ICM Unit 1 Day 3

Counting Principle, Combinations, & Permutations

Warm Up Day 3 ~ Unit 1

A survey of students at the University of Florida's film school revealed the following :

51 admire Moe
49 admire Larry
60 admire Curly
34 admire Moe and Larry
8 admire just Larry and Curly
36 admire Moe and Curly
24 admire all three of the Stooges
1 admires none of the Three Stooges

Remember to take out:
*Packet (opened to HW page)
*Calculator

A) How many people were surveyed?
B) How many admire Curly, but not Larry nor Moe?
C) How many admire Larry or Curly?
D) How many admire exactly one of the Stooges?
E) How many admire exactly 2 of the Stooges?

RIDDLE: If 1+9+8=1, what is 2+8+9?

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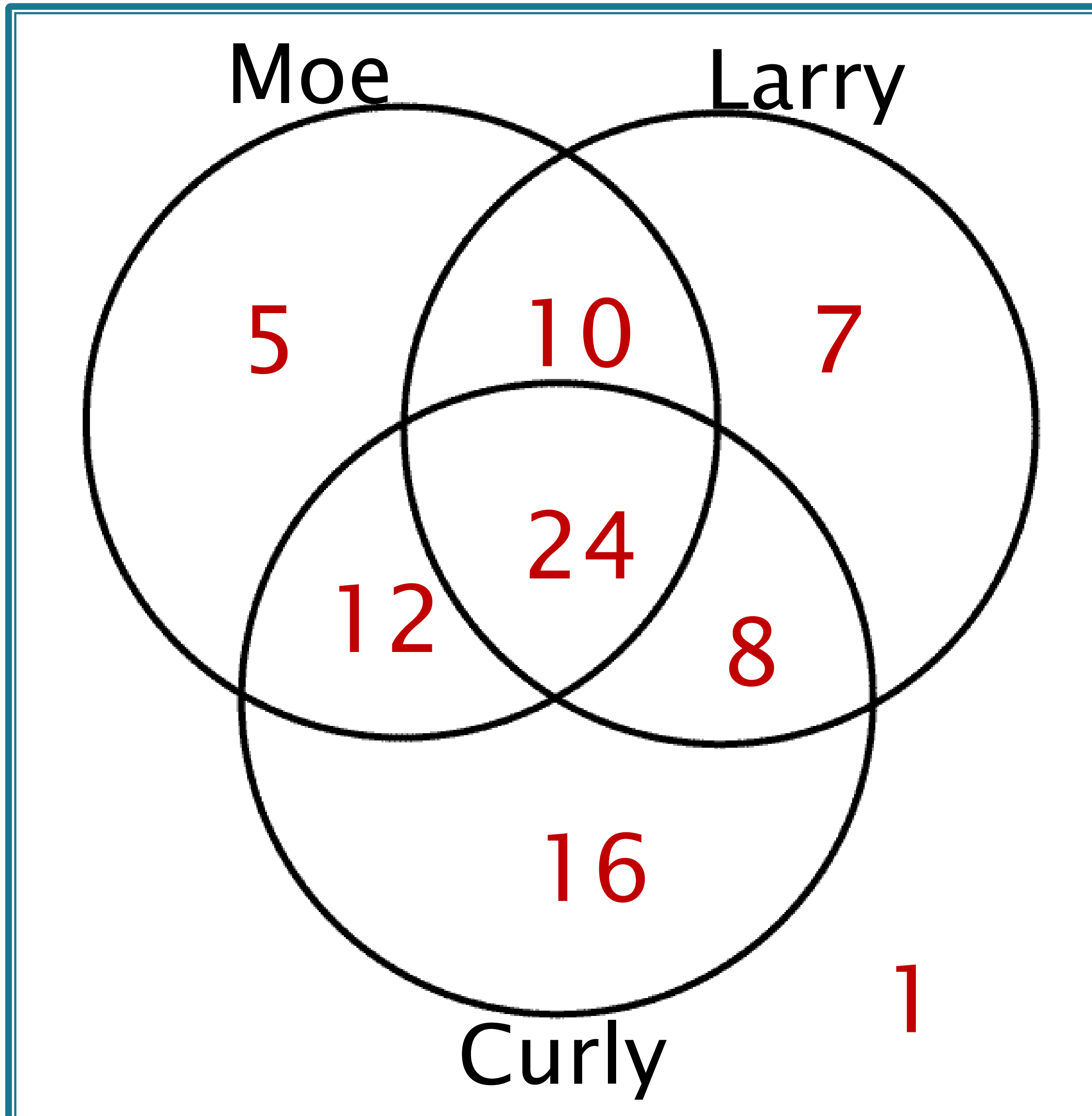
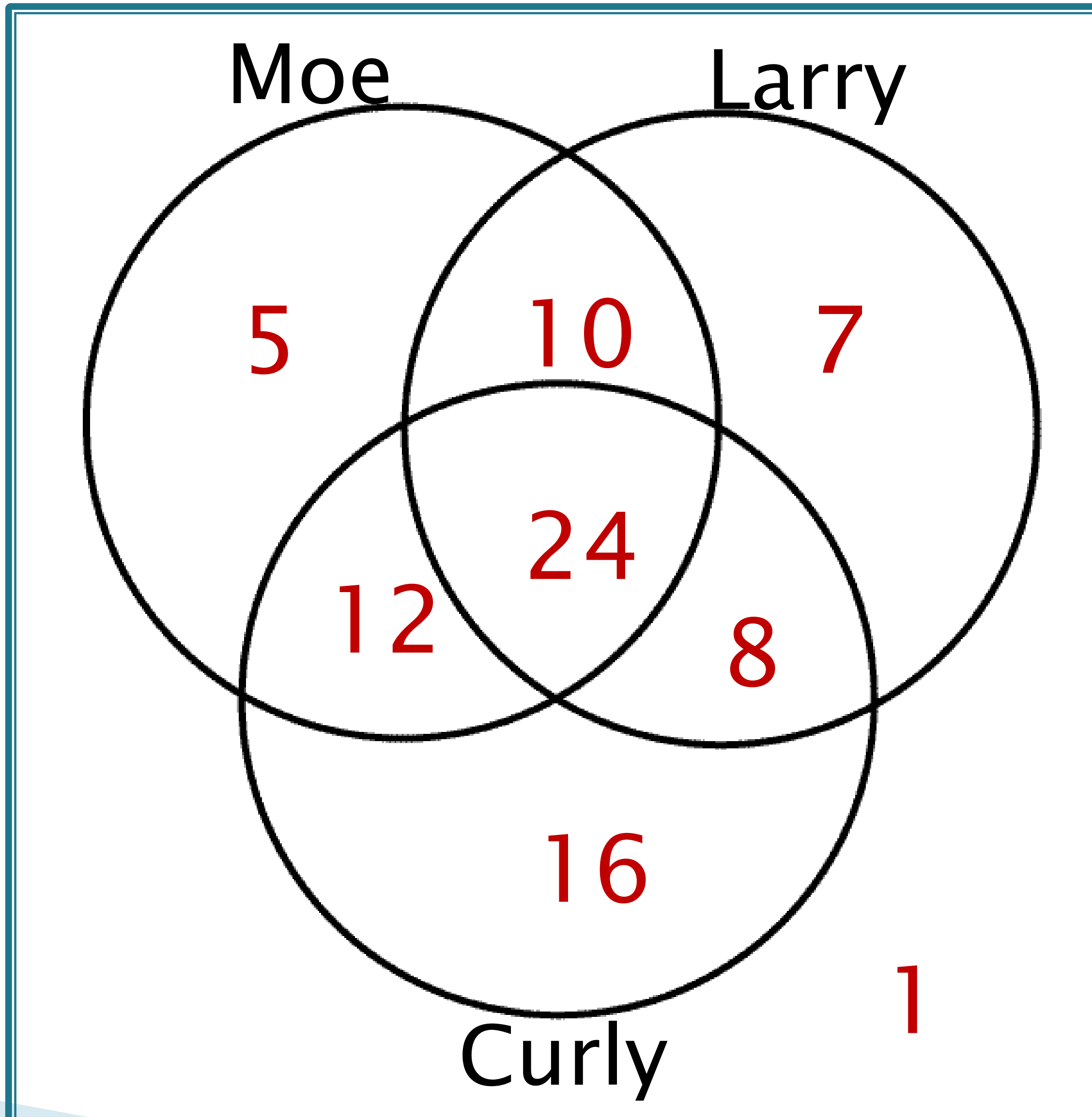


Warm Up ANSWERS:

A) How many people were surveyed? 83
B) How many admire Curly, but not Larry nor Moe? 16
C) How many admire Larry or Curly? 77
D) How many admire exactly one of the Stooges? 28
E) How many admire exactly 2 of the Stooges? 30



Let’s try a riddle!

- If 1+9+8=1, what is 2+8+9?

10, take the first letter of each spelled out number:

One+Nine+Eight= ONE
Two+Eight+Nine= TEN

Homework Questions?

Tonight's HW Day 3
p. 5-6

Be sure to show work for credit!

Ex: Write the formula you used, ${}_9P_5$ to show that you used permutation.

Reminders:

*Tutorials are Mon. and Wed. 1st half of lunch
& Most mornings ~7 AM & Thurs after school

*Quiz on Thursday
You'll need your graphing calculator!

Check-In:

Let’s see how we’re doing on Venn Diagrams and our Notation so far…

(Day 2 Venn Practice Half Sheet)

Counting Principle, Permutations and Combinations

Sections 6.3-6.4

Introduction to Probability

Probability Defined:

- General: Probability is the likelihood of something happening
- Mathematical expression:

$$\text{Probability} = \frac{\text{Number of desired outcomes}}{\text{Number of total outcomes}}$$

Today, we'll focus on counting techniques to help determine this total #!

Introduction to Probability: Counting methods

I. Fundamental Counting Principle
II. Permutations
III. Combinations

- Let's see what we remember about them….

With your group of 4…

<u>Identify which method is best AND explain why. Then Solve.</u>

A. Fundamental Counting Principle
B. Permutation OR
C. Combination

1. If you saw 10 movies in the last year, in how many ways can the three best be chosen and ranked?
2. There are 10 standbys who hope to get a seat on a flight, but only three seats are available on the plane. How many different ways can the 3 people be selected?
3. The model of the SUV you are considering buying comes in 10 different colors and three different styles (standard, limited, and luxury). In how many ways can you order the SUV?

Notes: Basic Counting Methods for Determining the Number of Possible Outcomes

Ex #1: LG will make 5 different cell phones: Ally, Extravert, Intuition, Cosmos and Optimus. Each phone comes in two different colors: Black or Red. Make a tree diagram representing the different products.

How many different products can the company display?

I. In general:

Fundamental Counting Principle.

- If there are ***m*** ways to make a first selection and ***n*** ways to make a second selection, then there are ***m times n*** ways to make the two selections simultaneously.

- Ex #1 above: 5 different cell phones in 2 different colors. How many different products?

$$5 \cdot 2 = 10$$

You Try! Practice

Ex #2: Elizabeth is going to completely refurbish her car. She can choose from 4 exterior colors: white, red, blue and black. She can choose from two interior colors: black and tan. She can choose from two sets of rims: chrome and alloy. How many different ways can Elizabeth remake her car? Make a tree diagram and use the Counting Principle.

Ex #3: License plates in NC consist of 3 letters and 4 numbers. How many different license plates can exist?

Ex # 4: Now how many different license plates can exist if they must have 3 unique letters and 4 unique numbers?

You Try! Practice ANSWERS

Ex #2: Elizabeth is going to completely refurbish her car. She can choose from 4 exterior colors: white, red, blue and black. She can choose from two interior colors: black and tan. She can choose from two sets of rims: chrome and alloy. How many different ways can Elizabeth remake her car? Make a tree diagram and use the Counting Principle.

$$4 \cdot 2 \cdot 2 = 16$$

You Try! Practice ANSWERS

Ex #3:

- License plates in NC consist of 3 letters and 4 numbers. How many different license plates can exist?

A	A	A	0	0	0	0
B	B	B	1	1	1	1
C	C	C	2	2	2	2
⋮	⋮	⋮	⋮	⋮	⋮	⋮
Z	Z	Z	9	9	9	9

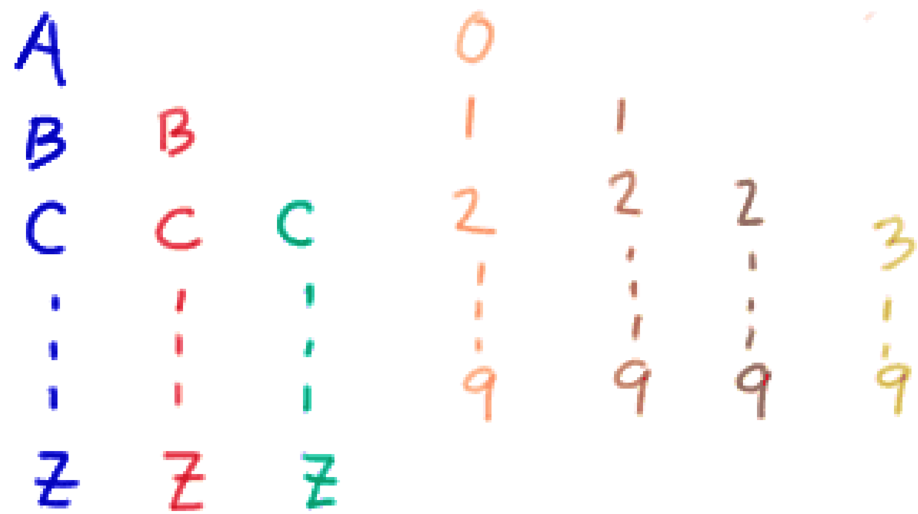
$26 \times 26 \times 26 \times 10 \times 10 \times 10 \times 10$

$= 175{,}760{,}000$

You Try! Practice ANSWERS

EX #4:

- Now how many different license plates can exist if they must have 3 unique letters and 4 unique numbers?



$$26 \times 25 \times 24 \times 10 \times 9 \times 8 \times 7$$

$$= 78,624,000$$

II. Permutations
Another way to "count" possibilities

Two characteristics:

1. Order IS important

2. No item is used more than once

Example #1

There are six “permutations”, or arrangements, of the numbers 1, 2 and 3.

What are they?

123	132
213	231
312	321

3 options and use all 3 = 6 arrangements

Example #2

How many ways can 10 cars park in 6 spaces? The other four will have to wait for a parking spot. ☺

(Use the Fundamental Counting Principle)

$$10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 = 151200$$

Is there a Formula?

If we have a large number of items to choose from, the fundamental counting principle would be inefficient. Therefore, a formula would be useful.

First: A reminder about **“factorials”.**

Notation: **n!** stands for **n factorial**

Definition of n factorial:

For any integer n>0,

$$n! = n(n-1)(n-2)(n-3)\ldots(3)(2)(1)$$

Example:

$$\mathbf{4! = 4 \cdot 3 \cdot 2 \cdot 1}$$

If n = 0, 0! = 1

Generally, the ***<u>Number of Permutations</u>*** of *n* items taken *r* at a time,

$$ {}_nP_r = \frac{n!}{(n-r)!} $$

How to do on the calculator:

n MATH PRB nPr r

Car example: n = 10 cars, r = 6 spaces

Example #2 (revisited):

$$_{n}P_{r} = \frac{n!}{(n-r)!}$$

Why does that formula work?

Looking again at the car example:

$$10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 = \frac{10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot \cancel{4 \cdot 3 \cdot 2 \cdot 1}}{\cancel{4 \cdot 3 \cdot 2 \cdot 1}}$$

$$= \frac{10!}{4!} = \frac{10!}{(10-6)!} = 151200$$

Why is there a 4 in the formula if we have 10 cars and 6 spaces?

EXAMPLE #3

In a scrabble game, Jane picked the letters A,D,F,V, E and I. How many permutations (or arrangements) of 4 letters are possible?

$$_6P_4 = 360$$

Order matters since arrangement of letters create different words!

Practice Problems

$${}_{n}P_{r} = \frac{n!}{(n-r)!}$$

1. Evaluate by hand: a. ${}_{10}P_{3}$ b. ${}_{9}P_{5}$

720 15120

2. How many ways can runners in the 100 meter dash finish 1st (Gold Medal), 2nd (Silver) and 3rd (Bronze Medal) from 8 runners in the final? NOTE: This is a permutation because the people are finishing in a position. ORDER matters!

336

III. Combinations

Two characteristics:

1. Order DOES NOT matter
2. No item is used more than once

Example: How many "combinations" of the numbers 1, 2 and 3 are possible?

There is just 1 combination of 1, 2, 3 because order doesn't matter so 123 is considered the same as 321, 213, etc.

Formula:

Number of Combinations of *n* items taken *r* items at a time is

$$ {}_nC_r = \frac{n!}{(n-r)! \bullet r!} $$

How to do on the calculator:

n MATH PRB nCr r

If a DJ for the prom has 6 hit songs to play and has time for 4 of them, in how many ways can he choose the songs to play?

$_6C_4 = 15$

Practice:

$$ {}_{n}C_{r} = \frac{n!}{(n-r)! \bullet r!} $$

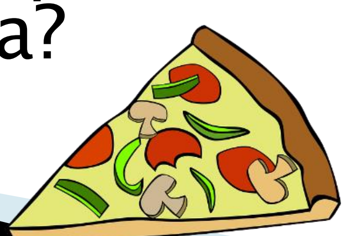
1. Evaluate by hand:

a. ${}_{4}C_{2}$ b. ${}_{7}C_{3}$

6 35

2. During the snow storm you ordered pizza. Papa John's is running a special: any large pizza with 3 veggie toppings for $5 since the Panthers won! Papa John's has 12 veggie toppings to choose from.

- How many options are there for your 3 topping veggie pizza?



$${}_{12}C_{3} = 220$$

Combining multiple possibility outcomes

- Recall “or” events mean addition because all of these could be possibilities:
  option1 + option2 + option3

- However “and” events means multiply because all of these will occur together:
  option1 * option2 * option3

Exampl Problems

$${}_nC_r = \frac{n!}{(n-r)!\cdot r!}$$

3. A local restaurant is offering a 3 item lunch special. If you can **choose 3 or fewer items** from a total of 7 choices, how many possible combinations can you select?

$${}_7C_3 + {}_7C_2 + {}_7C_1 + {}_7C_0 = 64$$

4. A hockey team consists of ten offensive players, seven defensive players, and three goaltenders. In how many ways can the coach select a starting line up of three offensive players, two defensive players, and one goaltender?

$${}_{10}C_3 \bullet {}_7C_2 \bullet {}_3C_1 = 7560$$

You Try! The members of a string quartet composed of 2 violinists, a violist, and a cellist are to be selected from a group of 6 violinists, 3 violists, and 2 cellists, respectively.

a) In how many ways could the string quartet be formed?

$$_6C_2 \text{ x } _3C_1 \text{ x } _2C_1 = 90$$

b) In how many ways can the string quartet be formed if one of the violinists is to be designated as 1st violinists and the other is to be designated as 2nd violinists?

$$_6P_2 \text{ x } _3C_1 \text{ x } _2C_1 = 180$$

Permutations with Repetitions

In how many ways can the letters of “SEA” be arranged?

6

In how many ways can the letters of “SEE” be arranged?

3

What’s the difference? How do you account for it?

Permutations of n objects, not all distinct

- The number of n objects of which p are alike and q are alike is:

$$\frac{n!}{p!q!}$$

- **n** is total number of objects
- **p** is the amount of one alike object
  …multiple e's or 7's, etc
- **q** is the amount of another alike object

- How many different 11-letter patterns can be formed with letters of MISSISSIPPI?

$$\frac{11!}{4!4!2!} = 34,650$$

- How many different 9-letter patterns can be formed with letters of TENNESSEE?

$$\frac{9!}{4!2!2!} = 3780$$

- How many different 7-letter patterns can be formed with the letters of WYOMING?

$$7! = 5040$$

Why is this third example different?

The letters of Wyoming are all distinct. ☺

Tonight's HW Day 3 p. 5–6

Be sure to show work for credit!

Ex: Write the formula you used, like ${}_9P_5$ to show that you used permutation.

Extra Practice on Next Slides…

How many outcomes are possible for a game that consists of rolling a die followed by flipping a fair coin?

How many possibilities when you roll a die?

How many possibilities when you flip a coin?

Total possibilities:

6x2 = 12