Key with Work

Basic Differentiation Rules
Day 3

Warm Up

- Rewrite using rational exponents.

1. $\dfrac{-2}{x}$

2. $\sqrt{x-5}$

3. $\sqrt[3]{8+4x}$

4. $\dfrac{5}{\sqrt[4]{x^3}}$

- 5. Use the limit definition to find the derivative of: $g(x)=\sqrt{3x}$

Warm Up

- Rewrite using rational exponents.

1. $\frac{-2}{x} = -2x^{-1}$

2. $\sqrt{x-5} = (x-5)^{1/2}$

3. $\sqrt[3]{8+4x} = (8+4x)^{1/3}$

4. $\frac{5}{\sqrt[4]{x^3}} = \frac{5}{x^{3/4}} = 5x^{-3/4}$

- 5. Use the limit definition to find the derivative of: $g(x)=\sqrt{3x}$

$\lim_{h\to 0} \frac{\sqrt{3(x+h)}-\sqrt{3x}}{h}$

$= \lim_{h\to 0} \frac{(\sqrt{3x+3h}-\sqrt{3x})(\sqrt{3x+3h}+\sqrt{3x})}{h(\sqrt{3x+3h}+\sqrt{3x})} = \lim_{h\to 0} \frac{\cancel{3x}+3\cancel{h}-\cancel{3x}}{\cancel{h}(\sqrt{3x+3h}+\sqrt{3x})}$

$\frac{3}{2\sqrt{3x}}$

Warm Up ANSWERS

- Rewrite using rational exponents.

1. $\frac{-2}{x}$ $-2x^{-1}$

2. $\sqrt{x-5}$ $(x-5)^{\frac{1}{2}}$

3. $\sqrt[3]{8+4x}$ $(8+4x)^{\frac{1}{3}}$

4. $\frac{5}{\sqrt[4]{x^3}}$ $5x^{-\frac{3}{4}}$

- 5. Use the limit definition to find the derivative of: $g(x)=\sqrt{3x}$

$\frac{3}{2\sqrt{3x}}$

Warm up #5

$$\lim_{h\to 0} \frac{g(x+h)-g(x)}{h} \qquad g(x)=\sqrt{3x}$$

$$\lim_{h\to 0} \frac{\sqrt{3(x+h)}-\sqrt{3x}}{h}$$

conjugate

$$\lim_{h\to 0} \frac{\left(\sqrt{3x+3h}-\sqrt{3x}\right)}{h}\cdot\frac{\left(\sqrt{3x+3h}+\sqrt{3x}\right)}{\left(\sqrt{3x+3h}+\sqrt{3x}\right)}$$

$$\left(\cancel{\sqrt{3x+3h}}\right)^{\cancel{2}} + \cancel{\sqrt{3x+3h}\cdot\sqrt{3x}} - \left(\cancel{\sqrt{3x}}\right)^{\cancel{2}}$$

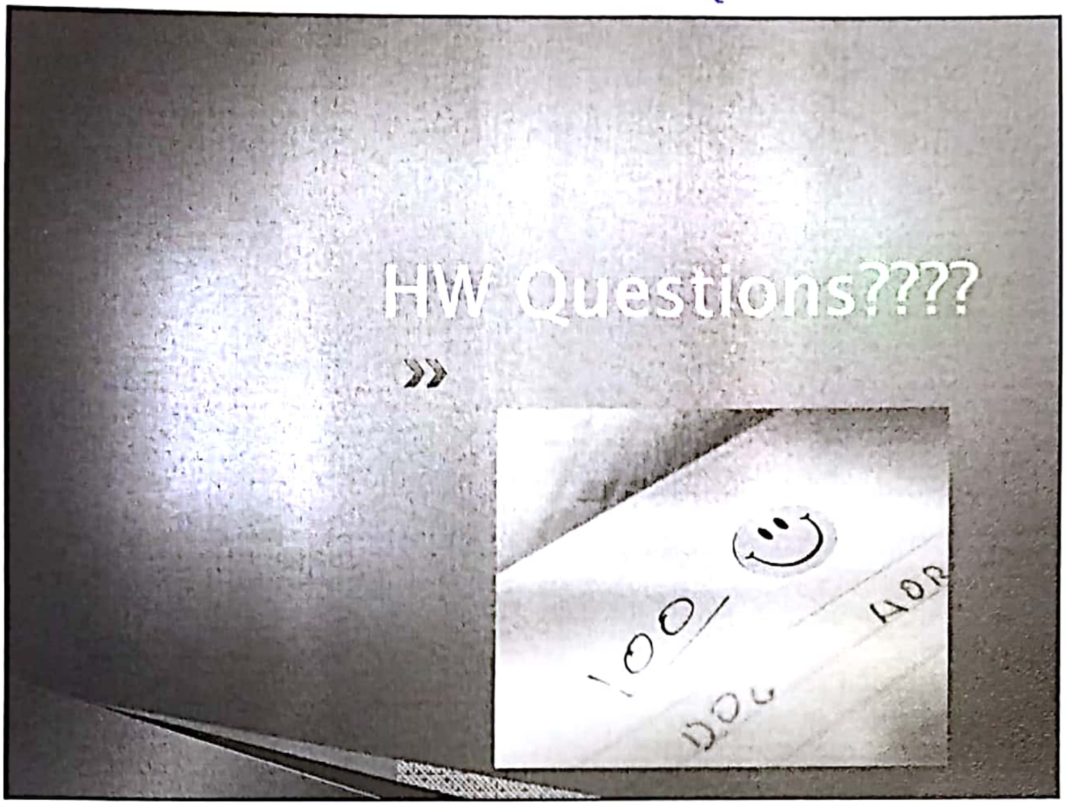
$$\lim_{h\to 0} \frac{(3x+3h) - \cancel{\sqrt{3x+3h}\cdot\sqrt{3x}} - 3x}{h\left(\sqrt{3x+3h}+\sqrt{3x}\right)}$$

$$\lim_{h\to 0} \frac{\cancel{3x}+3\cancel{h}-\cancel{3x}}{\cancel{h}\left(\sqrt{3x+3h}+\sqrt{3x}\right)} = \lim_{h\to 0}\frac{3}{\sqrt{3x+3h}+\sqrt{3x}}$$

$$g'(x)=\boxed{\frac{3}{2\sqrt{3x}}}$$

Notes Day 3: Power Rule and Tangent Equations

11/1/2017



Intro– Do you see the pattern?

$f(x)=7$ $\qquad f'(x)=0$

$g(x)=x^3$ $\qquad g'(x)=3x^2$

$h(x)=2x^4$ $\qquad h'(x)=8x^3$

$f(x)=-3x^5-2x^3$ $\qquad f'(x)=-15x^4-6x^2$

The Constant Rule

- The derivative of a constant function is 0. That is, if c is a real number, then

$$\frac{d}{dx}[c]=0$$

$$Ex: f(x)=-4$$

$$\boxed{f'(x)=0}$$

- What type of lines do constants make?

  Constants are horizontal lines – and their slope is zero. Remember, derivatives come from slope. ☺

The Power Rule

- If n is a rational number, then the function $f(x) = x^n$ is differentiable and

$$\frac{d}{dx}\left[x^n\right]=nx^{n-1}$$

$$Ex: \ f(x)=5x^3$$

$$\boxed{\frac{d}{dx}f(x)=15x^2}$$

Differentiable means you can take the derivative!

Examples

Write answers with only **positive** exponents!

- Find each derivative using the power rule.

1. $f(x) = -4x^2$ $\boxed{-8x}$

2. $f(x) = 3x^6 + 7$ $\boxed{18x^5}$

3. $f(x) = \frac{2}{x^2} = 2x^{-2}$
$-4x^{-3} = \boxed{\frac{-4}{x^3}}$

4. $f(x) = -\frac{7}{x^3} = -7x^{-3}$
$21x^{-4} = \boxed{\frac{21}{x^4}}$

5. $g(x) = 3\sqrt{x}$
$= 3x^{1/2} = \frac{3}{2}x^{-1/2}$
$= \boxed{\frac{3}{2\sqrt{x}}}$

6. $f(x) = 2\sqrt[3]{x^4} = 2x^{4/3}$
$\frac{8}{3}x^{1/3} = \boxed{\frac{8\sqrt[3]{x}}{3}}$ or $\boxed{\frac{8x^{1/3}}{3}}$

Examples ANSWERS

Write answers with only **positive** exponents!

- Find each derivative using the power rule.

1. $f(x) = -4x^2 = -8x$

2. $f(x) = 3x^6 + 7$
$= 18x^5$

3. $f(x) = \frac{2}{x^2} = \frac{-4}{x^3}$

4. $f(x) = -\frac{7}{x^3} = \frac{21}{x^4}$

5. $g(x) = 3\sqrt{x}$
$= \frac{3}{2\sqrt{x}}$

6. $f(x) = 2\sqrt[3]{x^4}$
$= \frac{8x^{\frac{1}{3}}}{3}$

The Sum and Difference Rules

- The sum (and difference) of two differentiable functions is differentiable
  and is the sum (or difference) of their derivatives.

$$\frac{d}{dx}[f(x)+g(x)] = f'(x)+g'(x)$$

$$\frac{d}{dx}[f(x)-g(x)] = f'(x)-g'(x)$$

Find each derivative:

Write answers with only **positive** exponents!

7. $f(x) = 3x^4 - 2x^3$

$\boxed{12x^3 - 6x^2}$

8. $f(x) = x^3 - 4x + 5$

$\boxed{3x^2 - 4}$

9. $f(x) = -\frac{x^4}{2} + 3x^3 - 2x$

$-\frac{4x^3}{2} + 9x^2 - 2$

$\boxed{-2x^3 + 9x^2 - 2}$

10. $f(x) = x^4 - 2x^3 + 4\sqrt{x}$

$4x^3 - 6x^2 + \frac{4}{2}x^{-1/2}$

$\boxed{4x^3 - 6x^2 + \frac{2}{\sqrt{x}}}$

Find each derivative ANSWERS:

Write answers with only **positive** exponents!

7. $f(x) = 3x^4 - 2x^3$

$= 12x^3 - 6x^2$

8. $f(x) = x^3 - 4x + 5$

$= 3x^2 - 4$

9. $f(x) = -\dfrac{x^4}{2} + 3x^3 - 2x$

$= -2x^3 + 9x^2 - 2$

10. $f(x) = x^4 - 2x^3 + 4\sqrt{x}$

$= 4x^3 - 6x^2 + \dfrac{2}{\sqrt{x}}$

What does the derivative tell us?

- The equation for the slope of the line tangent to the curve. ← Write this down!
- Is the slope of the line the same as we go across a curve? No!
- We can substitute in different x-values for our derivative equation to find the slope at specific points.

Know the difference!

- Question 1:

  Slope of a graph at a ***specific point, c.***
  ->Find the derivative (**difference quotient**)
  by substituting in c for x and simplify.

- Question 2:

  Finding a ***formula*** for the slope at ***any* point** on the graph is derivative

$$y - y_1 = \frac{dy}{dx}(x - x_1)$$

Find the slope at a point

- Find the slope of the graph of $f(x)=x^4$ when:

a. x=-1	b. x=0	c. x=1
f'(x)= $\boxed{4x^3}$	f'(x)= $\boxed{4x^3}$	f'(x)= $\boxed{4x^3}$
a. f'(-1)= $4(-1)^3$ $\boxed{-4}$	b. f'(0)= $4(0)^3$ $\boxed{0}$	c. f'(1)= $4(1)^3$ $\boxed{4}$

Remember a derivative is slope!

Find the slope at a point ANSWERS

- Find the slope of the graph of $f(x)=x^4$ when:

a. $x=-1$	b. $x=0$	c. $x=1$
$f'(x)= 4x^3$	$f'(x)= 4x^3$	$f'(x)= 4x^3$
a. $f'(-1)= -4$	b. $f'(0)= 0$	c. $f'(1)= 4$

Remember a derivative is slope!

Writing Equations of Tangent Lines

Given $f(x) = 3x^2 + 5x$. Write the equation of the tangent line at $x = 2$.

- We could say $f'(x) = 6x + 5$.
- We could say $f'(2) = 17$.

$6(2)+5$

- The derivative of $f(x)$ is $6x + 5$. The slope of the tangent line at $x = 2$ is 17.

m

- Can you find the equation of the tangent line??

*Substitute the x-value into the original equation to find y!!

$$f(2) = 3(2)^2 + 5(2)$$
$$12 + 10$$
$$22$$
$$(2, 22)$$
x_1 y_1

$$y - 22 = 17(x - 2)$$
y_1 m x_1

$$y = 17x - 12$$

$$\boxed{y = 17x - 12}$$

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Find the equation of the tangent line to the graph of $f(x) = -2x^2 + 9x + 1$ at $x = 3$.

First, find the derivative of the function.

$$f(x) = -2x^2 + 9x + 1$$

$$\boxed{f'(x) = -4x + 9}$$

Next, find the slope of the tangent line at $x = 3$.

$$f'(3) = -4(3) + 9 \qquad \boxed{m = -3}$$

$$-12 + 9$$

Finally, find the equation of the tangent line at the point (3, 10). $y - 10 = -3(x - 3)$

$f(3) = -2(3)^2 + 9(3) + 1$

$-18 + 27 + 1$ $\boxed{y = -3x + 19}$

10

How to find equation of tangent line when not given a point:

1. Take the derivative of the function
2. Substitute the given x-value into the derivative to find the slope
3. Substitute the given x-value into the ORIGINAL function to find the y-value of the point
4. Use point-slope formula with the slope and point that you found!

You Try: Equation of a Tangent Line

Ex. Find an <u>equation</u> of the tangent line to the graph of f(x) = x^2 when x = −2.

$f'(x) = 2x$

$f'(-2) = 2(-2) = -4 = m$

$f(-2) = (-2)^2$
$= 4$
$(-2, 4)$

$\boxed{y - 4 = -4(x+2)}$

Remember to write an equation of a line we need slope and a point!

You Try: Equation of a Tangent Line ANSWERS

Ex. Find an <u>equation</u> of the tangent line to the graph of f(x) = x^2 when x = −2.

- f'(x)= $2x$

  $m = -4$

*Substitute the x value into the original equation to find y!!

Write this down! $f(-2) = 4$

$\boxed{y - 4 = -4(x+2)}$

Remember to write an equation of a line we need slope and a point!