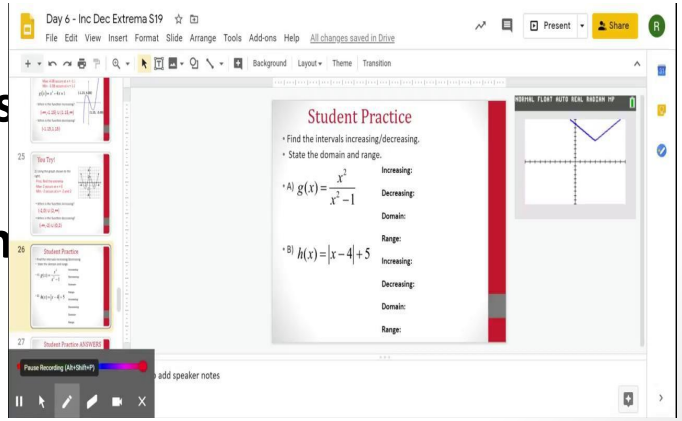
Increasing:

Decreas

Domain

Range:





Student Practice ANSWERS

- Find the intervals increasing/decreasing.
- State the domain and range.

- A) $g(x) = \dfrac{x^2}{x^2 - 1}$

Increasing: $(-\infty,-1) \cup (-1,0)$

Decreasing: $(0,1) \cup (1,\infty)$

Domain: $(-\infty, -1) \cup (-1, 1) \cup (1, \infty)$

Range: $(-\infty, 0] \cup (1, \infty)$

- B) $h(x) = |x - 4| + 5$

Increasing: $(4,\infty)$

Decreasing: $(-\infty,4)$

Domain: $(-\infty, \infty)$

Range: $[5, \infty)$

Practice! What is the EQUATION of the horizontal asymptote for the following functions? Then write the end behavior using limits.

$$g(x) = -\frac{2x^5}{5x^4 + 6}$$

Bottom > Top

y=**O**

Same = ratio

Top > Bottom

O ↑ No HA.

N

$$f(x) = \frac{2x^3 + 7}{5x - 7x^3 + 15}$$

$$h(x) = \frac{3x + 14}{9x^2}$$

Practice! What is the EQUATION of the horizontal asymptote for the following functions? Then write the end behavior using limits.

$$g(x) = -\frac{2x^5}{5x^4 + 6}$$

$$\lim_{x \to \infty} g(x) = -\infty$$

$$\lim_{x \to -\infty} g(x) = \infty$$

$H.A. : none$

Bottom > Top

y=**O**

Same = ratio

Top > Bottom

O ↑ No HA.

N

$$f(x) = \frac{2x^3 + 7}{5x - 7x^3 + 15}$$

$$\lim_{x \to \infty} f(x) = -\frac{2}{7}$$

$$\lim_{x \to -\infty} f(x) = -\frac{2}{7}$$

$H.A. : y = -2/7$

$$h(x) = \frac{3x + 14}{9x^2}$$

$$\lim_{x \to \infty} h(x) = 0$$

$$\lim_{x \to -\infty} h(x) = 0$$

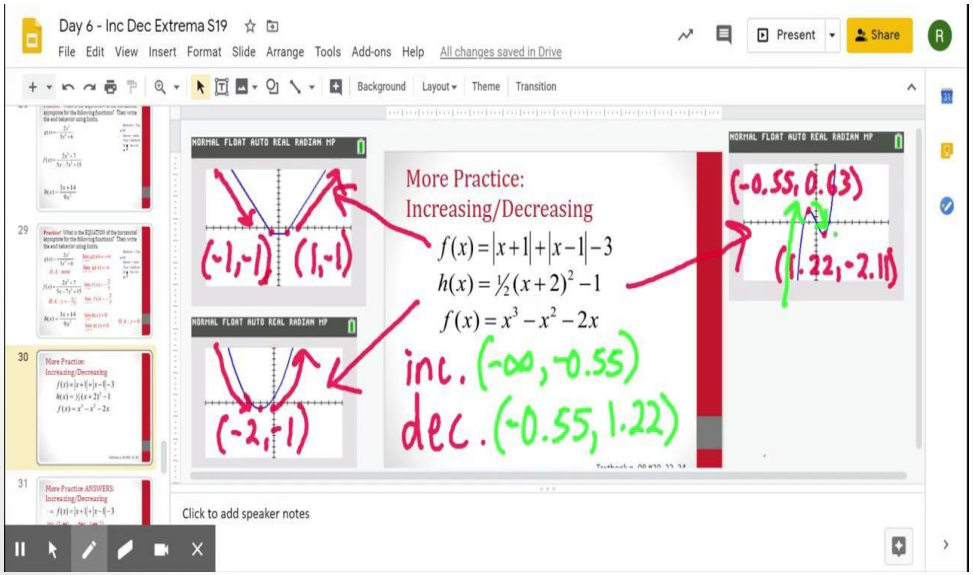
$H.A. : y = 0$

More Practice: Increasing/Decreasing

$$f(x) = |x+1| + |x-1| - 3$$

$$h(x) = \tfrac{1}{2}(x+2)^2 - 1$$

$$f(x) = x^3 - x^2 - 2x$$



Textbook p. 98 #30, 32, 34

More Practice ANSWERS: Increasing/Decreasing

- 30. $f(x) = |x+1| + |x-1| - 3$

inc. (1,∞) dec. (-∞,1)

- 32. $h(x) = \frac{1}{2}(x+2)^2 - 1$

inc. (-2,∞) dec. (-∞,-2)

- 34. $f(x) = x^3 - x^2 - 2x$

inc. (-∞,-0.55) U (1.22,∞)

dec. (-0.55,1.22)

Textbook p. 98 #30, 32, 34

Practice

Find the a) domain b) x & y intercepts c) range
d) discontinuities e) end behavior using limit notation

- 1) $h(x) = \dfrac{7x + 15}{2x^2}$

- 2) $g(x) = \dfrac{4x^3}{5x^2 + 9}$

Practice ANSWERS

- Find the a) domain b) x & y intercepts c) range d) discontinuities e) end behavior using limit notation

- 1)

$$h(x)=\frac{7x+15}{2x^2}$$

$D:(-\infty,0)\cup(0,\infty)$ $R:[-0.4,\infty)$

$x-\text{int}:(\frac{-15}{7},0)$ $y-\text{int}:none$

$\lim_{x\to\infty} f(x)=0$ $\lim_{x\to-\infty} f(x)=0$

- 2)

$$g(x)=\frac{4x^3}{5x^2+9}$$

$D:(-\infty,\infty)$ $R:(-\infty,\infty)$

$x-\text{int}:(0,0)$ $y-\text{int}:(0,0)$

$\lim_{x\to\infty} g(x)=\infty$ $\lim_{x\to-\infty} g(x)=-\infty$