ICM Day 4:

Experiments, Sample Spaces and Events

ICM Day 4 Arrival:

-Pick up Quiz Review Handout by door *IF you didn't on Friday*

-Open HW packet to p. 5-6

- Get out Name Tent

- Get out warm-up paper

Day 4 Warm-up:
Permutations vs. Combinations

1. If you have a standard deck of cards in how many different hands exists of: **a)** 5 cards **b)** 2 cards

2. Choose 3 desserts from a menu of 8 desserts

3. Choose a winner and a runner up from the 40 Miss Pickle Princess contestants

4. How many different 11-letter arrangements are there for a) PALINDROMES b) PRICELESSLY?

5. Assign the part of a play to the 4 different lead characters from a group of 20 who tried out and 3 backstage crew members (they all have the same job) from a group of 5.

Riddle: What integer between 1-100 when spelled out is in alphabetical order?

Warm-up: Permutations vs. Combinations

1. If you have a standard deck of cards in how many different hands exists of: **a)** 5 cards **b)** 2 cards

$${}_{52}C_5 = 2,598,960 \qquad {}_{52}C_2 = 1,326$$

2. Choose 3 desserts from a menu of 8 desserts

Combination ${}_8C_3 = 56$

3. Choose a winner and a runner up from the 40 Miss Pickle Princess contestants **Permutation** ${}_{40}P_2 = 1560$

4. How many different 11-letter arrangements are there for a) PALINDROMES b) PRICELESSLY?

$$11! = 39,916,800 \qquad \frac{11!}{(2!2!2!)} = 4,989,600$$

5. Assign the part of a play to the 4 different lead characters from a group of 20 who tried out and 3 backstage crew members (they all have the same job) from a group of 5. **Perm&Comb** ${}_{20}P_4 * {}_5C_3 = 1,162,800$

Warm Up

- Riddle:

What integer between 1-100 when spelled out is in alphabetical order?

- Forty!

Homework Questions?!

Homework Day 4

Tonight's HW
= Quiz Review Sheet
& Study for tomorrow's Quiz

Study your notations, Formulas, etc!! ☺

Check Review Sheet answers online tonight!

Let's Review Quiz Day Procedures

Announcements

- PreAssessment Corrections
  - are due by Quiz 2 day!
  - Rework ones you missed (show work) on NEW notebook paper
  - Get help, where needed, so you're ready for this stuff on Quiz 2 & Test
- Remember to Bookmark our class website
- Remember to sign up for the Remind class group

- The members of a string quartet composed of 2 violinists, a violist, and a cellist are to be selected from a group of 6 violinists, 3 violists, and 2 cellists, respectively.

a) In how many ways could the string quartet be formed?

$${}_6C_2 \times {}_3C_1 \times {}_2C_1 = 90$$

b) In how many ways can the string quartet be formed if one of the violinists is to be designated as 1st violinists and the other is to be designated as 2nd violinists?

$${}_6P_2 \times {}_3C_1 \times {}_2C_1 = 180$$

Experiments, Sample Spaces & Events

Section 7.1

Definition of probability

- **Probability** describes the chance that an uncertain event will occur.

$$\frac{\text{desired \#}}{\text{total \#}}$$

- Probability is always a number between 0 and 1. It is often given as a % between 0 and 100.

- Notation for probability:
  P(E) means probability of event E occurring.

Theoretical Probability of an event is the number of ways that the event can occur, divided by the total number of outcomes. It is finding the probability of events that come from a sample space of known equally likely outcomes.

Theoretical Probability Formula

$$P(E) = \frac{n(E)}{n(S)} = \frac{\#\ of\ outcomes\ in\ E}{total\ \#\ of\ outcomes\ in\ S}$$

$P(E)$ = probability that an event, E, will occur.
$n(E)$ = number of equally likely outcomes of E.
$n(S)$ = number of equally likely outcomes of sample space S.

Theoretical Probability = what *SHOULD* happen, in theory

# of ways desired event E occurs
total # in sample space

Cite: http://www.regentsprep.org/regents/math/algebra/apr5/theoprop.htm

Empirical Probability of an event is an "estimate" that the event will happen based on how often the event occurs after collecting data or running an experiment (in a large number of trials). It is based specifically on direct observations or experiences.

(Also known as Experimental Probability)

Empirical Probability Formula

$$P(E) = \frac{\#\ of\ times\ event\ E\ occurs}{total\ \#\ of\ observed\ occurrences}$$

$P(E)$ = probability that an event, E, will occur.
top = number of ways the specific event occurs.
bottom = number of ways the experiment could occur.

Empirical Probability = what *ACTUALLY happened* in an experiment

$$\frac{\#\ \text{of ways desired event E occurs}}{\#\ \text{of total trials}}$$

Cite:
http://www.regentsprep.org/regents/math/algebra/apr5/theoprop.htm

Terminology

- An Experiment is an activity with observable results. (called outcomes)

- Sample Space: The set of all possible outcomes
- Must use S = { , , , …}
  **must include ALL outcomes!

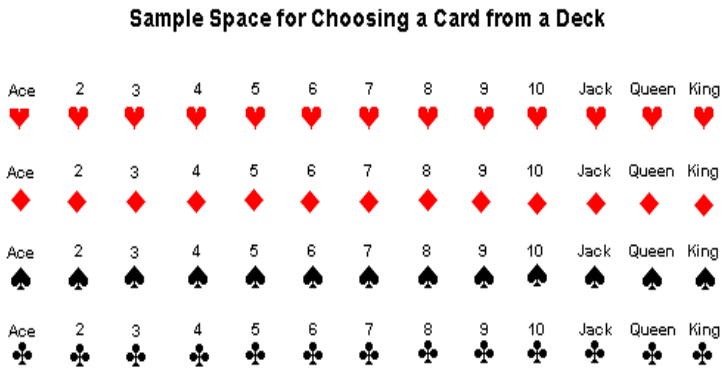
- Event: subset of a sample space
- List events using {…}, {…},

Ex. Rolling a die

- Outcomes: landing with a 1,2,3,4,5, or 6 face up
- Sample Space: S = {1,2,3,4,5,6}
- Events: $\varnothing$, {1}, {2}, {3}, {4}, {5}, {6}, S
- S is that certain event (contains all outcomes)
  - Like the Universal set so it must occur
- $\varnothing$ is an impossible event. (no elements or outcomes)

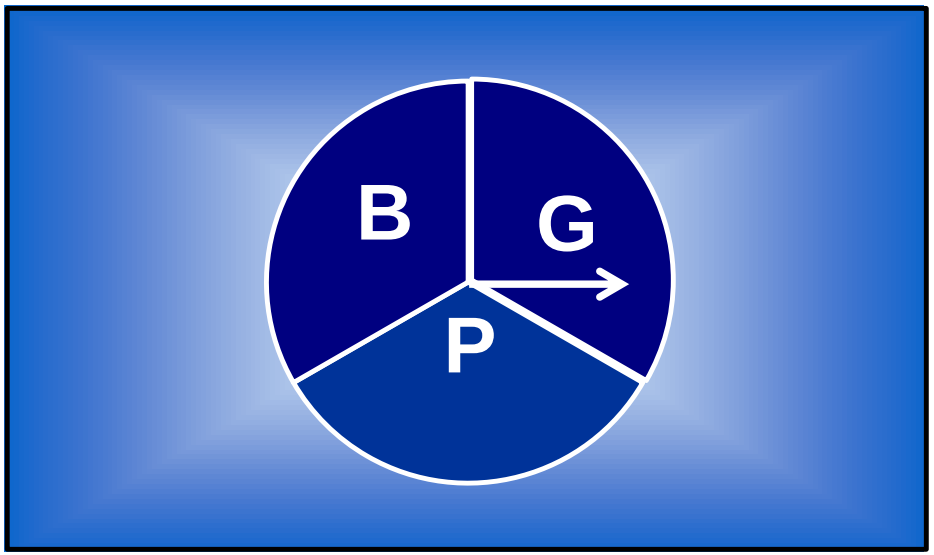
Sample Space

- Examples

a. Tossing a coin	b. Choosing a card from a deck of cards	c. Drawing a marble from a bag containing two red and three blue
S={Heads, Tails}		S={R, R, B, B, B}

This thing…

- An experiment consists of spinning the hand on the disk below twice. If it lands on a line, spin again. Find the sample space. Then determine the event E in which at least one B occurs.



Sample Space:

S = {BB, GB, BG, BP, PB, GG, GP, PG, PP}

Event E:

{BB}, {GB}, {BG}, {PB}, {BP}

Events: YOU TRY!

- Let S = {q, r, t} be a sample space of an experiment.

- List all of the events of this experiment.

  Ø, {q}, {r}, {t}, {q, r}, {q, t}, {r, t}, {q, r, t} or (S)

- ***Remember: When asked to write ALL events, include empty set and S.***

*Similar to #23 in your HW

Sample Space Example

- You have gone to the SPCA to adopt a puppy. You would like a poodle or cocker spaniel, that is brown or grey, and has either a red or orange collar. How many possible puppies fit your criteria? List the sample space.

S={PBR}, {PBO}, {PGR}, {PGO}, {CBR}, {CBO}, {CGR}, {CGO}

Events and Operations

- The union of events A & B is the event

$$A \cup B$$

- The intersection of events A & B is the event

$$A \cap B$$

- The complement of event A is the event

$$A^c$$

Review Example:
Rolling a die. S = {1,2,3,4,5,6}

Let A = rolling a number less than 4
B = rolling an odd number

Find: $A \cup B$

$A \cap B$

$A \cap B^c$

Review Example: ANSWERS

Rolling a die. S = {1,2,3,4,5,6}

Let A = rolling a number less than 4

B = rolling an odd number

Find: $A \cup B = \{1,2,3,5\}$

$A \cap B = \{1,3\}$

$A \cap B^c = \{2\}$

- Let P be any sample space and W, R, and S be any three events. Describe the given events using symbolic notation.

1. The event that S and W occur.

2. The event that R and S do not occur.

3. The event that W or R occur but not S.

4. Given events W and S, only one of the two occurs.

- Let P be any sample space and W, R, and S be any three events. Describe the given events using symbolic notation. ANSWERS

1. The event that S and W occur.

$$S \cap W$$

2. The event that R and S do not occur.

$$(R \cap S)^c \text{ or } R^c \cup S^c$$

3. The event that W or R occur but not S.

$$(W \cup R) \cap S^c$$

4. Given events W and S, only one of the two occurs.

$$(W \cap S^c) \cup (W^c \cap S)$$

Complementary Events

- **Complementary events** are two outcomes of an event that are the *only two possible outcomes*.

- Ex: Complementary:
  - Flipping a coin and getting heads or tails.

- Ex: Not Complementary:
  - Rolling a die and getting a 1 or 2

- All complementary events are mutually exclusive, but all mutually exclusive events are not necessarily complementary.

Events A & B are **mutually exclusive** if $A \cap B = \varnothing$

- **Mutually Exclusive Events (Disjoint Events)**: Two or more events that cannot occur at the same time.

Describe two events that are mutually exclusive.

Are these??

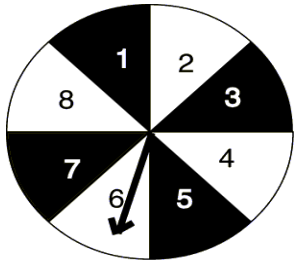
- Rolling an even # and rolling an odd # on a die

  Yes- Mutually Exclusive

- Drawing a single card from a deck of cards and having it be a diamond and a red card.

  No! They can occur at the same time so they are NOT Mutually Exclusive.

YOU TRY! Are the events mutually exclusive? Find the probability.



Spinner with numbers 1-8

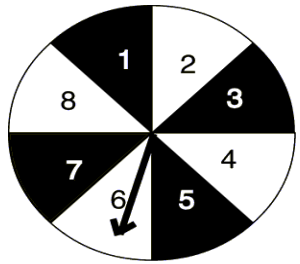
1) What is the probability of spinning a 4 and a 6 at the same time on a single spin.

2) Spinning an even number and a multiple of 3 at the same time on a single spin.

3) Spinning an even number and a prime number on a single spin.

4) Spinning an even number and a number less than 2 on a single spin.

YOU TRY! Are the events mutually exclusive? Find the probability. **ANSWERS**



Spinner with numbers 1-8

1) What is the probability of spinning a 4 and a 6 at the same time on a single spin. **Mutually Exclusive so the probability is 0.**

2) Spinning an even number and a multiple of 3 at the same time on a single spin.

NOT Mutually Exclusive (could be 6) so the probability is 1/8

3) Spinning an even number and a prime number on a single spin. **Not Mutually Exclusive (could be 2) so probability is 1/8 .**

4) Spinning an even number and a number less than 2 on a single spin.

Mutually Exclusive so the probability is 0.

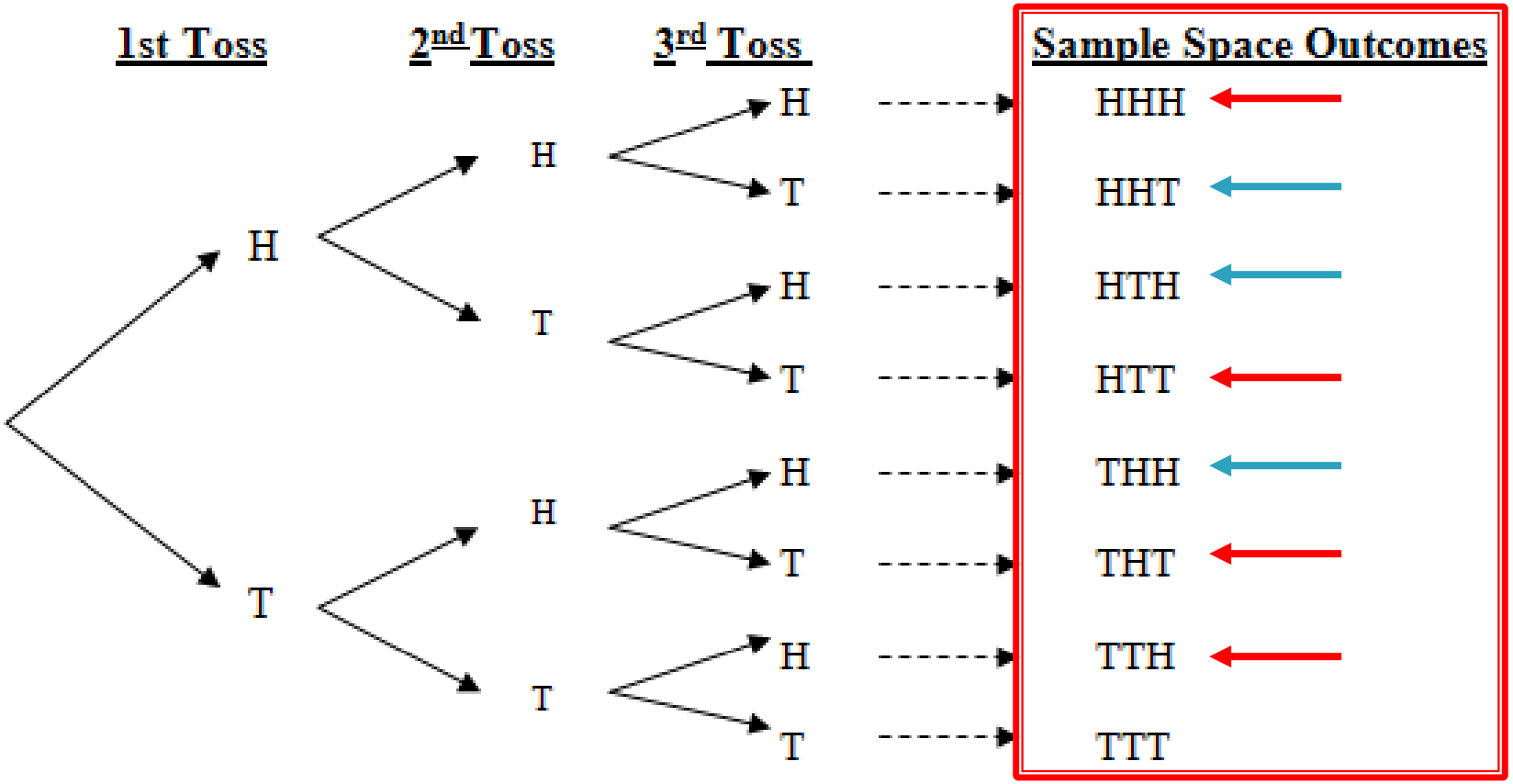
An experiment consists of tossing a coin 3 times and observing the resulting sequence of “heads” and “tails.”

- Find the sample space of the experiment.
  (Hint: you may need to draw a tree diagram ☺)

- Determine the event E that exactly two heads appear.

- Determine the event F that at least one head appears.

ANSWERS An experiment consists of tossing a coin 3 times and observing the sequence of "heads" and "tails."



- Determine the event E that exactly two heads appear.

{HHT}, {HTH}, {THH}

- Determine the event F that at least one head appears.

{HHH}, {HHT}, {HTH}, {HTT}, {THH}, {THT}, {TTH}

Remember, Events are Sets -> use Set notation!

- An experiment consists of casting a pair of dice and observing the number that falls uppermost on each die.
- Create the sample space S for this experiment. (Hint: Create a table or chart)

- Determine the events E_3 and E_7 that the sum of the numbers is 3 or 7, respectively.

Remember, Events are Sets -> use Set notation!

*Needed for #21 in your HW

- An experiment consists of casting a pair of dice and observing the number that falls uppermost on each die.
- Create the sample space S for this experiment. (Hint: Create a table or chart)

Die	1	2	3	4	5	6
1	(1, 1)	(1, 2)	(1, 3)	(1, 4)	(1, 5)	(1, 6)
2	(2, 1)	(2, 2)	(2, 3)	(2, 4)	(2, 5)	(2, 6)
3	(3, 1)	(3, 2)	(3, 3)	(3, 4)	(3, 5)	(3, 6)
4	(4, 1)	(4, 2)	(4, 3)	(4, 4)	(4, 5)	(4, 6)
5	(5, 1)	(5, 2)	(5, 3)	(5, 4)	(5, 5)	(5, 6)
6	(6, 1)	(6, 2)	(6, 3)	(6, 4)	(6, 5)	(6, 6)

- Determine the events E_3 and E_7 that the sum of the numbers is 3 or 7, respectively.

{1, 2}, {2, 1}, {3, 4}, {4, 3}, {2, 5}, {5, 2}, {6, 1}, {1, 6}

Remember, Events are Sets -> use Set notation!

*Needed for #21 in your HW

PRACTICE

- An experiment consists of casting a pair of dice and observing the number that falls uppermost on each die.

Die	1	2	3	4	5	6
1	(1, 1)	(1, 2)	(1, 3)	(1, 4)	(1, 5)	(1, 6)
2	(2, 1)	(2, 2)	(2, 3)	(2, 4)	(2, 5)	(2, 6)
3	(3, 1)	(3, 2)	(3, 3)	(3, 4)	(3, 5)	(3, 6)
4	(4, 1)	(4, 2)	(4, 3)	(4, 4)	(4, 5)	(4, 6)
5	(5, 1)	(5, 2)	(5, 3)	(5, 4)	(5, 5)	(5, 6)
6	(6, 1)	(6, 2)	(6, 3)	(6, 4)	(6, 5)	(6, 6)

a) What are the events of rolling a product less than 6?

b) What are the events of rolling an odd number on the first die and a 4 on the second die?

PRACTICE ANSWERS

- An experiment consists of casting a pair of dice and observing the number that falls uppermost on each die.

Die	1	2	3	4	5	6
1	(1, 1)	(1, 2)	(1, 3)	(1, 4)	(1, 5)	(1, 6)
2	(2, 1)	(2, 2)	(2, 3)	(2, 4)	(2, 5)	(2, 6)
3	(3, 1)	(3, 2)	(3, 3)	(3, 4)	(3, 5)	(3, 6)
4	(4, 1)	(4, 2)	(4, 3)	(4, 4)	(4, 5)	(4, 6)
5	(5, 1)	(5, 2)	(5, 3)	(5, 4)	(5, 5)	(5, 6)
6	(6, 1)	(6, 2)	(6, 3)	(6, 4)	(6, 5)	(6, 6)

a) What are the events of rolling a product less than 6?

{1, 1}, {2, 1}, {3, 1}, {4, 1}, {5, 1}, {1, 2}, {2, 2}, {1, 3}, {1, 4}, {1, 5}

b) What are the events of rolling an odd number on the first die and a 4 on the second die?

{1, 4}, {3, 4}, {5, 4}

In Groups of Four…

- Think of an experiment. Make it ***interesting***. Don't use anything we've discussed.
- Describe the sample space of the experiment.
- Construct two events, E and F, of the experiment.
- Find the union and intersection of E and F and the complement of E.
- Are E and F mutually exclusive? Explain.
- We'll share these with the rest of the class. (FUN!)

Homework Day 4

Tonight's HW
= Quiz Review Sheet
& Study for tomorrow's Quiz

Study your notations, Formulas, etc!! ☺

Check Review Sheet answers online tonight!